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An extension of API 653 Appendix B to unequally spaced, densely sampled data.

PEMY Consulting and Novlum Inc, Mar 15, 2019[1]

Summary

API 653, Annex B is the standard for evaluating tank bottom settlement. Section B2 requires shell settlement measurements (Z) with "maximum spacing of 32 ft around the circumference … (and) … there must be at least 4 equally spaced diametrical measurement lines[2]". It stipulates that out of plane settlement values (U) are computed as residuals from the best-fitting phase-shifted cosine curve. Tensile stress at the top of the shell above each measurement point is estimated by the negative second difference (S) of the U-series and compared to an upper bound (Smax) derived from Euler-Bernoulli beam theory.

Annex B warns against computing S-values at spacings less than 32 feet and suggests computing running 32-foot second differences when settlement measurements are less than 32-feet apart.

This suggestion is statistically inefficient in that it misses the opportunity to reduce the standard error of S-values by incorporating more data. Furthermore, it provides no way to deal with irregular spacing of Z measurements.

We have developed a statistical approach and software to efficiently estimate and hence, limit, tensile stress from a dense, somewhat irregularly spaced series of settlement measurements. Our method involves fitting a harmonic (sine-cosine) series to the deflection values (U). To avoid the overfitting problem recognized in Annex B, we do not allow components with wavelengths shorter than the height of the tank and we use the BIC criterion to prune out components that do not improve the model fit.

Finally, we approximate tensile stress as a multiple of the negative second derivative of the fitted model and compare it to the upper bound proposed in API 653 Annex B.

The API 653 standard for shell settlement.

According to API 653, section B.3.2.1, out of plane deflection, S, is permissible if for all i,

$$S_i = \left(U_i - \frac{U_{i+1}+U_{i-1}}{2}\right) \le \frac{L^2}{2} \cdot \frac{11 \cdot Y}{E \cdot H} \qquad (1.1)$$

Where $(U_1, \dots, U_n)$are out of plane settlement values (feet) at points, spaced about 32 feet apart around the tank perimeter. Places where elevations were measured are called "stations." Stations are identified by their arc length from a point of origin, as in Figure 1.

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[1] Novlum Inc., Calgary, Canada, provided actual 3D laser scan data for multiple tanks and jointly worked with PEMY Consulting to produce the results in this paper.

[2] We interpret this to mean that the number of measurement points is at least $n = \max\left(2 \cdot ceil\left(\pi R / 32\right), 4\right)$, where R is the tank's radius.

