Portfolio Size and Investment Risk: Empirical Evidence from the NYSE and NASDAQ Stock Markets

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Abstract

In the studies on risk diversification, the investment risk is usually measured in terms of standard deviation of the investment returns across time. Nonetheless, the investors with specific investment goals and/or obligations are concerned with the expected investment value at the end of the investment horizon and the dispersion of this terminal value rather than the "time series standard deviation" which measures the ups and downs of stock performance during the holding period. Radcliffe (1994) proposed "terminal wealth standard deviation" to measure the dispersion of the investment value at the end of the investment horizon, which is of utmost importance to these investors. Unfortunately, the use of this risk measure in financial research is somewhat limited. The objective of this study is to provide some empirical evidence on the relationship between portfolio size and the investors' "terminal wealth dispersion" from the investments in the U.S. stock markets.

The need for diversification is evident, as the stock markets have become increasingly volatile in recent years. Finance literature generally notes that most of the diversifiable risk may be eliminated while holding a portfolio with a relatively small number of stocks. Evans and Archer (1968) reported that only 10 stocks are required to form a well-diversified portfolio. Tole (1982) suggested that 25 to 40 brokerage firms recommended stocks might achieve a satisfactory level of diversification. Statman (1987) found that 30 to 40 randomly chosen stocks constitute a well-diversified portfolio. In these studies, the investment risk is measured in terms of the standard deviation of the investment returns over time. Nonetheless, institutional investors, insurance companies, pension funds, and individual investors with specific investment goals and/or obligations are concerned with the expected investment value at the end of the investment horizon and the dispersion of this terminal wealth rather than the ups and downs of stock performance during the holding periods. Radcliffe (1994) proposed "terminal wealth standard deviation" instead of "time series standard deviation" as a more appropriate risk measure for these investors. In essence, this proposed risk measure shows the dispersion of the investment values at the end of the investment horizon. O'Neal (1997) used "terminal wealth standard deviation" to measure the investment risk, and concluded that the risk may be reduced significantly by holding more than one mutual fund. Lee and Byrne (2002) investigated the "terminal wealth dispersion" of the real estate portfolios in the U.K. market, and found that a value-weighted portfolio contains greater "terminal wealth standard deviation" as compared to an equal-weighted portfolio of the same size. Although the "terminal wealth dispersion" is of extreme concern to the investors with fixed future investment goals and/or obligations, the use of this risk measure in financial research is somewhat limited. The objective of this study is to provide some empirical evidence on the relationship between the portfolio size and the investors' "terminal wealth dispersion" from the stock investments in the U.S. stock markets over the years from January 2010 through December 2014.

I. Data and Analysis

The sample in this study consists of 4,511 firms in the U.S. stock markets (2,441 firms listed in New York Stock Exchange (NYSE) and 2,070 firms in National Association of Securities Dealers Automated Quotations (NASDAQ)) with complete total return data over the period January 2010 through December 2014. The total return data are collected from the Center for Research of Security Prices (CRSP) US Stock Databases. The total return on a firm's common stock is the month-end to the month-end change in total investment of the security with ordinary dividend reinvested at the end of the month. The study adopts a naive diversification strategy via investing in randomly selected stocks in the U.S. stock markets.

To determine the "terminal wealth dispersion" of a portfolio, a simulation procedure similar to that of O'Neal (1997) is applied. Namely, one million dollars are invested in NYSE and NASDAQ stock markets in January 2010. The one million dollars are equally divided among n randomly chosen stocks (n = 1, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100). The n-stock portfolio is held for t years (t = 1, 2, 3, 4, 5). The value of a portfolio at the end of the investment horizon is calculated by compounding its annual returns over the holding periods. For example, if the total returns on a portfolio in years 2010, 2011, 2012, 2013, and 2014 are -5%, -8%, 6%, 7%, and 10% respectively, then the terminal value of the portfolio in December 2014 equals $1 million x 1.09 [= (.95) x (.92) x (1.06) x (1.07) x (1.10)]. It is clear that the level of the terminal period portfolio value depends on the stocks included in the portfolio as well as the length of the holding period. The value of the portfolio at the end of the investment horizon equals the investor's terminal period wealth. The investment risk faced by the investor is the dispersion of the terminal period wealth. The simulation is repeated 1,000 times for each n-stock portfolio with t-year holding period. The terminal wealth mean (dispersion) is then calculated as the average (standard deviation) of the 1,000 terminal wealth levels generated from the simulation for each investment. The "terminal wealth mean (TWM)" measures the average value of the n-stock portfolio investments at the end of t years, while the "terminal wealth standard deviation (TWSD)" measures the dispersion of the terminal wealth levels assumed by the investor for each investment in a n-stock portfolio for t years. The investors are aiming for greater rewards, measured in terms of the "terminal wealth means", but smaller risk, measured in terms of the "terminal wealth standard deviations."

