Mechanism Implementing Efficient Pure Strategy Nash Equilibrium in Distributed Systems

Tilak Sanyal[1], Uttiya Paul[2]

[1] *Department of Economics, Shibpur Dinobundhoo College, Howrah, India*

[2] *Department of Economics, University of Kansas, Lawrence, USA*

[1] Corresponding author: E-mail – sanyal.tilak@gmail.com

Abstract – This paper extends the 'access game' of Paul and Paul (2017) in a simplified dynamic framework consisting of two time periods. There are two users who desire to transmit data pack using a distributed medium. While the medium can be accessed by single user at a time, the users have waiting cost. They choose their actions non-cooperatively and this gives rise to a game form. We find that the pure strategy Nash equilibrium of the game is non-unique. Moreover, efficient outcome requires the user with the highest waiting cost to transmit in period I followed by the other user in period II.

Contrary to some technical control mechanisms envisaged in the existing literature, we design a strategyproof *ex ante* economic mechanism that enables truthful revelation of waiting cost by the respective users. Based on this, the slots are allocated efficiently to them. We also observe that the mechanism satisfies *ex post* individual rationality of all possible types of both the users.

Keywords – *Distributed systems, Organization and design – distributed systems, Mechanism design*

1. Introduction

The congestion problem in computer networks has been in the limelight for long time. Much of the initial endeavors have concentrated on understanding the nature of the problem and developing control techniques. The proposed congestion control mechanisms assume cooperation among the network users and require them to implement a particular flow control algorithm at the end hosts (Jacobson (1988), Jain and Ramakrishnan (1988) and Ramakrishnan and Jain (1990)). This approach relies on the users adopting a centrally mandated algorithm. But the recent papers consider that it is impossible to centrally mandate the behavior of end-users. They assume that users act selfishly to further their individual interest (see Demers, Keshav and Shenker (1990), for example). This particular approach has led to the diffusion of game theory in computer science to design algorithms for controlling congestion in distributed systems (Korilis, Lazar and Orda (1995); Orda, Rom and Shimkin (1993); Roughgarden and Tardos (2002); Papadimitriou (2001) and Wang, Xue and Yu (2011)).

Shenker (1994) studied the problem of controlling congestion from game theoretic perspective. He assumed that administrative control is exercised only at the network switches. In this framework he observed that the traditional FIFO service discipline is neither fair nor efficient. On the contrary, a service discipline called 'Fair Share' guarantees both. In another paper Akella et al (2002) show that the traditional environment – where end-points use loss recovery mechanism and routers use drop-tail queues, the Nash equilibria are efficient. On the other hand, if the loss recovery mechanism is replaced by a more recent variation of TCP then the Nash equilibria are very inefficient.

Rakshit and Guha (2005) consider an 'Access Game' where the selfish users have access to contention-based distributed medium. They show that such a game has infinite Nash equilibria and none of them is fair. But if a set of fairness constraints is imposed, then the Nash equilibrium is unique, fair and optimizes throughput. In another *access game* considered by Paul and Paul (2017), they use the fairness condition to obtain unique transmission probabilities where the unconstrained game yields multiple pure and mixed strategy Nash equilibria.

Our paper also considers an access game very similar to the one in Paul et al (2017). We believe that in the situation where all the users cannot successfully access the medium simultaneously and some have to wait, it can be best modeled as a dynamic game. Further, all the papers discussed above considered static games only. This fact motivates us to consider a simple yet appropriate dynamic version of the access game.

The theoretical model considers a situation where two agents (users) are to transmit data pack through a medium. It gives rise to a game form as the available network can support successful transmission of one pack at a time. So we assume a two-period model. The agents are impatient and each discounts the future payoff at a constant rate. The discount factor resembles the waiting cost of an agent. Higher the discount factor, lower is the waiting cost and hence more patient he is. It is quite legitimate to assume that the waiting costs of the users are private information i.e. each user perfectly knows his waiting cost but does not know the cost of other users. Thus we have a setting of a dynamic access game with asymmetric information.

There is no coordination between the agents. When both attempt to transmit simultaneously in period I, it results in a failed attempt and both get lower payoff. Since period II is the last period, so both would desperately attempt to transmit and yet again it would fail. Now, if both wait in period I (this possibility could arise from lack of communication between them) then they would surely transmit in period II. But that would again result in failure. It is clear by now that both could transmit successfully only if one transmits in period I and the other in period II. But as waiting is costly, so the issue is who would wait. Obviously, this decision making requires the agents to coordinate which is beyond the scope of the game.