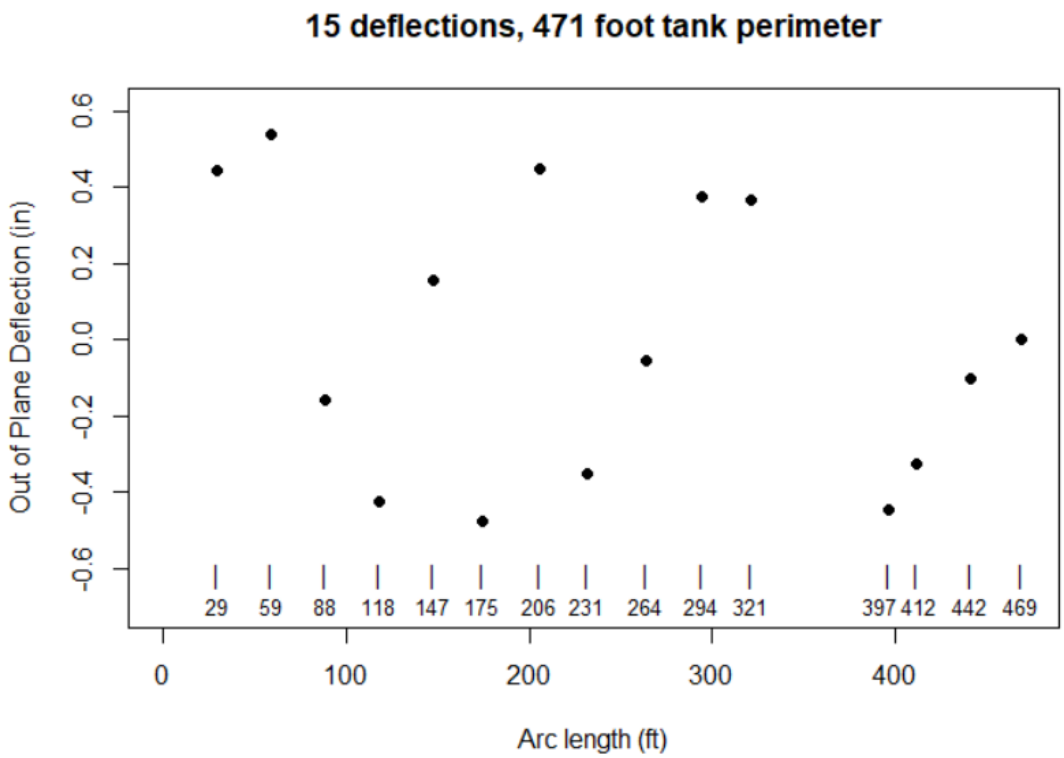

Figure 1. Out of plane deflections at 15 stations.

Dividing equation (1.1) L^2, we get

$$-1 \cdot \left[\frac{\frac{(U_{i+1}-U_i)}{L} - \frac{(U_i-U_{i-1})}{L}}{2 \cdot L} \right] \leq \frac{11 \cdot Y}{2 \cdot E \cdot H} \tag{1.2}$$

When spacing between stations is not uniform, equation (1.2) should be replaced by

$$-1 \cdot \left[\frac{\frac{(U_{i+1}-U_i)}{(l_{i+1}-l_i)} - \frac{(U_i-U_{i-1})}{(l_i-l_{i-1})}}{(l_{i+1}-l_{i-1})} \right] \leq \frac{11 \cdot Y}{2 \cdot E \cdot H} \tag{1.3}$$

where $(l_1, l_2, \dots, l_n)$ are positions of the n stations around the tank perimeter.

The expression in brackets is a second divided difference[3]. If deflections $(U_1, \dots, U_n)$ are thought of as values of a continuous deflection function, $U(l)$, then one common notation for the expression in brackets is

$$U[l_{i-1}, l_i, l_{i+1}] = \left[\frac{\frac{(U_{i+1}-U_i)}{(l_{i+1}-l_i)} - \frac{(U_i-U_{i-1})}{(l_i-l_{i-1})}}{(l_{i+1}-l_{i-1})} \right] \tag{1.4}$$

[3] Wikipedia contributors. (2019, February 19). Divided differences. In Wikipedia, The Free Encyclopedia. Retrieved 13:50, March 1, 2019, from https://en.wikipedia.org/w/index.php?title=Divided_differences&oldid=884031491

The mean value theorem for divided differences[4] says that,

$$U[l_{i-1}, l_i, l_{i+1}] = \frac{U''(l_*)}{2!} \tag{1.5}$$

where $U''(l_*)$ is the second derivative of $U(l)$ at a point l_* located somewhere between l_{i-1} and l_{i+1}.

So, API 653, section B.3.2.1 requires that at every point around the perimeter of the thank,

$$-U''(l) \leq \frac{11 \cdot Y}{E \cdot H} \tag{1.6}$$

The problem with using the second divided difference to estimate the second derivative is that out of plane deflection measurements are not exact,

$$U_i = U(l_i) + e_i \tag{1.7}$$

The noise term, e_i is a combination of measurement error and local roughness due to seams, dents, weld irregularities and the like.

Substituting equation (1.7) into the bracketed expression in equation (1.2) gives us the standard deviation for equally spaced measurements,

$$sd\left(\frac{\frac{(U_{i+1}-U_i)}{L} - \frac{(U_i-U_{i-1})}{L}}{2 \cdot L}\right) = sd\left(\frac{e_{i+1}}{2L^2} - \frac{e_i}{L^2} + \frac{e_{i-1}}{2L^2}\right) = \sigma_e \cdot \frac{\sqrt{1.5}}{L^2} \tag{1.8}$$

We'd expect roughness RMS amplitude to be on the order of a quarter inch, so, accurate risk assessment requires that this standard deviation should be an order of magnitude or two below the upper bound,

$$\frac{\sqrt{1.5}\sigma_e}{L^2} \ll \frac{11 \cdot Y}{2 \cdot E \cdot H} \tag{1.9}$$

