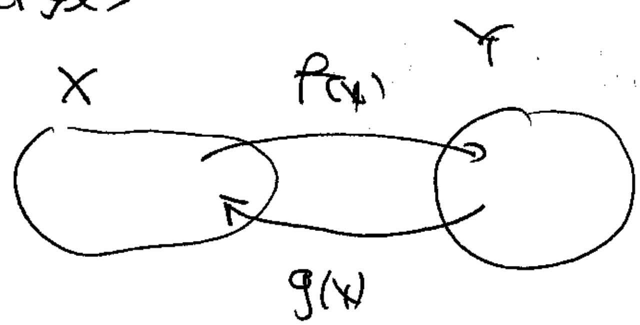
Precalc Review

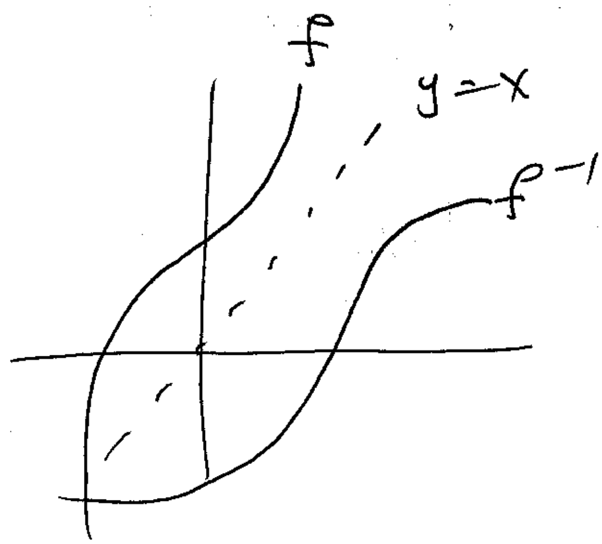
Inverses



Defined
$g(x) = f^{-1}(x)$

so $f(g(x)) = x$ and $g(f(x)) = x$

graphically



Ex $f(x) = 2x+1$ find $f^{-1}(x)$

1st $y = 2x+1$

interchange x & y $x = 2y+1$

so for y $\quad 2y = x-1$

$$y = \frac{x-1}{2}$$

so $\quad g(x) = f^{-1}(x) = \dfrac{x-1}{2}$

check

$$f(g(x)) = 2\,g(x)+1$$

$$= \cancel{2}\left(\frac{x-1}{\cancel{2}}\right)+1 = x-1+1 = x \checkmark$$

$$g(f(x)) = \frac{f(x)-1}{2} = \frac{2x+\cancel{1}-\cancel{1}}{2}$$

$$= \frac{\cancel{2}x}{\cancel{2}}$$

$$= x \checkmark$$

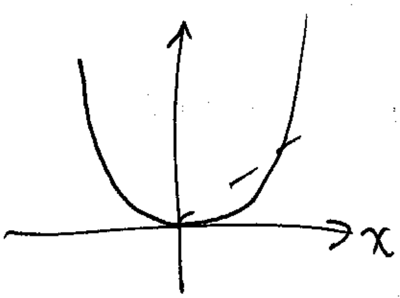
so $\quad f(x) = 2x+1, \quad g(x) = \dfrac{x-1}{2}$

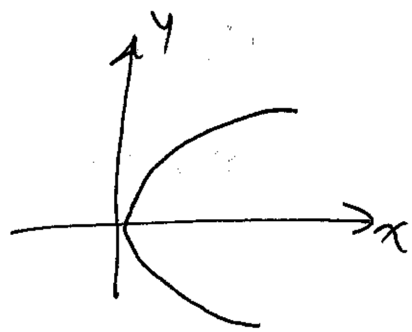
are inverses of each other

Existence of inverse

It's important to note that not every function has an inverse

Consider $y = x^2$





reflection in $y = x$

inverse $\quad x = y^2$ or $y = \pm\sqrt{x}$

$g(x) = \sqrt{x}$

check $\quad f(g(x)) = (g(x))^2 = (\sqrt{x})^2 = x$ ✓

$g(f(x)) = \sqrt{f(x)} = \sqrt{x^2} = |x|$ not always x

If f is one-to-one then

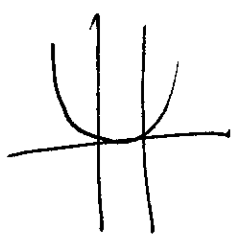
f has an inverse.

