

Chapter 7

Rational Functions

Section 7-2

Graphing Rational Functions

Graphing Simple Rational Functions

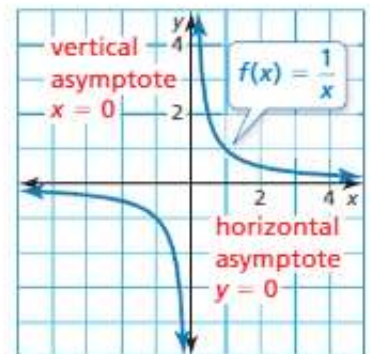
A **rational function** has the form $f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials and $q(x) \neq 0$. The inverse variation function $f(x) = \frac{a}{x}$ is a rational function. The graph of this function when $a = 1$ is shown below.

Core Concept

Parent Function for Simple Rational Functions

The graph of the parent function $f(x) = \frac{1}{x}$ is a *hyperbola*, which consists of two symmetrical parts called branches. The domain and range are all nonzero real numbers.

Any function of the form $g(x) = \frac{a}{x}$ ($a \neq 0$) has the same asymptotes, domain, and range as the function $f(x) = \frac{1}{x}$.

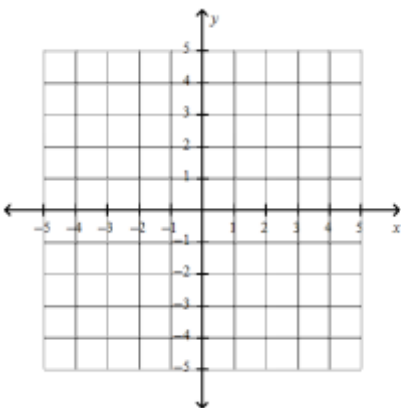


STUDY TIP

Notice that $\frac{1}{x} \rightarrow 0$ as $x \rightarrow \infty$ and as $x \rightarrow -\infty$. This explains why $y = 0$ is a horizontal asymptote of the graph of $f(x) = \frac{1}{x}$. You can also analyze y -values as x approaches 0 to see why $x = 0$ is a vertical asymptote.

EXAMPLE 1 Graphing a Rational Function of the Form $y = \frac{a}{x}$

Graph $g(x) = \frac{4}{x}$. Compare the graph with the graph of $f(x) = \frac{1}{x}$.



X	Y

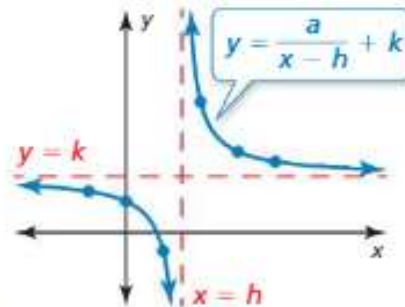
Translating Simple Rational Functions

Core Concept

Graphing Translations of Simple Rational Functions

To graph a rational function of the form $y = \frac{a}{x-h} + k$, follow these steps:

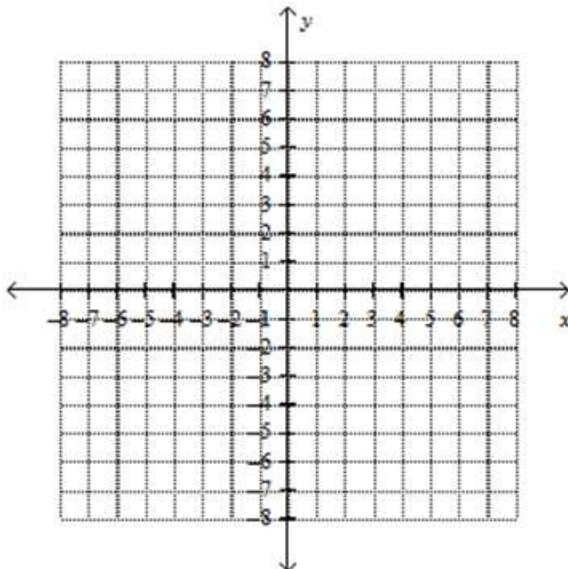
- Step 1** Draw the asymptotes $x = h$ and $y = k$.
- Step 2** Plot points to the left and to the right of the vertical asymptote.
- Step 3** Draw the two branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.



