

## Chaotic Properties of the Philippine Stock Exchange Index Returns

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### Abstract

This study found evidences of chaos in the Philippine Stock Exchange Index (PSEi) returns using three tests of chaotic behavior. The BDS test found that PSEi returns and ARMA residuals are not stochastic processes. With regards to the efficient market hypothesis (EMH), this paper cannot conclude the *iid* properties of GARCH residuals, except for the period of the Subprime Mortgage Crisis. The rescaled range (R/S) analysis showed that most Hurst exponents of the PSEi returns, ARMA and GARCH residuals have chaotic tendencies and have trend-reinforcing series, but showed anti-persistence during crisis. Furthermore, the correlation dimension analyses supplemented the first two initial tests and showed that the correlation dimension does not converge to a stable value as the embedding dimension increases. Both values increased in relation with each other, further confirming that PSEi returns, ARMA and GARCH residuals are consistent with chaos.

**Keywords:** Stock returns, Philippine Stock Exchange Index, Chaos process

### I. Introduction

The huge potential of the Philippine economy has been established with its inclusion as one of the world's top 16 economies by 2050. According to the global research department of Hong Kong and Shanghai Banking Corporation (HSBC) of January 2012, the Philippines will leapfrog by 27 notches from being the 41<sup>st</sup> ranking last 2010 to being the 16<sup>th</sup> largest economy by 2050. Based on the bank's report in the beginning of 2012, the Philippine economy is currently worth at \$122 billion, but is expected to expand to \$1.69 trillion by 2050. The projected gross domestic product (GDP) is based on current income per capita, rule of law, democracy, education levels and demographic change.

One of the economic indicators of the country is the Philippine Stock Exchange (PSE). The PSE is the national stock exchange of the country and is one of the oldest in Southeast Asia with its inception on August 8, 1927. The PSE was created from combining the Manila Stock Exchange and the Makati Stock Exchange. The PSE Composite Index (PSEi) is the main index of the bourse composed of thirty listed companies and additional six sector-based indices. The PSE, as of December 2010 had a total market capitalization of US\$202 billion with 253 listed companies and 184 listed trading participants. The PSEi is the most monitored index in the country and it is also one of the most watched economic indicators. The promising situation of the Philippines certainly gave a strong boost and confidence on the economic position of the country globally.

The efficient market hypothesis (EMH) of Fama (1970) has become a viable explanation on the difficulty or even impossibility of predicting stock prices because they behave independent and identically distributed (*iid*) or act on a random manner. This paper attempts to test the initial findings of Aquino (2006) that the Philippine stock market supports a weak-form efficient hypothesis using the autoregressive (AR) process. This study in particular wants to explore the possibility of finding nonlinear properties through detecting chaotic tendencies of the PSEi. According to Panas (2001), the importance of examining econometric time-series in a nonlinear framework is that nonlinearities communicate information about the inherent structure of the data series, which normally offers insight into the nature of the process that dominates the structure. Through these methods, it would be easy to distinguish between the stochastic and chaotic properties of the time-series, which is very difficult to determine using linear models. Recent researches have shown the presence of nonlinearity in economics and finance and in investment instruments in particular (e.g., Antoniou and Vorlow, 2005; Das and Das, 2007; Korkmaz et al., 2009; and Mariani et al., 2009), but none of these have carefully studied the nonlinear tendencies of the PSEi, specifically, its chaotic properties.

In earlier studies, Eldridge et al. (1993) and Opong et al. (1999) find similar findings of non-stochastic processes in the S&P 500 cash index and returns of FTSE index, respectively. This can be a good challenge to the idea that chaos effect assumes part of the underlying process is nonlinear. Hsieh (1991) defines chaos as a nonlinear time-series that appears to be random in nature, but also has the tendency to be a nonlinear deterministic system or a nonlinear stochastic system. The author argues that the chaotic process can be misinterpreted as a random process by conventional linear econometric method. This is the reason why appropriate modeling is necessary to evaluate the determinism of a financial time-series. Hsieh's (1991) seminal work provided an initial argument in the existence of chaos in financial markets. Chaotic tendencies of variables have been shown to be present in futures, foreign exchange and stock markets. For example, Blank (1991) and Kyrtsov et al. (2004) found nonlinear dynamics in futures prices, and also showed that short-term forecasting models may be improved by chaotic factors. Serletis and Gogas (1999), Panas and Ninni (2000) and Moshiri and Faezeh (2006) reported that the price sequence of natural gas and oil markets contain non-linear dynamics and that a chaos model can best capture these processes. Researches of Jin (2005), Das and Das (2007) and Yudin (2008) revealed that foreign exchange markets can also be characterized by chaotic nonlinear properties. Furthermore, Panas (2001) applied chaos models to London metal prices, and found that tin metal prices can be also modeled by the chaos process. In a related study regarding stock markets, Kyrtsov and Terraza (2002) found that the French CAC40 returns can be either modeled by a noisy chaotic or a pure stochastic process. In a later study, Ozer and Ertokatli (2010) found