Refer Table 1 and Figure 1

Table 1 and Figure 1 (Table 2) summarize the "terminal wealth means" ("terminal wealth standard deviations") of the simulated portfolios for each of the investment periods: January 2010-December 2010, January 2010-December 2011, January 2010-December 2012, January 2010-December 2013, and January 2010-December 2014. Basically, Table 1 and Figure 1 show that all simulated portfolios' "terminal wealth means" are above the initial investment of $1 million across all investment horizons during the study period. Table 1 and Figure 1 also illustrates that the "terminal wealth means" across all simulated portfolios are about the same within each investment horizon. Specifically, the terminal values of the simulated portfolios vary from $1.24 to $1.25 million over the investment period of January 2010-December 2010, $1.16 to $1.19 million over the period of January 2010-December 2011, $1.34 to $1.37 million over the period of January 2010-December 2012, $1.78 to $1.84 million over the period of January 2010-December 2013, and $1.82 to $1.85 million over the period of January 2010-December

2014. The simulated portfolios yield the highest "terminal wealth means" in January 2010-December 2014, and the lowest in January 2010-December 2011 investment period. The findings demonstrate that the naive diversification strategy is profitable with greater than $1 million terminal wealth means, and yields similar terminal values of the n-stock portfolios (n = 1, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100) in each of the t-year holding periods (t = 1, 2, 3, 4, 5) over the years 2010 through 2014.

Refer Table 2

In contrast, Table 2 reveals that the "terminal wealth standard deviations" decline as the number of stocks in the portfolios increases in each investment horizon. Namely, the "terminal wealth standard deviations" drop from $1.81 million to $0.23 million ($1.40 million to $0.22 million) when the number of stocks in the portfolio is increased from 1 to 50 in January 2010-December 2014 (January 2010-December 2013) investment period. Additionally, Table 2 exhibits that the "terminal wealth standard deviations" increase as the holding period increases. That is, the investors are subject to greater investment risk, measured in terms of the investor's "terminal wealth standard deviation," the longer the investment horizon.

Refer Figure 2

Figure 2 displays the "terminal wealth standard deviations" as % of the single-stock "terminal wealth standard deviation" in each of the investment periods. It shows that the "terminal wealth standard deviations" are reduced sharply, ranging from 65% to 71% (74% to 81%), when the portfolio size is increased from 1 to 10 (20) stocks across all investment periods. The "terminal wealth standard deviations" continue to decline, varying from 84% to 87%, when the portfolio size is increased from 1 to 50 stocks across all investment periods. The risk reduction benefits level off when the portfolio size exceeds 50 stocks across all investment periods.

It is of utmost importance to the investors with fixed future obligations and/or savings goals to sufficiently reduce the dispersion of terminal wealth without changing the terminal wealth level. In other words, it is desirable to reduce the "terminal wealth coefficient of variation (TWCV)" (= TWSD / TWM), which measures "terminal wealth standard deviation" per dollar of the "terminal wealth mean." Table 3 presents the simulated portfolios' "terminal wealth coefficients of variation" in each of the investment periods. The risk declining outcomes are evident and similar when the "terminal wealth dispersion" is measured in terms of coefficient of variation instead of standard deviation. Specifically, the "terminal wealth coefficients of variation" decrease from 0.979 to 0.127 (from 0.787 to 0.120) when number of stocks in the portfolios is changed from 1 to 50 in January 2010-December 2014 (January 2010-December 2013) investment period.

Refer Table 3 and Figure 3

Figure 3 shows the simulated portfolios' "terminal wealth coefficients of variation," expressed as % of the single-stock "terminal wealth coefficient of variation" in each of the investment periods. Similar to the results presented in Figure 2, Figure 3 shows that the "terminal wealth coefficients of variation" drop rapidly, ranging from 65% to 71%, across all the five investment periods when the portfolio size is increased from 1 to 10 stocks. Moreover, the "terminal wealth

coefficients of variation" continue to drop, ranging from 74% to 80% (84% to 87%), across the investment periods when the number of stocks in the portfolios is increased from 1 to 20 (50). The risk reduction benefit exhausts after the number of randomly chosen stocks in the portfolios goes beyond 50. The study concludes that the equity investors with fixed future obligations and/or savings goals should hold at least 50 randomly selected stocks in their portfolios in order to benefit from satisfactory reduction in "terminal wealth dispersion" without changing the terminal period wealth in the U.S. stock markets.