In this context efficient Nash equilibrium of the game warrants the user with highest waiting cost to transmit first. Suppose there is a mechanism designer who asks each agent to report his waiting cost. The paper observes that truth revelation is not dominant strategy incentive compatible (strategyproof) for each agent, except one particular type. Given this observation, the designer designs an *ex ante* strategyproof mechanism that makes truthful revelation strategyproof for all possible types of each agent. We explicitly characterize the mechanism. Next we show that such mechanism is also *ex post* individually rational.

This paper is unique in three aspects. First, the existing literature has dealt with technical control mechanisms only. Our paper introduces an economic mechanism.[1] Access to a particular medium involves user cost. This cost can be adjusted to include the incentive that we have characterized in our paper. Second, so far the papers like Rakshit et al (2005) and Paul et al (2017) have imposed constraints to find unique Nash equilibrium in the access game. Our mechanism finds an unconstrained Nash equilibrium which is not only unique but also efficient. Third, Paul et al (2017) find unique transmission probabilities by imposing fairness conditions. But interior values of probabilities simply imply that some outcomes are definitely inefficient. In contrast, this paper marks an improvement over their result as we find a mechanism that implements only efficient outcome in Nash equilibrium.

The rest of the paper proceeds as follows: we introduce the basic theoretical model in the next section which is followed by the section where the mechanism is designed. The section following concludes.

2. The Model

Consider a two-period non-cooperative game with the following components: let the set of agents be $N = \{1, 2\}$. The strategy set of $i \in N$ is denoted by S_i, where $S_i := \{Transmit, Wait\}$. The payoff of i is denoted as u_i, where $u_i : \times_{\forall i \in N} S_i \rightarrow \mathbb{R}$. Each agent discounts the payoff obtained in second period at a constant rate. Let Δ_i be the random variable that represents agent $i's$ discount factor. Both Δ_i and $\Delta_j; j \in N \setminus \{i\}$ are independently and identically distributed on the interval $(0,1)$ according to the increasing distribution function *F*. *F* admits a continuous density $f \equiv F'$ and has full support. Assume that the true discount factors of $i, j \in N; i \neq j$ are δ_i and δ_j respectively. The discount factor of an agent is private information of that individual, i.e. i knows the realization δ_i but does not know δ_j and vice-versa.

Now we define the payoffs of each agent for different strategy profiles. If both agents simultaneously choose to *Transmit* in first period, then their efforts are wasted and none is able to transmit successfully. Here we assume that each gets a payoff of ‘zero’. Since, in this case, second period is the last period, so both desperately choose to transmit again and they end up getting the same i.e. zero. If i chooses to *Transmit* and j chooses to *Wait*, then i successfully transmits in the first period and obtains a payoff of ‘unity’. Hence j transmits successfully in the second period and obtains a payoff of ‘unity’. But a payoff of unity in second period is equivalent to a discounted payoff of δ_j in first period. Similar arguments yield that i gets a payoff of δ_i and j gets unity when the former chooses to *Wait* and the latter chooses *Transmit*. Finally, if both agents decide to *Wait* in the first period, then also it is an inefficient outcome and each obtains ‘zero’. So in second period both would frantically choose to *Transmit* and again each would end up getting ‘zero’.[2] This is shown in the following normal form payoff matrix.

[1] The renowned Vickrey-Clarke-Groves (VCG) mechanism established the existence of an incentive compatible and efficient economic mechanism for a general class of *static* problems where agents’ values were private and preferences were quasi-linear. More recently, Bergemann and Valimaki (2010) proposed an efficient *dynamic* mechanism which is not budget-balanced. It relies on the assumption of independent types of the agents. In contrast, Athey and Segal (2013) construct an efficient, budget-balanced *Bayesian incentive compatible mechanism* under the assumptions of “private values” and “independent types”. Our paper designs a *dominant strategy incentive compatible (strategyproof) mechanism* in a dynamic setting.

[2] The payoffs are normalized as zero and one, representing failure and success respectively. These are assumed without losing any generality and to keep the algebra in the paper tractable.