API 653, section B.3.2.1 specifies $L \approx 32 feet$ so the implication is that the framers of API 653 assumed that

$$\sigma_e \ll \frac{32^2}{\sqrt{1.5}} \cdot \frac{11 \cdot 36{,}000}{2 \cdot 29{,}000{,}000 \cdot H} \cong \frac{5.7}{H} ft \tag{1.10}$$

For a 50-foot high tank, the right side of equation (1.10) amounts to about 1.5 inches. Conventional wisdom has it that the roughness component of σ_e is on the order of 0.25 to 0.5 inches RMS, so spacing observations at 32 feet makes the upper bound 3 to 6 sigma above noise and reduces the false positive rate to about 1 in 740, provided measurement error is much smaller than RMS roughness.

[4] Wikipedia contributors. (2015, March 13). Mean value theorem (divided differences). In Wikipedia, The Free Encyclopedia. Retrieved 15:54, March 1, 2019, from https://en.wikipedia.org/w/index.php?title=Mean_value_theorem_(divided_differences)&oldid=651202397

Densely Spaced Observations

Although API 653 stipulates that point spacing, L, should be as close to 32 feet as possible, it also states that denser spacing is better for estimating rigid tilt, but suggest throwing away data when testing for excessive out of plane deflections,

> *When determining the optimum cosine curve described in B.2.2.4 e), taking additional measurements around the shell will result in a more accurate cosine curve fit. However, using all of the measurement points in the equation shown in B.3.2.1 will result in very small allowable out-of-plane settlements, Smax since the arc length L between measurement points is small. It is acceptable to use all measurement points to develop the optimum cosine curve, but only use a subset of these points spaced no further than 32 ft (8 minimum) when calculating Si and Smax. The points used must include the points furthest from the optimum cosine curve. For example, if 8 points are required, but 16 measurements are taken, and the arc length between measurement points is only 15 ft, calculate the optimum cosine curve using all 16 points, but use only 8 points to calculate Si. The equations in Figure B.3* (our equation (1.1)) *would be revised to read:*

$$S_i = U_i - \frac{U_{i+2} + U_{i-2}}{2} \tag{2.1}$$

Such inefficient use of data is not best statistical practice and we propose taking numerical second derivatives of data that has been smoothed to filter out high-frequency noise due to roughness and measurement error.

Smoothing to reduce standard error

Broadly, there are two ways to estimate second derivatives from noisy data: either fit a global model and analytically calculate the second derivative of that model or take the second derivative of a moving smoother. Here we describe our R application to fit parsimonious harmonic models

A kth order harmonic model of out of plane deflections is,

$$\widehat{U}(l) = \sum_{k=2}^{K} \beta_{k,c} \cdot cos\left(k \cdot \frac{l}{R}\right) + \beta_{k,s} \cdot sin\left(k \cdot \frac{l}{R}\right) \tag{2.2}$$

where l is arc length in feet from the point of origin and $R$is the radius of the tank.

The maximum order, K, of the model fit (maximum number of cycles) is automatically limited to avoid fitting noise. First, the program sets a maximum frequency limit depending on point spacing and the ratio of tank height to perimeter; the maximum quarter-cycle wavelength is,

$$min.Q\,.cycle = max\begin{cases} \min\limits_{1\le i\le n}(l_i - l_{i-1}) \\ H/4 \end{cases} \tag{2.3}$$

where $l_0 = l_n - \text{tank.perimeter.feet}$.

The maximum number of cycles is,

$$K = \text{tank.perimeter.feet}/4 \cdot min.Q\,.cycle \tag{2.4}$$

(Note that under the API 653, section B.3.2.1 protocol, $min. Q . cycle \cong 32 feet$and the maximum number of cycles is half the number of observations.)

Second, individual frequencies (k's) are retained or removed, using the BIC criterion, to select the most parsimonious model[5]. In the most parsimonious model some of the beta's in equation (2.5) will be a-priori zero. Selection of the final model of the data in Figure 1 is detailed in Table 1. The most parsimonious model (with highest adjusted R^2 and lowest BIC) was found at step 10. Sine and cosine terms in the model are all variables selected up to and including that step.