II. Conclusion

The study highlights the relationship between portfolio size and investment risk using the "terminal wealth dispersion" as the relevant risk measure. The study finds that naive diversification among stocks in NYSE and NASDAQ stock markets is profitable, and results in similar "terminal wealth means" across the simulated portfolios in each investment horizon over the periods from January 2010 to December 2014. Furthermore, the study reveals that the "terminal value dispersion" declines sharply at first and then tails off after the portfolios contain more than 50 stocks in each of the investment horizons. Moreover, the study shows that the "terminal wealth dispersion" increases with the holding period. The findings imply that the investors are subject to greater investment risk the longer the investment horizon. The study concludes that the equity investors with fixed future obligations and/or savings goals should hold at least 50 randomly selected stocks in their portfolios in order to benefit from satisfactory reduction in "terminal wealth dispersion" without changing the terminal period wealth in the U.S. stock markets.

Table 1. Simulated Portfolios' Terminal Wealth Means (in $million) for each of the Holding Periods: January 2010-December 2010, January 2010-December 2011, January 2010-December 2012, January 2010-December 2013, and January 2010-December 2014.

Number of Stocks in the Portfolio	Holding Period				
	2010-2010	2010-2011	2010-2012	2010-2013	2010-2014
1-Stock	$ 1.24	$ 1.19	$ 1.34	$ 1.78	$ 1.85
10-Stock	$ 1.25	$ 1.17	$ 1.37	$ 1.81	$ 1.82
20-Stock	$ 1.25	$ 1.17	$ 1.35	$ 1.83	$ 1.83
30-Stock	$ 1.24	$ 1.16	$ 1.36	$ 1.83	$ 1.82
40-Stock	$ 1.25	$ 1.17	$ 1.35	$ 1.82	$ 1.83
50-Stock	$ 1.25	$ 1.17	$ 1.36	$ 1.82	$ 1.83
60-Stock	$ 1.24	$ 1.17	$ 1.36	$ 1.82	$ 1.82
70-Stock	$ 1.24	$ 1.17	$ 1.36	$ 1.84	$ 1.83
80-Stock	$ 1.25	$ 1.17	$ 1.37	$ 1.83	$ 1.83
90-Stock	$ 1.24	$ 1.16	$ 1.36	$ 1.83	$ 1.83
100-Stock	$ 1.25	$ 1.17	$ 1.36	$ 1.82	$ 1.83

All simulated portfolios' "terminal wealth means" are above the initial investment of $1 million, and are about the same across all simulated portfolios within each investment horizon.

Table 2. Simulated Portfolios' Terminal Wealth Standard Deviations (in $million) for each of the Holding Periods: January 2010-December 2010, January 2010-December 2011, January 2010-December 2012, January 2010-December 2013, and January 2010-December 2014.

Number of Stocks in the Portfolio	Holding Period				
	2010-2010	2010-2011	2010-2012	2010-2013	2010-2014
1-Stock	$ 0.44	$ 0.58	$ 0.72	$ 1.40	$ 1.81
10-Stock	$ 0.15	$ 0.19	$ 0.25	$ 0.47	$ 0.52
20-Stock	$ 0.11	$ 0.14	$ 0.19	$ 0.32	$ 0.35
30-Stock	$ 0.09	$ 0.11	$ 0.16	$ 0.28	$ 0.30
40-Stock	$ 0.08	$ 0.09	$ 0.13	$ 0.24	$ 0.25
50-Stock	$ 0.07	$ 0.08	$ 0.12	$ 0.22	$ 0.23
60-Stock	$ 0.06	$ 0.08	$ 0.11	$ 0.20	$ 0.20
70-Stock	$ 0.05	$ 0.07	$ 0.10	$ 0.18	$ 0.19
80-Stock	$ 0.05	$ 0.07	$ 0.10	$ 0.18	$ 0.17
90-Stock	$ 0.05	$ 0.06	$ 0.09	$ 0.16	$ 0.17
100-Stock	$ 0.05	$ 0.06	$ 0.08	$ 0.14	$ 0.16