So what is 1-1? Given x_1 & x_2

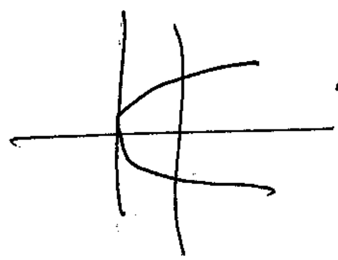
if $f(x_1) = f(x_2)$ then $x_1 = x_2$

Vertical Line Test

A relation is a function if it passes the vertical line test. ~~the~~ Each graph intersect a vertical line only once



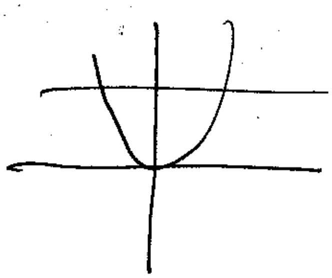
✓



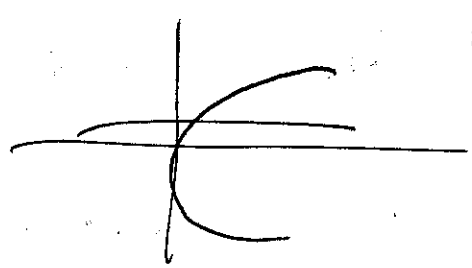
X

Horizontal Line Test

Each relation intersect a horizontal line at most once



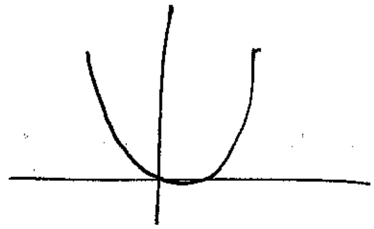
X



✓

A function has an inverse if it passes the horizontal line test

Ex $f(x) = x^2$



passes VL test ✓
fails HL test

$f(x_1) = f(x_2)$ so $x_1^2 = x_2^2$

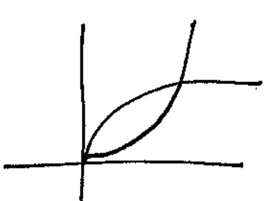
so $x_1^2 - x_2^2 = 0$ $(x_1 - x_2)(x_1 + x_2) = 0$

$\Rightarrow$ $x_1 = x_2$ or $x_1 = -x_2$

so $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ X

so we consider only a part of the graph

so $f(x) = x^2$ $x \geq 0$ only. This eliminates $x_1 = -x_2$



$g(x) = \sqrt{x}$

$f(g(x)) = x$ ✓ $g(f(x)) = \sqrt{x^2} = x$, since $x \geq 0$ ✓

More Functions

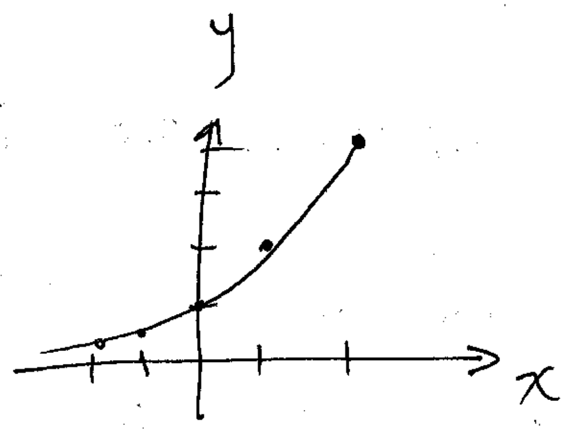
Exponential Function

ex

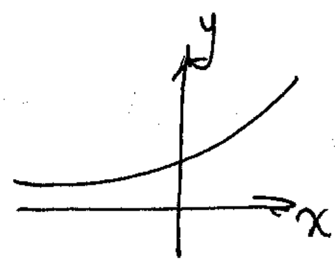
$y = 2^x$ 2-base x power

T of V

x	y
-2	$2^{-2} = \frac{1}{2^2} = \frac{1}{4} = .25$
-1	$2^{-1} = 1/2 = .5$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$

