Class – XI
Mathematics

PERMUTATIONS AND COMBINATIONS

1. How many 3-digit numbers can be formed from the digits 1,2,3,4 and 5 assuming that
   i. Repetition of the digits is allowed?
   ii. Repetition of the digits is not allowed?

Ans.

i. There will be as there are ways of filling 3 vacant places

☐ ☐ ☐ in succession by the given five digits. In this case, repetition of digits is allowed. Therefore, the units place can be filled in by any of the given five digits.

Similarly, tens and hundreds digits can be filled in by any of the given five digits.

Thus, by the multiplication principal, the number of ways in which three-digit numbers can be formed from the given digits is $5\times5\times5=125$.

   iii. In this case, repetition of digits is not allowed. Here, if units place is filled in first, then it can be filled in by any of the given five digits.
   Therefore, the number of ways of filling the units place of the three-digit number is 5.
   Then, the tens place can be filled with any of the remaining four digits and the hundreds place can be filled with any of the remaining three digits.
   Thus, by the multiplication principal, the number of ways in which three-digit numbers can be formed without repeating the given digits is $5\times 4 \times3=60$.
2. The coin is tossed 3 times and the out comes are recorded. How many possible outcomes are there?
   Ans. When a coin is tossed once, the number of outcomes is 2(Head and tail)i.e., in each throw, the number of ways of showing a different face is 2.
   Thus, by the multiplication principal, the required number of possible outcomes is $2\times2\times2=8$.
   HOME WORK:
   NCERT BOOK:EX-**7.1:** 2,6
3. 8!

Ans.

$8! =1\times2\times3\times4\times5\times6\times7\times8$
$=40320$

4. Is $3!+4!=7!$?
   Ans. $3!=1\times2\times3=6$
   $4!= 1\times2\times3\times4=24$
   $\therefore 3!+4!=6+24=30$
   $7! =1\times2\times3\times4\times5\times6\times7=5040$
   $\therefore 3!+4!\neq7!$
5. Evaluate $\frac{n!}{(n-r)!}$, when n=6, r=2.

Ans. $\frac{n!}{(n-r)!}$=$\frac{6!}{(6-2)!}$=$\frac{6!}{4!}$=$\frac{6\times5\times4!}{4!}$=30

HOME WORK:

NCERT BOOK: EX-**7.2:** 1(ii),3,4,5(ii).

6. How many 3-digit numbers can be formed by using the digits 1 to 9 if no digit is repeated?

Ans. 3-digit numbers have to be formed using the digits 1 to 9.

Here, the order of the digits matters.

Therefore, there will be as many 3-digit numbers as there are permutations of 9 different digits taken 3 at a time.

Therefore, required number of 3-digits numbers

=9_{P_3}=$\frac{9!}{(9-3)!}$=$\frac{9!}{6!}$

=$\frac{9\times8\times7\times6!}{6!}$=504

7. Find n if $n-1_{P_3}$:n_{P_4}=1:9

Ans. $n-1_{P_3}$:n_{P_4}=1:9

$\Rightarrow \frac{n-1_{P_3}}{n_{P_4}}$=$\frac{1}{9}$

$\Rightarrow \frac{\frac{(n-1)!}{(n-1-3)!}}{\frac{n!}{(n-4)!}}$=$\frac{1}{9}$

$\Rightarrow \frac{(n-1)!}{(n-4)!}\times\frac{(n-4)!}{n!}$=$\frac{1}{9}$

$\Rightarrow \frac{(n-1)!}{n\times(n-1)!}$=$\frac{1}{9}$

$\Rightarrow \frac{1}{n}$=$\frac{1}{9}$

$\therefore n$ =9

8. How many words, with or without meaning , can be formed using all the letters of the word EQUATION, using each letter exactly once?

Ans. There are 8 different letters in the word EQUATION.

Therefore, the numbers of words that can be formed using all the letters of the word EQUATION, , using each letter exactly once, is the number of permutations of 8 different objects taken 8 at a time, which is 8_{P_8}=8! .

Thus, required number of words that can be formed =8! =40320

HOME WORK:

NCERT BOOK:EX-**7.3**: 2,7,9,11 & Theorem 6.

9. Determine n if $2n_{C_3}$:n_{C_3}=12:1

Ans. $\frac{2n_{C_3}}{n_{C_3}}=\frac{12}{1}$

$\Rightarrow\frac{(2n)!}{3!(2n-3)!}\times\frac{3!(n-3)!}{n!}=\frac{12}{1}$

$\Rightarrow\frac{(2n)(2n-1)(2n-2)(2n-3)!}{(2n-3)!}\times\frac{(n-3)!}{n(n-1)(n-2)(n-3)!}=12$

$\Rightarrow\frac{2(2n-1)(2n-2)}{(n-1)(n-2)}=12$

$\Rightarrow\frac{4(2n-1)(n-1)}{(n-1)(n-2)}=12$

$\Rightarrow\frac{(2n-1)}{(n-2)}=3$

⇒ 2n-1=3(n-2)

⇒ 2n-1=3n-6

⇒ 3n-2n=6-1

⇒ n=5

10. Determine the number of 5 card combinations out of deck of 52 cards if there is exactly one ace in each combination.

Ans. In a deck of 52 cards, there are 4 aces. A combination of 5 cards have to be made in which there is exactly one ace.

Then, one ace can be selected in 4_{C_1} ways and the remaining 4 card can be selected out of the 48 cards in 48_{C_4}ways.

Thus, by multiplication principle, required number of 5 card combinations

$=48_{C_4}\times4_{C_1}$

$=\frac{48!}{4!44!}\times\frac{4!}{1!3!}$

$=\frac{48\times47\times46\times45}{4\times3\times2\times1}\times4$

=778320

11. How many chords can be drawn through 21 points on a circle?

Ans. For drawing one chord on a circle, only 2 points are required.

To know the number of chords that can be drawn through the given 21 points on a circle, the number of combinations have to be counted.

Therefore, there will be as many chords as there are combinations of 21 points taken 2 at a time.

Thus, required number of chords=$21_{C_2}=\frac{21!}{2!(21-2)!}=\frac{21!}{2!19!}=\frac{21\times20}{2}=210$.

HOME WORK:

NCERT BOOK:EX-**7.4:** 1,2(ii),5,7,8,9