that the Istanbul Stock Exchange (ISE) Index returns have chaotic properties.

The research applies three different approaches in testing chaotic tendencies of the PSEi returns, namely, Brock, Dechert, and Scheinkman (BDS) test, Rescaled Range (R/S) analysis and Correlation Dimension (CD) analysis. According to Frank and Stengos (1989), who are one of the pioneers in finding nonlinear structures in econometric data, financial time-series exhibits irregular behavior wherein a process response is not relative to the stimulus provided. Linear models were already proven to offer simple solutions that are often inadequate to the growing complexities of financial time-series. Sometimes large price changes are not followed by relatively huge movements and at some instances even small reactions trigger great changes. These irregular phenomena lead to the solid conclusion that market volatilities are not constant over time.

To the best of our knowledge, no research yet has been done to determine chaos in the returns of the main Philippine stock index. This research contributes to the literature of emerging stock markets and aims to:

- 1) provide additional evidence on the nonlinearities in financial time-series, through examining the chaotic properties of the PSEi returns;
- 2) challenge the validity of the EMH based on the chosen data sets of the Philippine index through the identification of structural breaks using Chow (1960) test in the recent Sub-prime mortgage crisis; and
- 3) examine if there are changes in the chaotic properties of the exchange before, during and after the recent crisis.

The great prospect that the Philippines is projecting as far as its current economic situation will inevitably increase the interest of the international investing community in the next decades. This renewed momentum from both traders and investors is a good opportunity and motivation to provide guidance on some of the financial characteristics of the PSEi. Also, proper modeling of its nonlinearities can yield better results that will benefit the investing community to understand emerging stock market behavior. The paper is also motivated by the fact that providing new understanding in the tendencies of the PSEi returns creates a considerable amount of knowledge for both academicians and researchers in providing potential avenues for research.

The research is structured as follows. Section II explains the data and methodology of the three tests of chaos utilized; Section III interprets the empirical findings; and Section IV provides the conclusion.

## II. Data and Methodology

Daily closing prices of the PSEi stock market returns during the last decade were obtained from the Yahoo Finance Website. This study divided the original data into four separate time-series using the Chow breakpoint test and the US National Bureau of Economic Research (NBER) business cycles: (1) All data set, which covers January 4, 2000 to December 29, 2011; (2) before sub-prime mortgage crisis, which spans January 4, 2000 to Nov. 29, 2007; (3) time of crisis, which covers December 3, 2007 to June 30, 2009; and (4) after crisis, which spans July 7, 2009 to December 29, 2011.

### II.2. Chaos Methodologies

This paper utilizes three different approaches in testing the chaotic tendencies of an underlying time series data of the PSEi. In order for a process to become chaotic, the existence of a fractal dimension and sensitive dependence on initial conditions are the two necessary conditions (Peters, 1994). The detailed three methodologies presented in this research are as follows:

#### II.2.1. Brock, Dechert, and Scheinkman Test

The Brock, Dechert, and Scheinkman (BDS) test of Brock et al. (1996) provides a distinction between a random series from deterministic chaos or from nonlinear stochastic series. As initially defined by Hsieh (1991), chaos is a nonlinear deterministic series that appears to be random in nature and cannot be determined as nonlinear deterministic system or a nonlinear stochastic system. The BDS test statistic computes for the significant value of the correlation dimension and detects nonlinear dependence. This test is proven powerful by Opong et al. (1999) when they discovered that the time-series of the FTSE stock index returns is not stochastic because of frequent showing of patterns. However, the BDS test has been observed to have a low power against the AR and autoregressive conditional heteroscedasticity (ARCH) models (Hsieh, 1991). Thus, this study deemed it necessary to pre-filter the data series with linear filter such as the autoregressive moving average (ARMA) or autoregressive integrated moving average (ARIMA) and a nonlinear filter such as the generalized ARCH (GARCH) before proceeding with the BDS test.