The second derivative of the fitted model is,

$$\widehat{U}''(l) = -\left[\sum_{k=1}^{K} \beta_{k,c} \cdot k^2 \cdot cos\left(k \cdot \frac{l}{R}\right) + \beta_{k,s} \cdot k^2 \cdot sin\left(k \cdot \frac{l}{R}\right)\right] \cdot R^{-2} \quad (2.5)$$

Equation (1.6) says that the maximum permitted value of the negative second derivative is,

$$-\widehat{U}''(l) \le \frac{11 \cdot Y}{E \cdot H} \quad (2.6)$$

Selection Step	Adjusted R^2	BIC	Variable Added	Included in model
1	0.274	-3.12992	sin(2· ℓ/R)	yes
2	0.409317	-4.62754	sin(6· ℓ/R)	yes
3	0.563972	-7.67373	sin(3· ℓ/R)	yes
4	0.592128	-7.27214	cos(3· ℓ/R)	yes
5	0.638085	-7.78691	cos(7· ℓ/R)	yes
6	0.685911	-8.78525	sin(7· ℓ/R)	yes
7	0.74503	-10.9719	cos(6· ℓ/R)	yes
8	0.781534	-12.5845	cos(5· ℓ/R)	yes
9	0.814369	-14.6318	cos(4· ℓ/R)	yes
10	0.834681	-16.3969	sin(4· ℓ/R)	yes
11	0.811285	-15.0505	cos(2· ℓ/R)	no
12	0.76295	-13.2373	sin(5· ℓ/R)	no

Table 1. Selection of most parsimonious model.

The parsimonious fit and its second derivative are plotted in Figure 2 with the maximum allowable second derivative indicated by the horizontal dashed line. Since the second derivative (solid red) stays under the red dotted line, the tank passes the test.

[5] Joseph B. KADANE and Nicole A. LAZAR, **Methods and Criteria for Model Selection,** *Journal of the American Statistical Association*, March 2004, Vol. 99, No. 465, Review Article.
https://pdfs.semanticscholar.org/9e41/c21329fdad0887460b3ca312d017ebad8763.pdf