		Agent 2: Transmit	Agent 2: Wait
Agent1	Transmit	0,0	1, δ_2
	Wait	δ_1, 1	0, 0

Figure 1

Observe that this game has two pure strategy Nash equilibria viz. (Transmit, Wait) and (Wait, Transmit). Thus the concept of Nash equilibrium fails to predict a unique outcome. Moreover, an important question here is: who would transmit and who would wait. Obviously, that requires coordination between the agents. But it is beyond the scope of the environment of the game as it has purely non-cooperative setup. What follows is: unless some restrictions are imposed on the environment of the game, unique Nash equilibrium cannot be obtained.

In an almost similar setting with complete information, Paul and Paul (2017) used the 'fairness' condition to find unique transmission probabilities of the static game that they considered. But we find no mention regarding implementation of unique pure strategy equilibrium. So the objective of this paper is to find a mechanism that implements unique and efficient pure strategy equilibrium.

Definition 2.1 *An allocation rule* $f_i^e : (0,1)^2 \to S_i$ *is efficient iff* $\forall i \in N$

$$f_i^e \in \arg\max_{f_i \in S_i} \sum_i u_i(s_i, s_j);\, s_i \in S_i \text{ and } s_j \in S_j \ \forall j \in N \setminus \{i\}$$

The efficient allocation rule suggests that if $\delta_i > \delta_j$ then i should wait and transmit in second period and j must transmit in first period. The reason is simple. Higher the discount factor, higher is the patience of the agent and hence lower is the waiting cost. So an agent having the highest waiting cost should be served first.

Definition 2.2 *A mechanism M is a triplet* $\langle \delta_i, \delta_j, g_i \rangle$ *where* $i, j \in N; i \neq j$ *and* $g_i : (0,1)^2 \to S_i$, *implements the efficient pure strategy Nash equilibrium if*

$g_i = f_i^e$ *for all* $i \in N$ *and all* $\delta_i, \delta_j \in (0,1)$; *provided* g_i *is a Nash equilibrium outcome.*

Let us consider a mechanism say M_0, that asks each agent to announce their discount factors. In this situation it is important to see whether the agents have any incentive to report the truth.

Let the announcement vector of the agents i and j be $(\delta_i', \delta_j') \in (0,1)^2$. Then the expected payoff of i is: [3]

$$Eu_i = \delta_i \Pr(\delta_j' \le \delta_i') + 1.\Pr(\Delta_j = \delta_j' | \delta_j' \ge \delta_i') = \delta_i F(\delta_i') + \int_{\delta_i'}^{1} (1 - F(\delta_i')) f(t)\, dt \qquad (1)$$

From equation (1) we have the following proposition.

Proposition 1: *Truthful revelation of the discount factor by agent* $i \in N$ *is strategyproof only for a unique* $\delta_i = \delta_i^* \in (0,1)$.

Proof: After a bit of algebra, equation (1) reduces to:

$$Eu_i = \delta_i F(\delta_i') + (1 - F(\delta_i'))^2 \qquad (2)$$

If truth revelation has to be strategyproof i.e. dominant strategy incentive compatible for all $i \in N$ and all $\delta_i' \in (0,1)$ then we must have $\frac{\partial Eu_i}{\partial \delta_i'} = 0$ for all i and δ_i'.

[3] The integral in equation (1) is defined over the open interval $(0,1)$ and hence is improper in nature. The correct way of representing it is to write $\lim_{\varepsilon \to 0^+} \int_{\delta_i' + \varepsilon}^{1-\varepsilon} (1 - F(\delta_i')) f(t) dt$. But that would not alter the result. Hence we make slight abuse of the notation.

Partially differentiating both sides of equation (2) with respect to δ_i' we obtain:

$$\frac{\partial Eu_i}{\partial \delta_i'} = f\left(\delta_i'\right)\left[\delta_i - 2 + 2F\left(\delta_i'\right)\right] \tag{3}$$

Clearly, $f(.) > 0$. Now as $\delta_i' \to 0$, $F(.) \to 0$. Since $\delta_i \in (0,1)$, equation (3) reveals $\frac{\partial Eu_i}{\partial \delta_i'} < 0$. Again as $\delta_i' \to 1$, $F(.) \to 1$ and hence $\frac{\partial Eu_i}{\partial \delta_i'} > 0$. Thus there exists a unique $\delta_i' = \delta_i^* \in (0,1)$ where $\frac{\partial Eu_i}{\partial \delta_i'} = 0$. To state in another way, $\frac{\partial Eu_i}{\partial \delta_i'} \neq 0$ for all δ_i'. Hence the statement of the proposition follows.■