The correlation integral provides the basis to compute for the BDS test statistic and is computed as:

$$C_N(l, T) = \frac{2}{T_N(T_N - 1)} \sum_{t < s} I_l(x_t^N, x_s^N), \quad (1)$$

where  $T_N = T - N + 1$ .

The correlation integral is dependent on a sequence  $\{x_t : t = 1, \dots, T\}$  of observations which are independent and identically distributed (*iid*), and N-dimensional vectors

$[x_t^N = (x_t, x_{t+1}, \dots, x_{t+N-1})]$ , called the “N-histories”.

The null hypothesis  $\{x_t\}$  is *iid* with a non-degenerative density  $F$ ,  $C_N(l, T) \rightarrow C_1(l)^N$  with probability of one, as  $T \rightarrow \infty$ , for any fixed  $N$  and  $l$ ; and that  $\sqrt{T}[C_N(l, T) - C_1(l, T)^N]$  has a normal distribution with zero mean and variance (Brock et al., 1996).

$$\sigma_N^2(l) = 4 \left[ K^N + 2 \sum_{j=1}^{N-1} K^{N-1} C^{2j} + (N-1)^2 C^{2N} - N^2 K C^{2N-2} \right], \quad (2)$$

where  $C = C(l) = \int [F(z+1) - F(z-1)] dF(z)$ ,  $K = K(l) = \iint [F(z+1) - F(z-1)]^2 dF(z)$ .

Furthermore,  $C_1(l, T)$  is a consistent estimate of  $C(l)$ , and

$$K(l, T) = \frac{6}{T_N(T_N - 1)(T_N - 2)} \sum_{t < s < r} I_t(x_t, x_s) I_t(x_s, x_r). \quad (3)$$

The expression  $\sigma_N(l)$  can be estimated consistently by  $\sigma_N(l, T)$ , which  $C_1(l, T)$  and  $K_1(l, T)$  can replace  $C(l)$  and  $K(l)$  in the equation, because Eq. (3) is also a consistent estimate of  $K(l)$ . The BDS test statistic following a normal distribution can be calculated as follows:

$$w_N(l, T) = \sqrt{T} [C_N(l, T) - C_1(l, T)^N] / \sigma_N(l, T), \quad (4)$$

where  $\sigma_N(l, T)$  represents the standard deviation of the correlation integrals.

### II.2.2. Rescaled Range Analysis: Hurst Exponent

The range and standard deviation (R/S statistic) or the so-called rescaled range defines the R/S analysis. Mandelbrot and Wallis (1969) and Wallis and Matalas (1970) made some improvements on the rescaled range procedure first developed by Hurst (1951). According to Lo (1991), the major limitation of the traditional rescaled range analysis is that it can determine range dependencies without discriminating short and long dependencies in the data series. This shortcoming made the modified R/S analysis stronger because it was able to remove short-term dependencies and also able to detect long-term dependencies. Following Peters (1994) and Opong et al. (1999) in their procedures on how to perform the R/S analysis, this study initially transformed the PSEi into logarithmic returns and is given by:

$$S_t = \ln(P_t / P_{t-1}), \quad (5)$$

where  $S_t$  = logarithmic returns at time  $t$ , and  $P_t$  = price index at time  $t$ . In order to minimize the effect of linear dependency and non-stationarity the  $S_t$  series undergoes a process called

pre-whitening by adopting an AR(1) model to  $S_t$  which is computed as follows:

$$S_t = \alpha + \beta S_{t-1} + \varepsilon_t, \quad (6)$$

where  $S_{t-1}$  is the logarithmic return at time period  $t-1$  and  $\alpha$  and  $\beta$  are the parameters to be estimated and  $\varepsilon_t$  represent the residual.