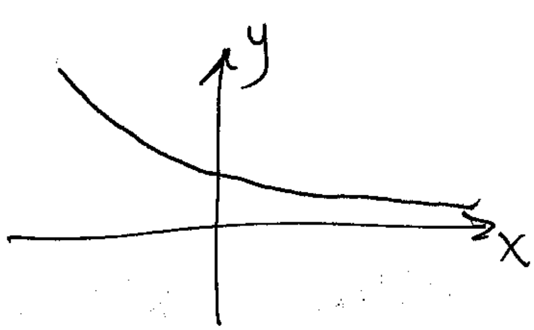


In general $y = a^x$

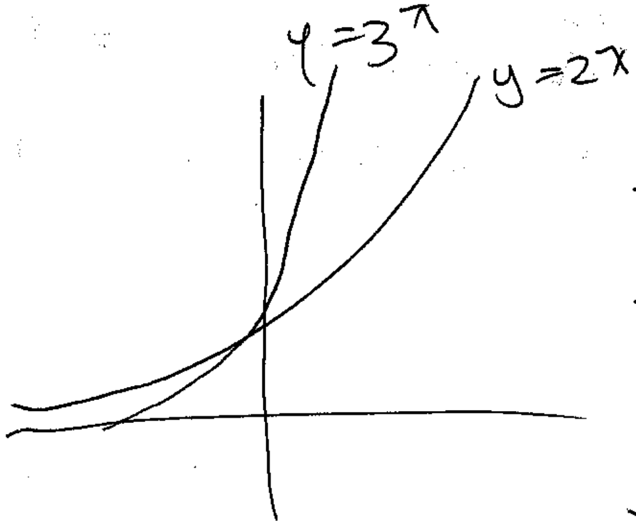


Exponential Growth

Also $y = a^{-x} = \left(\frac{1}{a}\right)^x = \frac{1}{a^x}$

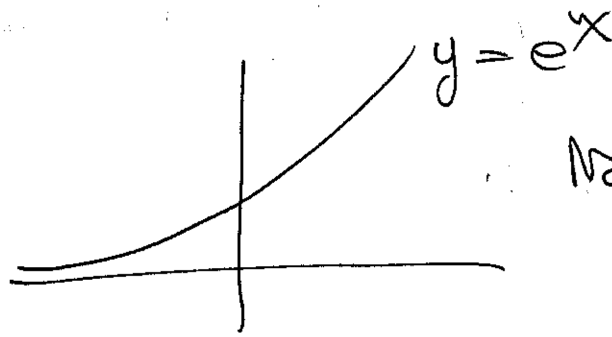


Exponential decay



there is a special exponential function where the base a is between 2 & 3

$a = 2.71828 = e$ ← we will explore this more later



Natural ~~Log~~ Exp function

Some properties a, b +ve #'s $x, y \in \mathbb{R}$

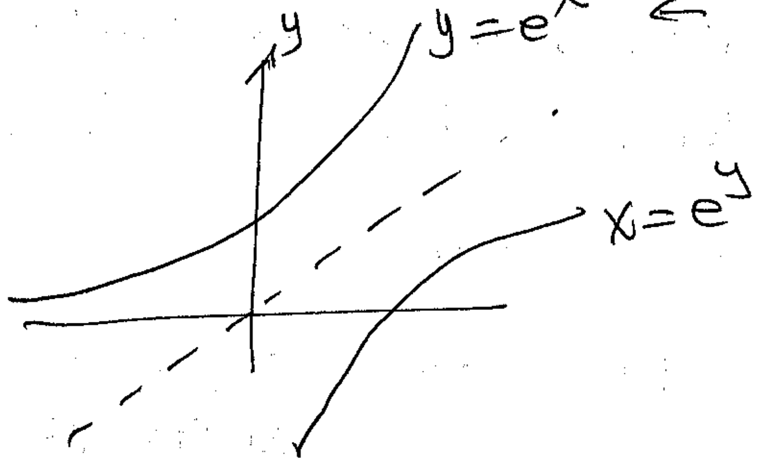
(i) $a^0 = 1$, (ii) $a^x \cdot a^y = a^{x+y}$ (iii) $(a^x)^y = a^{xy}$

(iv) $a^x \cdot b^x = (ab)^x$ (v) $\dfrac{a^x}{a^y} = a^{x-y}$

(vi) $\left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}$ (vii) $a^{-x} = \dfrac{1}{a^x}$

Inverse of the natural ~~log~~ exp functn

$y = e^x$ ← it does have an inverse



if we solve this inverse for y, we call it the natural log funct

$$y = \ln x$$

Note $\ln e^x = x$ & $e^{\ln x} = x$

properties - let $x, y, z \in \mathbb{R}$ $x, y > 0$

(1) $\ln xy = \ln x + \ln y$

2) $\ln \frac{x}{y} = \ln x - \ln y$

3) $\ln x^n = n \ln x$