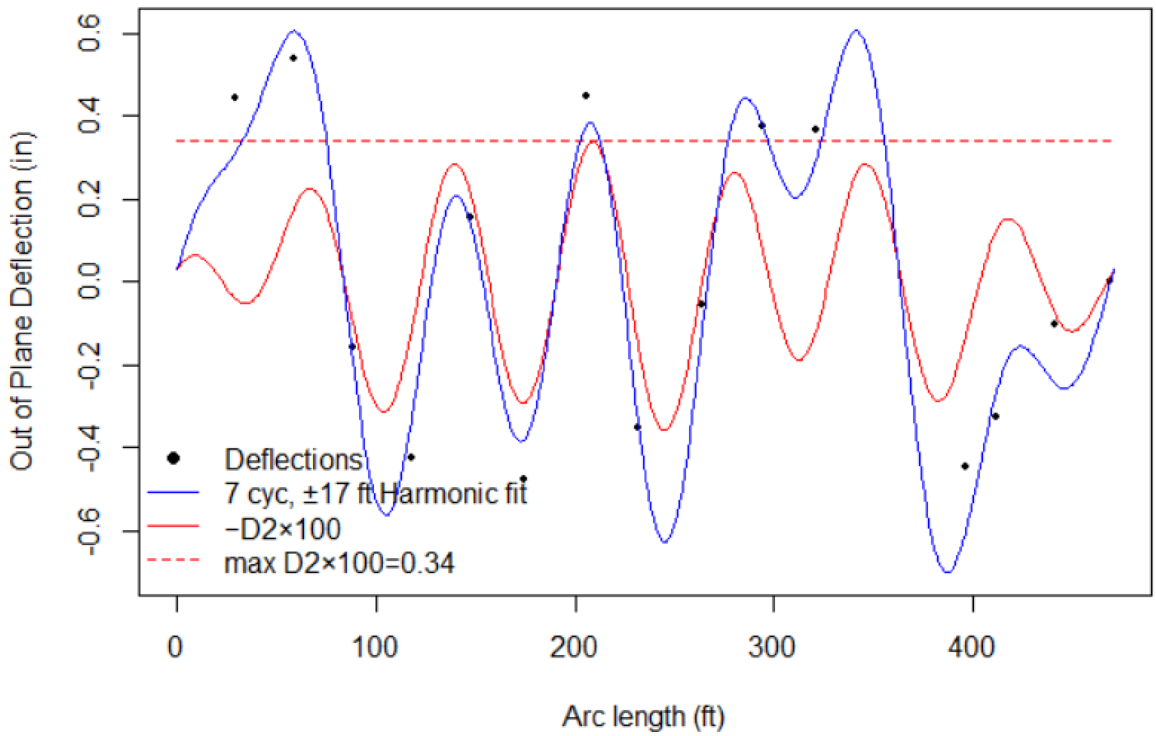

Figure 2. Most parsimonious harmonic model of out of plane deflections.

The tank barely passes the test in equation (2.6), having maximum second derivative just tangent to the limit line in Figure 2. However, it slightly exceeds the API 653, section B.3.2.1 limit in equation (1.1), presumably due to unsmoothed, noise.

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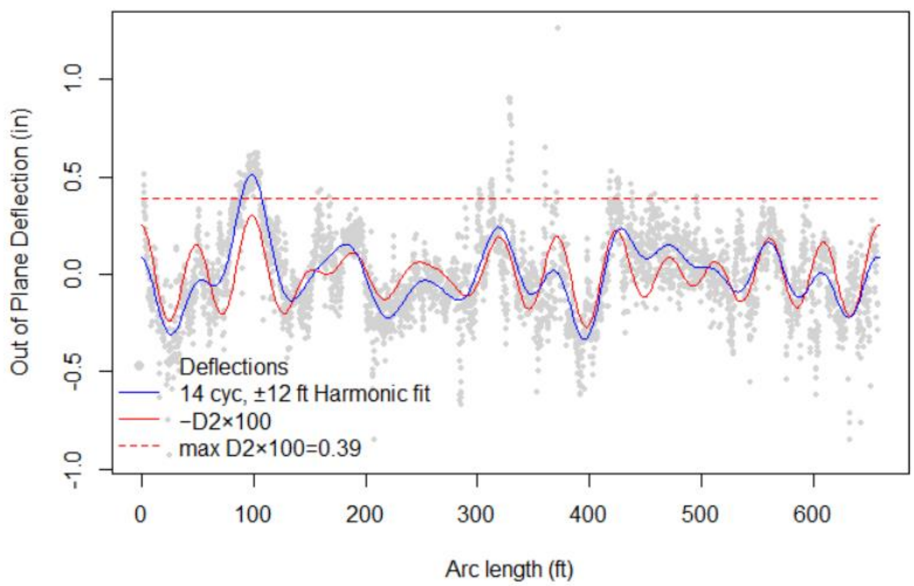
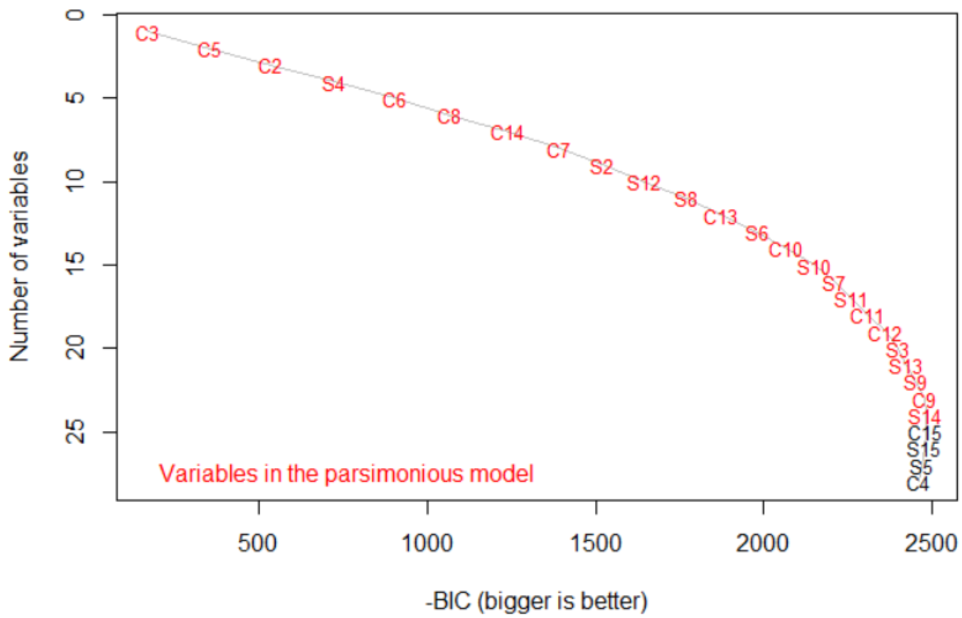
Examples of the harmonic smooth method applied to densely spaced observations.