The time series is separated into  $A$  adjacent sub-periods of length  $n$ , such that  $A \times n = N$ , where  $N$  denotes the extent of the series  $N_t$ , which follows the processes of Peters (1994) and Opong et al. (1999). Each sub-period is labeled  $I_a$ ,  $a=1,2,3,\dots,A$ . The data series in  $I_a$  is marked  $N_{k,a}$ ,  $k=1,2,3,\dots,n$  and the average value  $e_a$  for each  $I_a$  of length  $n$  is

$$e_a = \left(\frac{1}{n}\right) \times \sum_{k=1}^n N_{k,a}, \quad (7)$$

$$R_{I_a} = \max(X_{k,a}) - \min(X_{k,a}), \text{ where } 1 \leq k \leq n, 1 \leq a \leq A. \quad (8)$$

Within each sub-period  $I_a$ , the range  $R_{I_a}$  is the difference between the maximum and minimum value  $X_{k,a}$ , which is expressed as:

$$X_{k,a} = \sum_{i=1}^k (N_{i,a} - e_a), \quad k=1,2,3,\dots,n \text{ representing the elements for each sub-period of}$$

departures from the mean value. The R/S analysis sets the necessary condition that  $R_{I_a}$  should be normalized by dividing the sample by the standard deviation  $S_{I_a}$  corresponding to it and is computed below:

$$S_{I_a} = \left[ \frac{1}{n} \times \sum_{k=1}^n (N_{k,a} - e_a)^2 \right]^{0.50}. \quad (9)$$

The average R/S value for length  $n$  is calculated as:

$$\left(\frac{R}{S}\right)_n = \left(\frac{1}{A}\right) \times \sum_{a=1}^A (R_{I_a} / S_{I_a}). \quad (10)$$

The last step in the analysis applies an ordinary least squares (OLS) regression with  $\log(n)$  as the independent variable and  $\log(R/S)$  as the dependent variable. The Hurst exponent,  $H$  is obtained from the slope from the regression. The three values of the  $H$  exponent are expected to be:  $H = 0.5$ , which means that the series is a random walk;  $0 \leq H < 0.5$ , which denotes an anti-persistent series; and  $0.5 < H < 1$ , which represents a persistent series, or a trend-

reinforcing series. The R/S analysis is assessed through the computation of the expected values of the R/S statistics calculated as:

$$E(R/S) = \left[ \left( \frac{n-0.5}{n} \right) \times \left( \frac{n \times \pi}{2} \right) \right]^{-0.50} \times \sum_{r=1}^{n-1} \sqrt{\frac{(n-r)}{r}}. \quad (11)$$

The value of the Hurst exponent is from the slope of the regression of  $E(\log(R/S))_n$  on  $\log(n)$ . The variance of the Hurst exponent is illustrated as:

$$Var(H)n = \frac{1}{T}, \quad (12)$$

where  $T$  represents the total number of observations in the series.

### II.2.3. Correlation Dimension Analysis

The correlation dimension (CD) offers a diagnostic process to differentiate deterministic and stochastic time series  $\{x_t\}$ . As introduced by Grassberger and Procaccia (1983), the analysis determines the amount of complexity of a time-series data, which can be help in identifying possible signs of chaotic properties. The CD analysis was utilized by Kyrtsov and Terraza (2002) in their empirical study with regards to the French CAC40 returns. The authors showed evidence that the series can be either generated through a noisy chaotic or a pure random process. Grassberger and Procaccia (1983) and Hsieh (1991) suggested that the analysis requires the initial filtering of the observations through the ARMA and GARCH processes to remove autocorrelation and conditional heteroscedasticity, respectively, which can negatively impact on tests for evaluating chaos.

The filtering is followed by creating  $n$ -histories of the filtered data, which can be shown as follows:

$$1\text{-history: } x_t^1 = x_t, \quad (13)$$

$$2\text{-history: } x_t^2 = (x_{t-1}, x_t), \quad (14)$$

:

$$n\text{-history: } x_t^n = (x_{t-n+1}, \dots, x_t). \quad (15)$$

where  $n$ -history corresponds to a particular point in the  $n$ -dimensional space.

The correlation integral is then computed to define the correlation dimension which can be illustrated as:

$$C_n(\varepsilon) = \lim_{T \rightarrow \infty} \frac{\#\{(t, s), 0 < t, s < T : \|x_t^n - x_s^n\| < \varepsilon\}}{T^2}, \quad (16)$$

where # characterizes the number of points in the set, and  $\| \|$  represents the sup- or max-norm making the correlation integral  $C_n(\varepsilon)$  the fraction of pairs  $(x_s^n, x_t^n)$ , which are close to each other, based on the limit:

$$\max_{i=0, \dots, n-1} \{ |x_{s-i} - x_{t-i}| \} < \varepsilon . \quad (17)$$

The final step computes the slope of  $\log C_n(\varepsilon)$  on  $\log \varepsilon$  for small values of  $\varepsilon$  with the following equation:

$$v_n = \lim_{\varepsilon \rightarrow 0} \log C_n(\varepsilon) / \log \varepsilon . \quad (18)$$

The chaotic behavior of the series is present if the value of correlation dimension ( $v_n$ ) does not converge to a stable value as the embedding dimension increases.