To understand proposition 1, let $\delta_i^* \in (0,1)$ be such that $\frac{\partial Eu_i}{\partial \delta_i'} = 0$. Then for $\delta_i \in (0, \delta_i^*)$, the dominant strategy of agent i is to under-report his discount factor. Conversely, for $\delta_i \in (\delta_i^*, 1)$, over-reporting is beneficial. The agent reports the truth only when his discount factor is δ_i^*; because in this situation manipulation does not provide any additional benefit. Thus, in a nutshell, the mechanism M_0 that asks each agent to announce their respective discount factors, fails to achieve efficiency as truth revelation is not dominant strategy for all types of both the agents. In the following section we describe an *ex ante* efficient strategyproof mechanism.

3. A Strategyproof Direct Revelation Mechanism

Definition 2.3 *An efficient and strategyproof mechanism* M_{SP} *is a tuple* $\left\langle \delta_i, \tau_i; \delta_j, \tau_j; g_i \right\rangle$ *where* $\tau_i : (0,1) \to \mathbb{R}_{++}$ *and* $g_i : (0,1)^2 \to S_i$ *for all* $i \in N$ *such that*

$g_i = f_i^e$ *and* $u_i\left(g_i\left(\delta_i, \delta_j\right), \tau_i\left(\delta_i\right); \delta_i\right) \geq u_i\left(g_i\left(\delta_i', \delta_j\right), \tau_i\left(\delta_i'\right); \delta_i\right)$ *for all* $\delta_i, \delta_i', \delta_j \in (0,1)$ *and* $\delta_i \neq \delta_i'$

Consider a transfer $\tau_i : (0,1) \to \mathbb{R}_{++}$ to agent i. The purpose of this transfer is to make truthful revelation of δ_i strategyproof for $i \in N$. We call this mechanism as $M_{SP} := \left\langle \delta_i, \tau_i\left(\delta_i\right); \delta_j, \tau_j\left(\delta_j\right); g_i \right\rangle$. Based on the announcement δ_i', the value of transfer $\tau_i\left(\delta_i'\right)$ is chosen. Then the expected payoff of agent i becomes:

$$Eu_i = \delta_i F\left(\delta_i'\right) + \int_{\delta_i'}^{1} \left(1 + \tau_i\left(\delta_i'\right)\right)\left(1 - F\left(\delta_i'\right)\right) f(t)\, dt \tag{4}$$

Equation (4) leads us to the next proposition.

Proposition 2: *A transfer* $\tau_i : (0,1) \to \mathbb{R}_{++}$ *makes truth revelation strategyproof for all* $i \in N$ *if;*

(I) *For all* $\delta_i \in (0,1)$, $\tau_i\left(\delta_i\right)$ *solves the equation*

$$1 + \tau_i\left(\delta_i\right) = \frac{1}{\left(1 - F\left(\delta_i\right)\right)^2}\left[1 - \delta_i F\left(\delta_i\right) - \int_{\delta_i}^{1} F(t)\, dt\right]$$

(II) *(i)* *For all* $\delta_i \in (0, \delta_i^*)$, $\tau_i\left(\delta_i\right)$ *is monotonically positively sloped with* $\inf \tau_i\left(\delta_i\right) = 0$, *and*

(ii) *For all* $\delta_i \in (\delta_i^*, 1)$, $\tau_i\left(\delta_i\right)$ *is monotonically negatively sloped with* $\sup \tau_i\left(\delta_i\right) = 0$, *provided* $\delta_i > 2\left(1 - F\left(\delta_i\right)\right)\left(1 + \tau_i\left(\delta_i\right)\right)$.