EXAMPLE 2

Graphing a Translation of a Rational Function

Graph $g(x) = \frac{-4}{x+2} - 1$. State the domain and range.



X	Y

Graphing Other Rational Functions

All rational functions of the form $y = \frac{ax + b}{cx + d}$ also have graphs that are hyperbolas.

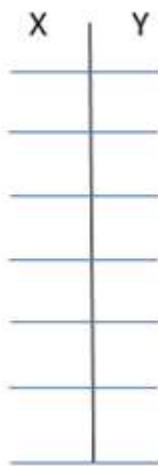
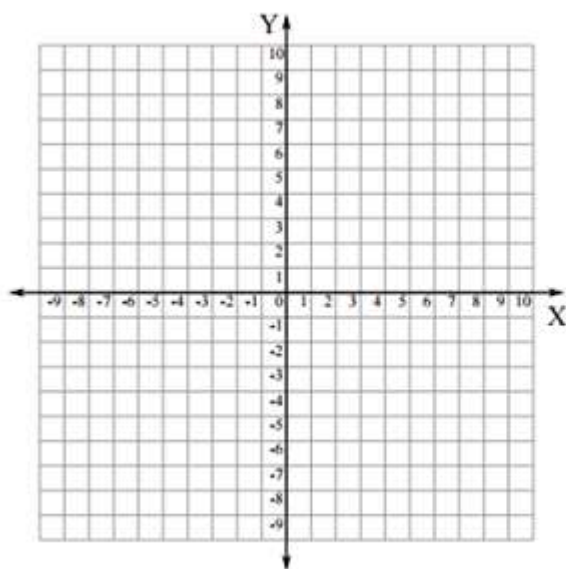
- The vertical asymptote of the graph is the line $x = -\frac{d}{c}$ because the function is undefined when the denominator $cx + d$ is zero.
- The horizontal asymptote is the line $y = \frac{a}{c}$.

EXAMPLE 3

Graphing a Rational Function of the

Form $y = \frac{ax + b}{cx + d}$

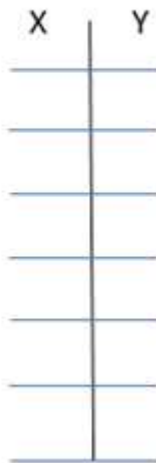
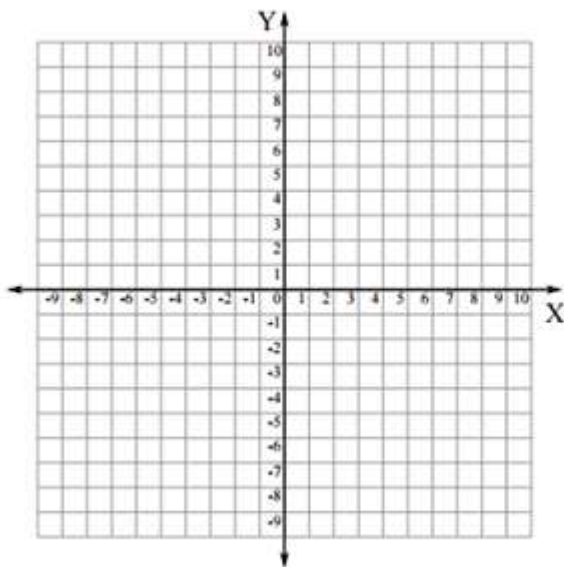
Graph $f(x) = \frac{2x + 1}{x - 3}$. State the domain and range.



EXAMPLE 4 Rewriting and Graphing a Rational Function

Rewrite $g(x) = \frac{3x + 5}{x + 1}$ in the form $g(x) = \frac{a}{x - h} + k$. Graph the function. Describe

the graph of g as a transformation of the graph of $f(x) = \frac{a}{x}$.



EXAMPLE 5 Modeling with Mathematics

A 3-D printer builds up layers of materials to make three-dimensional models. Each deposited layer bonds to the layer below it. A company decides to make small display models of engine components using a 3-D printer. The printer costs \$1000. The material for each model costs \$50.

- Estimate how many models must be printed for the average cost per model to fall to \$90.
- What happens to the average cost as more models are printed?



USING A GRAPHING CALCULATOR

Because the number of models and average cost cannot be negative, choose a viewing window in the first quadrant.