### III. Empirical Results

Table 1 shows that the PSEi has an average of 1.27% positive returns for the whole data sample. The index also has positive returns before and after the crisis, while experienced average losses of 10% during the recession period. The highest positive return was experienced in the time when the economy was reviving, posting an average return of 9.46% 2.5 years after the recession ended. It is also noticeable that the highest volatility was experienced during the crisis period with the 1.942 standard deviation. Following the Modern Portfolio Theory of Markowitz (1952), we can tell that with the greater dispersion the PSEi returns had during the sub-prime mortgage crisis, a higher their risk was experienced which lead to higher losses. The lowest volatility was felt by the country after the recession with 1.123. All of the sample periods are negatively skewed and the kurtosis coefficients have leptokurtic distributions. The Jarque-Bera statistic for residual normality shows that the PSEi returns are under a non-normal distribution assumption.

Table 2 illustrates the initial filtering done by this research. The Augmented Dickey-Fuller (ADF) test established the stationarity of the data. The minimum value of the Akaike Information Criterion is utilized to identify the orders of the models for the ARMA model, ARMA residual and GARCH residuals. All data period of the PSEi passed the serial correlation examination based on the results of the Lagrange Multiplier (LM) test. This paper used the ARCH-LM process to test the ARCH effect and shows that we can apply GARCH filtering models for each of the designed periods, because the null hypothesis was rejected. The final test for heteroscedasticity showed that all data sets have already constant variance upon the determination of the GARCH residual models.



This research utilized a series of test to detect chaos on the returns and residuals of the PSEi. The BDS test is first of the three tests to identify chaos and remove the possibility that the time-series behave *iid*. Table 3 illustrates that the BDS statistics are significant at the 1% level for most values of  $\varepsilon/\sigma$  for the PSEi returns and ARMA residuals. Therefore, this study concludes that these time-series are not *iid* or not pure random series, and conventional linear methodologies are not appropriate for their analyses. In earlier studies, Eldridge et al. (1993) and Opong et al. (1999) have the same findings of non-stochastic processes in the returns of S&P 500 cash index and FTSE index, respectively. These results also validated the recent study of Ozer and Ertokali (2010) regarding the chaotic properties of emerging stock returns index like the Istanbul Stock Exchange Index. This study does not conform to the EMH of Fama (1970), and to the initial conclusions of Aquino (2006) regarding the weak-form efficiency of the PSEi. However, BDS test cannot conclude the *iid* properties for all the GARCH residuals, except for the Recession Period, wherein the presence of significant result at  $\varepsilon/\sigma=6$  cannot be discounted and may hint a possibility of having a stochastic process. Since BDS test is just the beginning in testing for chaos, this research further tests its validity and utilizes R/S and correlation dimension analyses to supplement this initial examination.

Peters (1994) earlier explained that if a time-series is determined by a chaotic process, the Hurst exponent would be much closer to 0.5 after scrambling the data than the one before the procedure. Table 4 shows that most Hurst exponents of the PSEi returns, ARMA and GARCH residuals are way below 0.5, however, after scrambling the data, all Hurst exponents are above 0.5. These findings are again consistent with Opong et al. (1999), and in the expectations of this study. This research also concludes that the PSEi returns have persistent, and trend-reinforcing series, wherein having an upward (downward) trend in the last period, will continue to be positive (negative) in the next period. However, the characteristics of the PSEi returns changed during the recession period, wherein it was characterized by an anti-persistent series. Table 5 presents the correlation dimension estimates for PSEi data series. The CD analysis is a procedure necessary for confirming chaotic behavior as defined by Wei and Leuthold (1998). This paper observed that as the embedding dimensions gradually increased from 1 to 10, the correlation dimension generally increases. This tendency tells that the PSEi returns, ARMA and GARCH residuals are consistent with chaos and these findings also conforms to the study of Kyrtsov et al. (2004) of the French CAC40 index returns. These results warn technical investing analysts to be cautious in utilizing linear processes in modeling the PSEi; and provide an understanding to academicians and researchers that in the previous decade, the PSEi returns signified chaotic tendencies that opposes the EMH.

