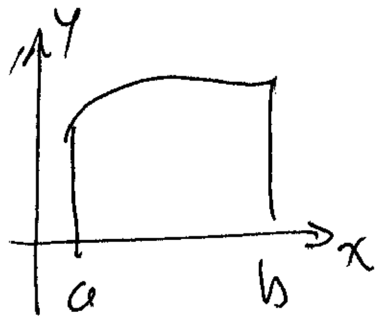
Math 1496 - Calc 1

Last class we introduced the definite integral



$$A = \int_a^b f(x)\,dx$$

Properties

(1) $$\int_a^b k\, f(x)\,dx = k \int_a^b f(x)\,dx$$

(2) $$\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

(3) $$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

if $a \le c \le b$

(4) $$\int_a^b f(x)\,dx = - \int_b^a f(x)\,dx$$

So Calculate

$$\int_1^4 (4x - x^2)\,dx$$

given $\int_1^4 x\,dx = \frac{15}{2}$ & $\int_1^4 x^2\,dx = 21$

From our properties

$$\int_1^4 (4x - x^2)\,dx = 4\int_1^4 x\,dx - \int_1^4 x^2\,dx$$

$$= 4\left(\frac{15}{2}\right) - 21$$

$$= 30 - 21 = 9 \qquad \text{(pretty easy)}$$

ex Find $\displaystyle\int_{-1}^{1} |x|\,dx$

given $\displaystyle\int_{-1}^{0} x\,dx = -\frac{1}{2}$ & $\displaystyle\int_0^1 x\,dx = 1/2$

so $$\int_{-1}^{1} |x|\,dx = \int_{-1}^{0} |x|\,dx + \int_0^1 |x|\,dx$$

$$= \int_{-1}^{0} -x\,dx + \int_0^1 x\,dx$$

$$= -\left(-\frac{1}{2}\right) + \frac{1}{2} = 1$$

But we needed formula's for

$\displaystyle\int_0^1 x\,dx$ & $\displaystyle\int_1^4 x\,dx$ & $\displaystyle\int_1^4 x^2\,dx$?

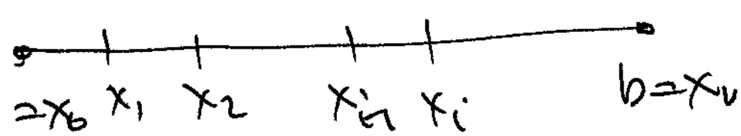
Fundemental Th^m of Calculus

If $f(x)$ is a cont^s function on $[a,b]$
and $F(x)$ is the anti derivative $(F'(x)=f(x))$

then $$\int_a^b f(x)\,dx = F(b)-F(a)$$

Proof we subdivide interval $[a,b]$ into n

pieces



So $a=x_0<x_1<x_2 \cdots <x_{i-1}<x_i \cdots <x_n=b$

the Rieman Sum is

$$\lim_{\substack{n\to\infty \\ \Delta x_i\to 0}} \sum_{i=1}^{n} f(c_i)\Delta x_i = \int_a^b f(x)\,dx$$

From the MVT for $F(x)$ on $[x_{i-1},x_i]$ there exists
a c_i in (x_{i-1},x_i) such that

$$\frac{F(x_i)-F(x_{i-1})}{x_i-x_{i-1}} = F'(c_i) = f(c_i)$$

so $f(c_i)\Delta x_i = F(x_i) - F(x_{i-1})$

so $\int_a^b f(x)\,dx = \lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} F(x_i) - F(x_{i-1})$

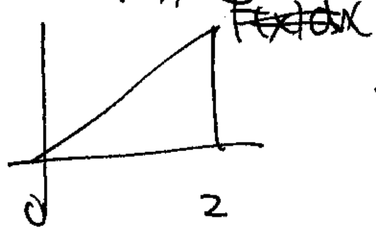
$= \lim\limits_{n\to\infty} [F(x_1)-F(x_0)] + [F(x_2)-F(x_1)] + \dots [F(x_n)-F(x_{n-1})]$

$= \lim\limits_{n\to\infty} F(x_n) - F(x_0) = F(b) - F(a)$

so $\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$

$f(x) = x$

ex



$A = \int_0^2 x\,dx = \frac{1}{2}(2)(2) = 2$

using fund. Thm

$A = \int_0^2 x\,dx = \frac{x^2}{2}\Big|_0^2 = \frac{2^2}{2} - \frac{0^2}{2} = 2$ same

Ex $\int_1^4 (4x - x^2)\,dx = 4\int_1^4 x\,dx - \int_1^4 x^2\,dx$

$= 4\left.\frac{x^2}{2}\right|_1^4 - \left.\frac{x^3}{3}\right|_1^4$

$= \cancel{4}2(4^2-1) - \frac{4^3-1}{3} = 2\cdot 15 - \frac{64-1}{3}$ $\qquad \frac{63}{3} = 21$

$= 30 - 21$

$= \cancel{2}9$

ex pg 328 # 14

$\int_{-2}^{-1} \left(u - \frac{1}{u^2}\right) du$

$\int_{-2}^{-1} (u - u^{-2})\,du = \left.\frac{u^2}{2} - \frac{u^{-1}}{-1}\right|_{-2}^{-1}$

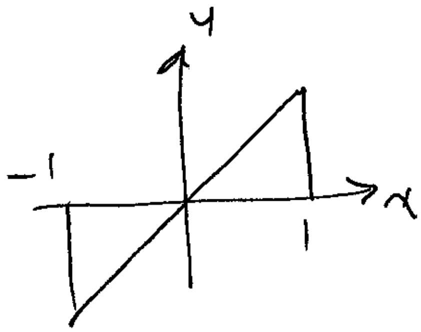
$= \left.\frac{u^2}{2} + \frac{1}{u}\right|_{-2}^{-1}$

$= \left(\frac{(-1)^2}{2} + \frac{1}{-1}\right) - \left(\frac{-2^2}{2} + \frac{1}{-2}\right)$

$= \frac{1}{2} - 1 - 2 + \frac{1}{2} = -2$

Ex Find the area between $y = x$ & the y axis on $[-1, 1]$

so $A = \int_{-1}^{1} x\,dx = \frac{x^2}{2}\Big|_{-1}^{1} = 0$???



Problem

$y = x \leq 0$ on $[-1, 0]$

$y = x \geq 0$ on $[0, 1]$

$\int_{-1}^{0} x\,dx + \int_{0}^{1} x\,dx$ cancel each other.

So what we want

$$-\int_{-1}^{0} x\,dx + \int_{0}^{1} x\,dx$$

$$-\frac{x^2}{2}\Big|_{-1}^{0} + \frac{x^2}{2}\Big|_{0}^{1} = -\left(0 - \frac{1}{2}\right) + \frac{1}{2} - 0$$

$$= \frac{1}{2} + \frac{1}{2} = 1 \checkmark$$