Math 1497 - Calc 2 lec 12/1

Strategy for integration

So in this chapter we've seen several techniques for integrating integrals

(1) u - substitution

$\int f(g(x))\, g'(x)\, dx$ let $u = g(x)$
$du = g'(x)dx$

$\int f(u)\, du$

Ex' $\int \frac{e^x}{(e^x+1)^2}\, dx$ let $u = e^x+1$ so $du = e^x dx$

most complicated piece

$\int \frac{du}{u^2} = \int u^{-2} du = \frac{u^{-1}}{-1} + C = \frac{-1}{u} + C$

back sub $= -\frac{1}{e^x+1} + C$

(2) Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

Kinda like a product rul for $\int$

Ex2 $\int \frac{\ln x}{x^2}\,dx$ $u = \ln x$ $v = -\frac{1}{x}$

$du = \frac{1}{x}dx$ $dv = \frac{1}{x^2}dx$

$$= \ln x\left(\frac{-1}{x}\right) - \int \frac{-1}{x}\cdot\frac{1}{x}\,dx$$

$$= -\frac{\ln x}{x} + \int \frac{1}{x^2}\,dx = -\frac{\ln x}{x} - \frac{1}{x} + C$$

Usually if $\int x^m f(x)\,dx$

choose $u = x^m$

$dv = f(x)\,dx$

unless there a $\ln x$

$\int x^m \ln x\,dx$ $u = \ln x$

$dv = x^m$

x's will cancel.

(3) Trig integrals

$\int \sin^m x \cos^n x\,dx$ ← Depending on powers
- m even n odd
- m odd n even
- m, n odd
- m, n even

$\int \tan^m x \sec^n x\,dx$

m odd
n even
only $\tan x$, only $\sec x$

$\boxed{\text{sub } u = \sin x,\ \cos x \quad u = \sec x,\ \tan x}$

Ex 3 $\displaystyle\int \sin^3 x \cos^4 x\,dx = \int \sin^2 x \cos^4 x \sin x\,dx$

$u = \cos x$
$du = -\sin x\,dx$

$\displaystyle\int \sin^2 x\, u^4 (-du)$ ← $\sin^2 x = 1 - \cos^2 x$

$\displaystyle -\int (1-u^2)\, u^4\,du$

$\displaystyle = \int (u^6 - u^4)\,du = \frac{1}{7}u^7 - \frac{1}{5}u^5 + c$

$\displaystyle = \frac{1}{7}\cos^7 x - \frac{1}{5}\cos^5 x + c$

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Ex4 $\int \tan^2 x \sec^4 x \, dx$

new $\int \tan^2 x \sec^2 x \sec^2 x \, dx$

$u = \tan x$

$du = \sec^2 x \, dx$

$\sec^2 x = 1 + \tan^2 x$

$\int u^2(1+u^2)\, du = \int u^2 + u^4 \, du$

$= \frac{1}{3}u^3 + \frac{1}{5}u^5 + C = \frac{1}{3}\tan^3 x + \frac{1}{5}\tan^5 x + C$

(4) Trig Sub

for integrals that look like

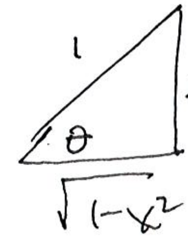
$\int \sqrt{a^2 - x^2}\, dx$ $\quad x = a\sin\theta \quad \sqrt{a^2-x^2} = \sqrt{a^2 - a^2\sin^2\theta} = a\cos\theta$

$\int \sqrt{a^2 + x^2}\, dx$ $\quad x = a\tan\theta \quad \sqrt{a^2+x^2} = \sqrt{a^2 + a^2\tan^2\theta} = a\sec\theta$

$\int \sqrt{x^2 - a^2}\, dx$ $\quad x = a\sec\theta \quad \sqrt{x^2-a^2} = \sqrt{a^2\sec^2\theta - a^2} = a\tan\theta$

Ex5 $\int \frac{\sqrt{1-x^2}}{x^2}\, dx$ $\quad x = \sin\theta \quad \sqrt{1-x^2} = 1-\sin^2\theta = \cos\theta$

$dx = \cos\theta \, d\theta$



$\int \frac{\cos^2\theta}{\sin^2\theta}\, d\theta = \int \csc^2\theta \, d\theta = -\cot\theta + C$

$= -\frac{\sqrt{1-x^2}}{x} + C$

Ex6 $\int_0^1 \frac{x^3}{\sqrt{1+x^2}}\,dx$ $x = \tan\theta$ $dx = \sec^2\theta$ $1+x^2 = 1+\tan^2\theta = \sec^2\theta$ 12/5

$x=0$ $\tan\theta = 0$ $\theta = 0$
$x=1$ $\tan\theta = 1$ $\theta = \pi/4$

$$\int_0^{\pi/4} \frac{\tan^3\theta \sec^2\theta\, d\theta}{\sec\theta} = \int_0^{\pi/4} \tan^3\theta \sec\theta\, d\theta$$

$$\int_0^{\pi/4} \tan^2\theta \sec\theta \tan\theta\, d\theta$$ $u = \sec\theta$ $du = \sec\theta\tan\theta\, d\theta$

$$\int_1^{\sqrt{2}} (u^2-1)\,du$$ ← this we can do

(5) <u>Partial Fractions</u>

$\int \frac{p(x)}{q(x)}\,dx$ p, q polynomials

split into separate simpler $\int$'s

$$\int \frac{2x+1}{(x+1)(x+2)}\,dx \qquad \frac{A}{x+1} + \frac{B}{x+2} = \frac{2x+1}{(x+1)(x+2)}$$

$$\int \frac{2x+1}{(x+1)^2(x+2)}\,dx \qquad \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x+2} = \frac{2x+1}{(x+1)^2(x+2)}$$

$$\int \frac{2x+1}{x(x^2+1)}\,dx \quad \text{so} \quad \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

$Bx+C$ ← linear

x^2+1 ← good

(6) Improper integrals:

(A) $\int_a^b f(x)\,dx$ f singularity at $x=a, b$ or inside (a,b)

B) $\int_a^{\infty} f(x)\,dx \qquad \int_{-\infty}^{b} f(x)\,dx \qquad \int_{-\infty}^{\infty} f(x)\,dx$

(infinite limits.)

Turn these integrals into limit problem.

Ex $$\int_0^4 \frac{dx}{x-4} = \lim_{b\to 4} \underbrace{\int_0^b \frac{dx}{x-4}}_{\text{integral this}}$$

then do limit ? converges?

or diverges?