#### IV. Conclusion

This study using three tests for chaotic behavior: BDS, R/S Analysis and Correlation Dimension concluded that the PSEi returns showed evidences of chaos. The BDS test concluded that PSEi returns and ARMA residuals are not *iid*, and conventional linear methodologies are not suited for their analysis; and are not consistent to the initial conclusions of Aquino (2006) regarding the weak-form efficiency of the PSEi. This test initially cannot conclude the *iid* properties for GARCH residuals, except for the duration of recent Subprime Mortgage Crisis. The R/S analysis was conducted to verify the initial results and showed that most Hurst exponents of the PSEi returns, ARMA and GARCH residuals became closer to 0.5 after scrambling the time-series, which means that chaotic tendencies are present. This paper also concluded that the data have persistent and trend-reinforcing series, except for the ARMA residuals of the crisis period. The CD analysis was also used to supplement the first two tests and observed that as the embedding dimensions gradually increased from 1 to 10, the correlation dimension generally increases. This further confirmed that PSEi returns, ARMA and GARCH residuals are consistent with chaos.

These findings caution the investing community in the risk of using linear processes in modeling the main Philippine stock index; and give further research avenue to academicians and researchers in the chaotic tendencies of PSEi returns. Differences in findings with regards to the non-linear characteristics can be affected by the volume of the data, particularly on the sub-data sets; Harrison et al. (1999) explains that the accuracy of test results improves with the increase in the length of the time series. For future studies, it is suggested that researchers use larger (maybe 5,000 or more) data sets. Researches can also consider developed and other emerging economies and compare the differences of results.

**Table 1: The Sample Size and Periods of the Philippine Stock Exchange Index**

Currency ETNs	Data Period	Obs.	Mean	Std. Dev.	Skew.	Kurt.	J-Bera
All Data Period	Jan. 4, 2000 - Dec. 29, 2011	3013	0.0127	1.3197	-0.5719	8.8031	4392.0300***
Before Recession Period	Jan. 4, 2000 - Nov. 29, 2007	2018	0.0093	1.2241	-0.1988	5.2096	423.8134***
RecessionPeriod	Dec. 3, 2007 - Jun. 30, 2009	384	-0.0999	1.9422	-0.9803	9.2749	691.4866***
After Recession Period	Jul. 7, 2009 - Dec. 29, 2011	611	0.0946	1.1234	-0.1693	5.2195	128.3261***

Source: Yahoo Finance – various inception dates up to December 29, 2011; <http://www.yahoo.com/finance>.

**Table 2: Summary Statistics of Unit Root test, and ARMA-GARCH filtering**

ETNs	ADF	ARMA	AIC	LM-test	ARMA Res.	AIC	LM-test	ARCH-LM	GARCH Res.	AIC	ARCH-LM
All	-47.276***	(2,1)	3.371	1.037	(0,1)	3.369	2.213	96.986***	(1,2)	3.217	0.083
Before	-39.384***	(2,1)	3.221	1.509	(2,1)	3.218	1.539	16.040***	(2,1)	3.149	0.124
Recession	-16.228***	(1,0)	4.141	0.804	(2,2)	4.120	0.103	16.349***	(0,2)	3.898	0.156
After	-15.168***	(1,2)	3.059	0.662	(2,2)	3.049	0.183	35.590***	(1,2)	2.938	0.114

Note: \*, \*\* and \*\*\* are significance at 10, 5 and 1% levels, respectively; p-values are in parentheses.

**Table 3: BDS test for the PSEi data sets**

ALL $\varepsilon/\sigma$	returns				ARMA residuals				GARCH residuals			
	0.5	1.0	1.5	2.0	0.5	1.0	1.5	2.0	0.5	1.0	1.5	2.0
2	0.005*** (0.000)	0.0125*** (0.000)	0.014*** (0.000)	0.010*** (0.000)	0.005*** (0.000)	0.013*** (0.000)	0.015*** (0.000)	0.011*** (0.000)	-0.000 (1.000)	0.001 (0.653)	0.001 (0.653)	0.001 (0.416)
3	0.006*** (0.000)	0.023*** (0.000)	0.030*** (0.000)	0.026*** (0.000)	0.005*** (0.000)	0.023*** (0.000)	0.031*** (0.000)	0.026*** (0.000)	-0.000 (0.982)	0.001 (0.695)	0.001 (0.695)	0.001 (0.485)
4	0.004*** (0.000)	0.026*** (0.000)	0.044*** (0.000)	0.041*** (0.000)	0.004*** (0.000)	0.026*** (0.000)	0.044*** (0.000)	0.041*** (0.000)	0.000 (0.989)	0.000 (0.807)	0.000 (0.807)	0.002 (0.529)
5	0.002*** (0.000)	0.026*** (0.000)	0.053*** (0.000)	0.056*** (0.000)	0.002*** (0.000)	0.025*** (0.000)	0.054*** (0.000)	0.056*** (0.000)	-0.000 (0.912)	0.000 (0.924)	0.000 (0.924)	0.002 (0.587)
6	0.001*** (0.000)	0.023*** (0.000)	0.060*** (0.000)	0.069*** (0.000)	0.001*** (0.000)	0.023*** (0.000)	0.059*** (0.000)	0.069*** (0.000)	0.000 (0.757)	0.000 (0.894)	0.000 (0.894)	0.002 (0.575)