For all $\delta_i \in (\delta_i^*, 1)$ *and* $\delta_i < 2\left(1 - F\left(\delta_i\right)\right)\left(1 + \tau_i\left(\delta_i\right)\right)$, *there is no strategyproof mechanism.*

Proof: ***First part*** The mechanism M_{SP} would make truth revelation strategyproof if $\left.\frac{dEu_i}{d\delta_i'}\right|_{\delta_i'=\delta_i}=0$. So differentiating both sides of equation (4) with respect to δ_i' and then substituting $\delta_i'=\delta_i$ and setting it equal to zero we obtain:

$$\delta_i f(\delta_i)+\tau_i'(\delta_i)(1-F(\delta_i))^2-2(1+\tau_i(\delta_i))(1-F(\delta_i))f(\delta_i)=0 \qquad (5)$$

Re-arranging the terms, equation (5) reduces to:

$$\delta_i f(\delta_i)d\delta_i+d\left[(1+\tau_i(\delta_i))(1-F(\delta_i))^2\right]=0$$

Integrating both sides over the interval $[\delta_i,1)$ we arrive at the following:

$$1+\tau_i(\delta_i)=\frac{1}{(1-F(\delta_i))^2}\left[1-\delta_i F(\delta_i)-\int_{\delta_i}^{1}F(t)dt\right]$$

This completes the proof of first part of the proposition. For the second part we proceed in two steps.

Second part We rewrite equation (5) as:

$$\delta_i f(\delta_i)-2(1-F(\delta_i))f(\delta_i)-2\tau_i(\delta_i)(1-F(\delta_i))f(\delta_i)=-\tau_i'(\delta_i)(1-F(\delta_i))^2 \qquad (6)$$

Step I: Recall from equation (3) when $\delta_i\in(0,\delta_i*)$, $\delta_i f(\delta_i)-2(1-F(\delta_i))f(\delta_i)<0$. It implies that the left hand side of equation (6) is negative. To maintain equality we must have $\tau_i'(\delta_i)>0$ i.e. the transfer function is monotonically positively sloped. Remember in this situation we had $\frac{dEu_i}{d\delta_i}<0$. Thus agent i was better off in under-reporting. Hence an agent with true discount factor close enough to zero i.e. extremely impatient agent, is reporting the truth; since he has no further scope to under-report. This agent does not require any incentive for truthful revelation. In other words, the mechanism designer should choose $\lim_{\delta_i\to 0}\tau_i=0$.

Step II: Now consider the situation where $\frac{dEu_i}{d\delta_i}>0$. This implies from equation (3) $\delta_i f(\delta_i)-2(1-F(\delta_i))f(\delta_i)>0$.

From equation (6) we have two possibilities.

Case I: $\delta_i>2(1-F(\delta_i))(1+\tau_i(\delta_i))$

This time the left hand side of equation (6) is positive. So we have $\tau_i'(\delta_i)<0$. Observe in this situation an agent over-reports. Thus an agent with true $\delta_i\to 1$ is already reporting the truth and does not need any incentive in the form of transfer. This means $\lim_{\delta_i\to 1}\tau_i=0$.

Case II: $\delta_i<2(1-F(\delta_i))(1+\tau_i(\delta_i))$

Then left hand side of equation (6) is negative. Hence we have $\tau_i'(\delta_i)>0$. Observe in this situation an agent over-reports. Following the same argument as case I we get $\lim_{\delta_i\to 1}\tau_i=0$. Now a monotonically positively sloped transfer function having zero as supremum of the co-domain set is consistent only with $\tau_i<0$. But by definition 2.3 we have $\tau_i>0$. Thus we have two mutually inconsistent situations. This, basically, confirms that there is no such inter-consistent mechanism here.

This completes the proof of second part of the proposition.■

Consider the situation where an agent has incentive to under-report his discount factor. Here the magnitude of transfer received by the agent increases with the true discount factor. The reason is obvious. Higher the true value, higher is the deviation from the under-reported value and hence, higher should be the incentive provided for truthful revelation. But, an agent who has extremely low discount factor, has no scope to under-report. Thus he would be reporting truth only. So incentivizing truth telling becomes redundant for such agent. For this type of individual, the transfer amount vanishes in the limit. Likewise we can explain the intuition behind the mechanism described for the situation of over-reporting. The mechanism described in proposition 2 is depicted in figure 2 below.