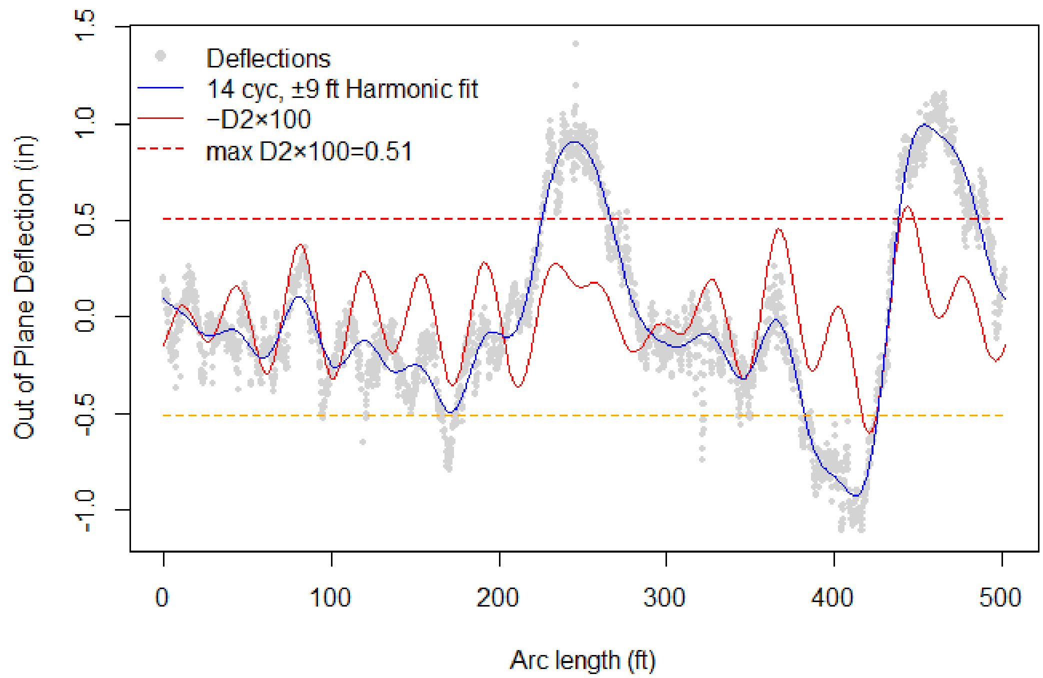
This tank had diameter 220 feet and height 42 feet. It exhibits out of plane deflections of about ± 0.5 in. The solid blue curve is the best harmonic fit. The shortest quarter-wavelength is 12 feet, analogous to the 32-foot limit in API 653 Annex B. The red curve is the negative second derivative, analogous to $\frac{S_i}{L^2}$in Annex B, and the dotted red line is the upper stress limit, analogous to $\frac{S_{MAX}}{L^2}$ in Annex B. Maximum stress, at about 100 feet from the point of origin does not exceed the limit. Note that the red curve and line are blown up by a factor of 100 for visibility the blow-up factor is the nearest power of 10 that makes the visual range of the blue and red curves comparable.



This model selection display shows how the 14-cycle parsimonious model was selected.



Our R program allows the user to elect to plot a “lower bound” (the mirror image of the API 653 upper bound). The tank in this plot is has excessive tensile stress at the top of the shell at about 450 feet from the point of origin. The second derivative (in red) crosses the lower bound as well, suggesting that there is potentially excess tensile stress at the bottom of the shell at about 420 feet. Although API 653 is silent on negative deflections, it is prudent to report them.



This tank stays well within spec