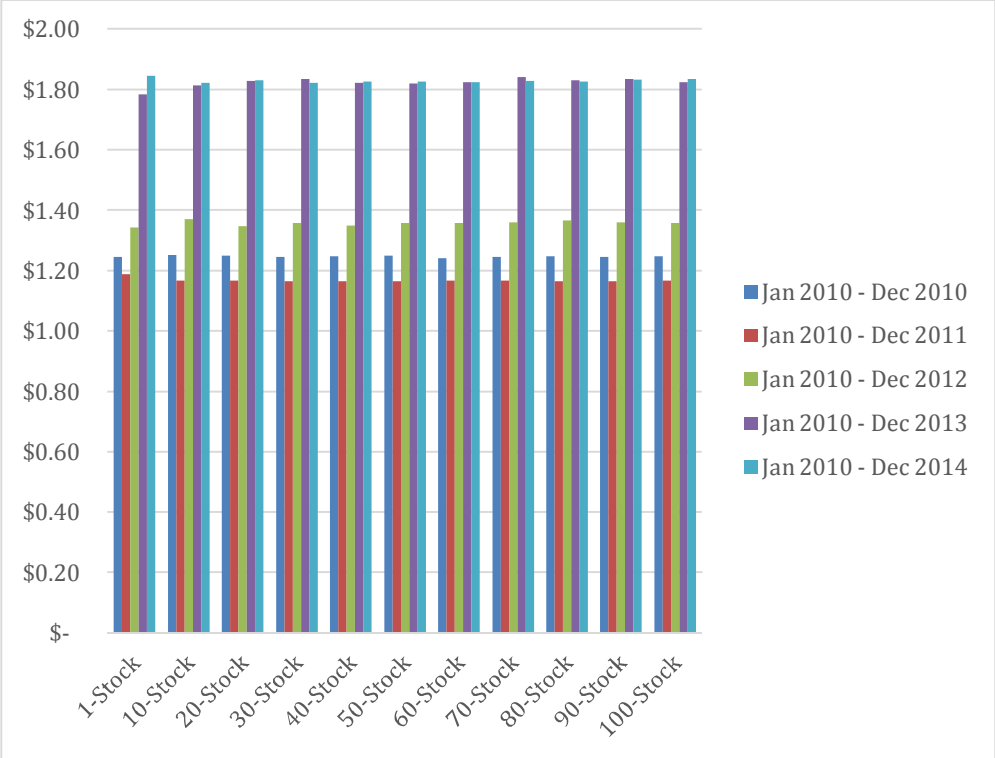
The "terminal wealth standard deviations" of the simulated portfolios decline sharply at first and then level off after the number of stocks in the portfolios goes beyond 50 in each of the investment horizons.

Table 3. Simulated Portfolios' Terminal Wealth Coefficients of Variation, defined as Various Portfolios' Terminal Wealth Means / Various Portfolios' Terminal Wealth Standard Deviations for each of the Holding Periods: January 2010-December 2010, January 2010-December 2011, January 2010-December 2012, January 2010-December 2013, and January 2010-December 2014.

Number of Stocks in the Portfolio	Holding Period				
	2010-2010	2010-2011	2010-2012	2010-2013	2010-2014
1-Stock	0.351	0.491	0.533	0.787	0.979
10-Stock	0.116	0.164	0.185	0.259	0.285
20-Stock	0.085	0.116	0.138	0.177	0.193
30-Stock	0.069	0.093	0.118	0.154	0.164
40-Stock	0.060	0.080	0.100	0.131	0.138
50-Stock	0.055	0.073	0.085	0.120	0.127
60-Stock	0.047	0.069	0.080	0.107	0.111
70-Stock	0.044	0.063	0.075	0.097	0.105
80-Stock	0.043	0.057	0.072	0.096	0.096
90-Stock	0.038	0.055	0.069	0.088	0.095
100-Stock	0.038	0.051	0.062	0.079	0.087