  

Before $\varepsilon/\sigma$	returns				ARMA residuals				GARCH residuals			
	0.5	1.0	1.5	2.0	0.5	1.0	1.5	2.0	0.5	1.0	1.5	2.0
2	0.003*** (0.001)	0.008*** (0.000)	0.009*** (0.000)	0.007*** (0.000)	0.003*** (0.000)	0.008*** (0.000)	0.009*** (0.000)	0.007*** (0.000)	-0.000 (0.754)	0.000 (0.857)	0.001 (0.465)	0.000 (0.322)
3	0.004*** (0.000)	0.015*** (0.000)	0.020*** (0.000)	0.017*** (0.000)	0.003*** (0.000)	0.014*** (0.000)	0.019*** (0.000)	0.016*** (0.000)	-0.000 (0.487)	-0.000 (0.982)	0.002 (0.540)	0.002 (0.376)
4	0.002*** (0.000)	0.018*** (0.000)	0.030*** (0.000)	0.028*** (0.000)	0.002*** (0.000)	0.017*** (0.000)	0.030*** (0.000)	0.027*** (0.000)	-0.000 (0.532)	0.000 (0.972)	0.002 (0.582)	0.002 (0.483)
5	0.001*** (0.000)	0.017*** (0.000)	0.038*** (0.000)	0.039*** (0.000)	0.001*** (0.000)	0.017*** (0.000)	0.038*** (0.000)	0.039*** (0.000)	-0.000 (0.819)	0.000 (0.934)	0.002 (0.569)	0.002 (0.557)
6	0.001*** (0.000)	0.016*** (0.000)	0.044*** (0.000)	0.050*** (0.000)	0.001*** (0.000)	0.015*** (0.000)	0.043*** (0.000)	0.049*** (0.000)	0.000 (0.846)	0.000 (0.777)	0.000 (0.465)	0.003 (0.514)

(continued)