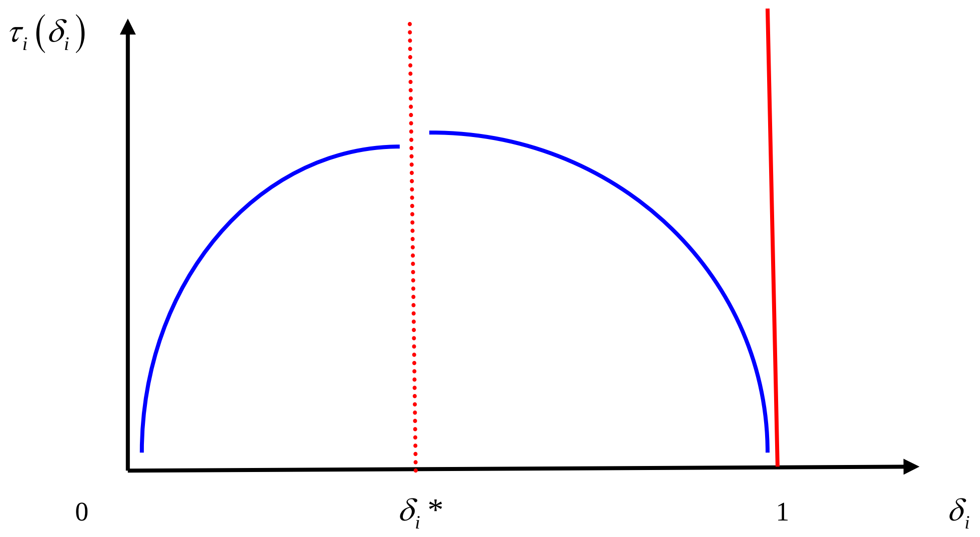

Figure 2: A strategyproof transfer mechanism with $\delta_i > 2\left(1 - F(\delta_i)\right)\left(1 + \tau_i(\delta_i)\right)$

In chronological sense, the mechanism designer announces the mechanism before asking the agents to report their respective discount factors. The argument is: given that the mechanism is *ex ante* strategyproof, it would induce the agents to report the truth only. Now it is interesting to see whether the mechanism is individually rational in *ex post* situation. We explore this by defining individual rationality in the following section.

3.1 Ex post Individual Rationality

Definition 2.4 *An efficient and strategyproof mechanism* M_{SP} *is ex post individually rational* $\forall i \in N$ *iff*

$$u_i\left(f_i^e(\delta_i, \delta_j), \tau_i(\delta_i); \delta_i\right) \geq 0$$

In other words, individual rationality warrants that participation in the mechanism is beneficial, at least weakly.

Assume that given the strategyproof mechanism, the agents report their true discount factors i.e. δ_i and δ_j respectively. Without loss of generality, let $\delta_i < \delta_j$. So agent i is relatively impatient and has higher waiting cost. Thus the efficient outcome is agent i *Transmits* in period 1 while agent j *Waits* and transmits in period 2. Accordingly, the payoffs of the respective agents in ex post equilibrium are:

$$u_i{*} = 1 + \tau_i(\delta_i) \tag{7}$$

$$u_j{*} = \delta_j + \tau_j(\delta_j) \tag{8}$$

Proposition 2 points out that the efficient and strategyproof transfer mechanism has $\tau_i, \tau_j > 0$ for all $\delta_i, \delta_j \in (0,1)$. So it is clearly evident from equations (7) and (8) that $u_i{*}, u_j{*} > 0$ i.e. ex post individual rationality is satisfied for both the agents. We note this finding in our next proposition.

Proposition 3: *The ex ante efficient and strategyproof mechanism designed for a distributed system achieves ex post individual rationality for all possible types of the agents.*

Proof: See the discussion above.■

4. Conclusion

This paper considers the *Access Game* in a dynamic setting of two time periods. There is a medium which can be accessed by one user at a time and there are two users or agents. The agents are impatient and have waiting costs, which are private information for each agent. So if both the agents attempt to transmit in period I, they end up in failure. Again they desperately try to repeat the same act in period II which is the last period, and yet again they fail. On the other hand, if each decides to wait for the other in period I and transmit in period II, then also the outcome is a failure. Both agents transmit successfully if one transmits in period I and the other does so in period II. But the situation becomes complex as there is no room for the agents to coordinate. In this context, the most impatient agent i.e. the one with the highest waiting cost transmitting in period I followed by the relatively patient agent in period II, is the most efficient Nash equilibrium outcome. Our goal in this paper is to design a mechanism that makes the efficient outcome Nash implementable.

We begin with a simple mechanism that asks each agent to announce his waiting cost. We find truth revelation is not always strategyproof. So we characterize an *ex ante* mechanism that makes truth revelation strategyproof. We further go on to show that such mechanism is also *ex post* individually rational.

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