Recession $\varepsilon/\sigma$	returns				ARMA residuals				GARCH residuals			
	0.5	1.0	1.5	2.0	0.5	1.0	1.5	2.0	0.5	1.0	1.5	2.0
2	0.006*** (0.004)	0.014*** (0.001)	0.015*** (0.001)	0.008*** (0.008)	0.008*** (0.000)	0.020*** (0.000)	0.019*** (0.000)	0.010*** (0.000)	0.000 (0.768)	0.000 (0.893)	-0.001 (0.809)	0.000 (0.977)
3	0.008*** (0.000)	0.034*** (0.000)	0.045*** (0.000)	0.034*** (0.000)	0.008*** (0.000)	0.038*** (0.000)	0.049*** (0.000)	0.034*** (0.000)	0.000 (0.862)	0.000 (0.999)	-0.003 (0.608)	-0.001 (0.780)
4	0.005*** (0.000)	0.038*** (0.000)	0.066*** (0.000)	0.058*** (0.000)	0.005*** (0.000)	0.040*** (0.000)	0.069*** (0.000)	0.058*** (0.000)	-0.000 (0.977)	-0.000 (0.903)	-0.004 (0.594)	-0.001 (0.865)
5	0.003*** (0.000)	0.036*** (0.000)	0.079*** (0.000)	0.080*** (0.000)	0.003*** (0.000)	0.036*** (0.000)	0.082*** (0.000)	0.080*** (0.000)	-0.000 (0.380)	-0.001 (0.742)	-0.003 (0.674)	-0.000 (0.960)
6	0.001*** (0.000)	0.031*** (0.000)	0.087*** (0.000)	0.099*** (0.000)	0.001*** (0.000)	0.030*** (0.000)	0.088*** (0.000)	0.097*** (0.000)	-0.000** (0.015)	-0.000 (0.871)	-0.000 (0.765)	-0.000 (0.974)
After $\varepsilon/\sigma$	returns				ARMA residuals				GARCH residuals			
0.5	1.0	1.5	2.0	0.5	1.0	1.5	2.0	0.5	1.0	1.5	2.0	
2	0.007*** (0.000)	0.015*** (0.000)	0.016*** (0.000)	0.014*** (0.000)	0.006*** (0.000)	0.014*** (0.000)	0.014*** (0.000)	0.012*** (0.000)	0.001 (0.247)	0.000 (0.875)	0.000 (0.908)	0.000 (0.875)
3	0.006*** (0.000)	0.023*** (0.000)	0.031*** (0.000)	0.028*** (0.000)	0.005*** (0.000)	0.021*** (0.000)	0.028*** (0.000)	0.026*** (0.000)	0.001 (0.389)	0.001 (0.793)	0.001 (0.851)	0.001 (0.689)
4	0.003*** (0.000)	0.024*** (0.000)	0.041*** (0.000)	0.043*** (0.000)	0.003*** (0.000)	0.021*** (0.000)	0.036*** (0.000)	0.039*** (0.000)	-0.000 (0.877)	-0.001 (0.761)	-0.000 (0.976)	0.002 (0.691)
5	0.001*** (0.000)	0.021*** (0.000)	0.047*** (0.000)	0.055*** (0.000)	0.001*** (0.000)	0.018*** (0.000)	0.041*** (0.000)	0.050*** (0.000)	-0.000 (0.505)	-0.001 (0.648)	-0.001 (0.897)	0.003 (0.706)
6	0.001*** (0.000)	0.017*** (0.000)	0.048*** (0.000)	0.064*** (0.000)	0.001*** (0.000)	0.014*** (0.000)	0.042*** (0.000)	0.058*** (0.000)	-0.000 (0.107)	-0.000 (0.576)	-0.000 (0.789)	0.002 (0.834)

Note: \*, \*\* and \*\*\* are significance at 10, 5 and 1% levels, respectively; p-values are in parentheses.

**Table 4: Hurst exponents of the PSEi data sets**

Stock returns	All	Before	Recession	After
Original Series	0.00178	0.00150	0.00379	0.00101
Scrambled Series	0.54960	0.55920	0.52310	0.51300
ARMA residuals	All	Before	Recession	After
Original Series	-0.00024	-0.00036	0.000506	-0.00099
Scrambled Series	0.52770	0.52130	0.45620	0.50240
GARCH residuals	All	Before	Recession	After
Original Series	-0.00004	-0.00018	-0.00027	-0.00032
Scrambled Series	0.55000	0.50100	0.51180	0.52270

**Table 5: Correlation Dimension Analysis estimates of the PSEi data sets**

Correlation Dimensions	Embedding Dimensions									
	1	2	3	4	5	6	7	8	9	10
1. All returns	1.018	1.74	2.104	2.505	2.798	3.035	3.17	3.178	3.516	3.454
ARMA residuals	1.021	1.82	2.408	2.501	3.079	3.02	3.484	3.519	3.439	3.784
GARCH residuals	1.024	1.911	2.602	3.16	3.587	3.925	4.081	4.164	4.662	4.049
2. Before returns	1.017	1.853	2.304	2.613	2.986	2.931	3.364	3.4	3.368	3.7
ARMA residuals	1.042	1.85	2.36	2.851	2.925	3.457	3.565	3.619	3.642	3.98
GARCH residuals	1.01	2.018	2.847	3.337	3.635	4.25	4.232	4.71	4.789	4.766
3. Recession returns	1.033	2.326	2.655	3.45	3.628	4.34	4.137	4.628	3.981	4.317
ARMA residuals	0.985	2.191	3.029	3.399	3.842	3.805	3.905	3.751	4.033	3.827
GARCH residuals	1.049	2.047	2.88	3.341	3.822	4.645	5.572	5.336	6.076	5.541
4. After returns	0.966	1.967	2.712	3.352	3.638	3.919	4.263	4.711	4.401	4.79
ARMA residuals	1.045	1.993	3.075	3.598	4.106	4.708	4.727	5.273	5.563	5.479
GARCH residuals	0.989	2.038	2.874	3.357	3.958	4.823	4.623	5.129	5.306	5.842

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