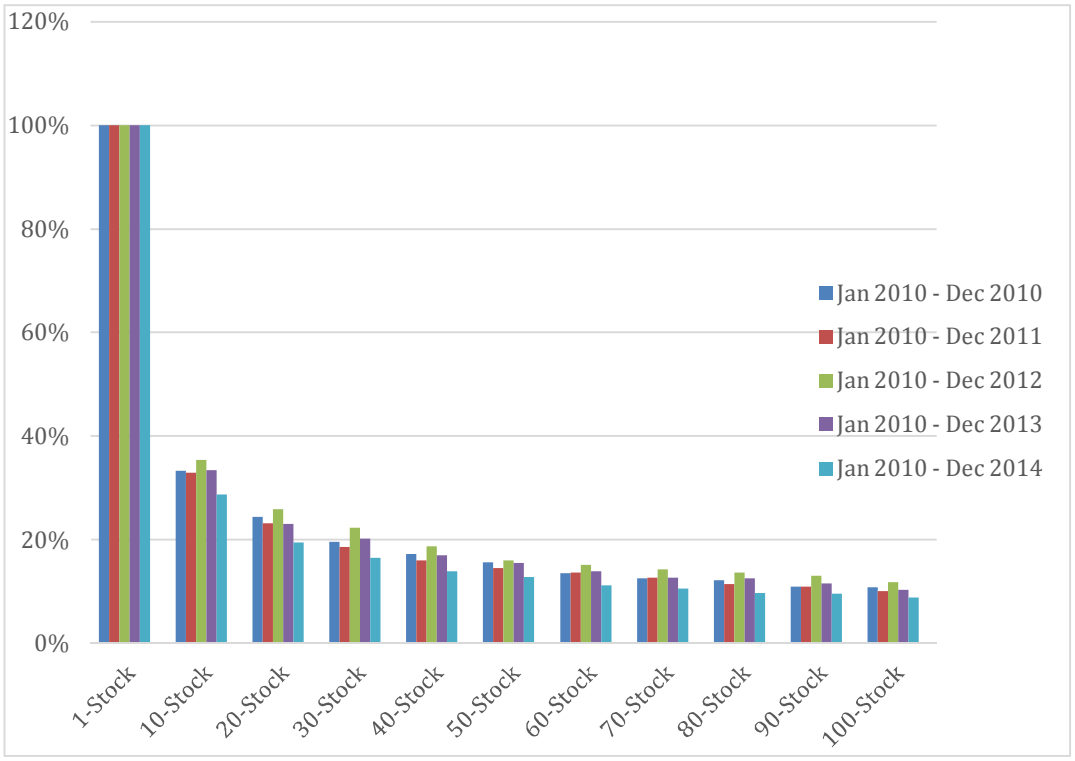
The "terminal wealth coefficients of variation" of the simulated portfolios drop rapidly at first and then tail off after the portfolio size exceeds 50 stocks in each of the investment horizons.

Figure 1. Simulated Portfolios' Terminal Wealth Means (in $million) for each of the Holding Periods: January 2010-December 2010, January 2010-December 2011, January 2010-December 2012, January 2010-December 2013, and January 2010-December 2014.



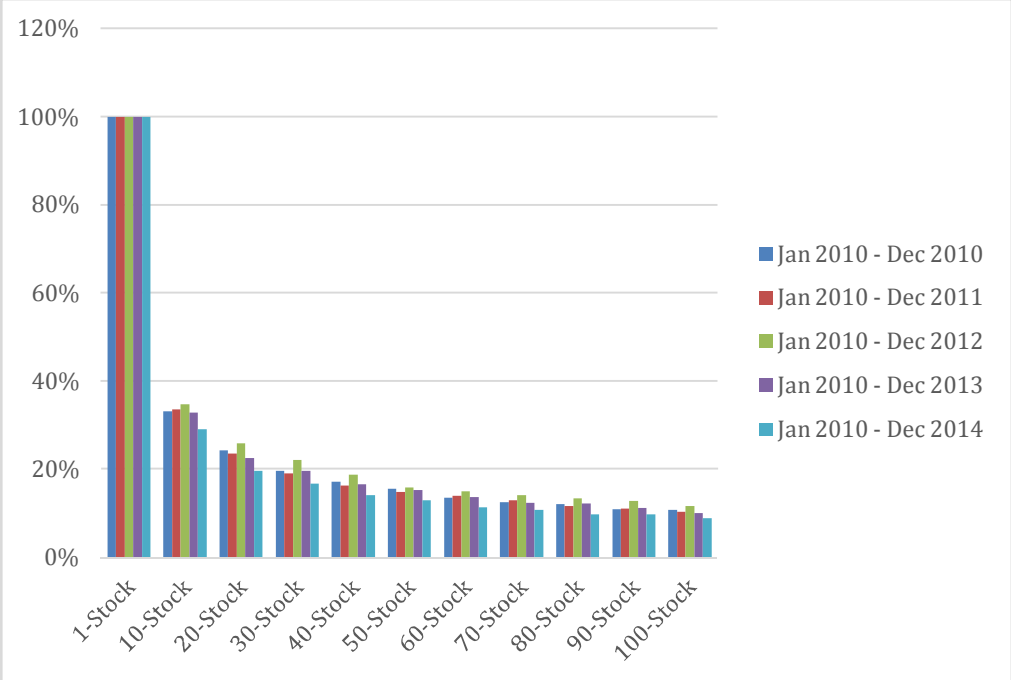
All simulated portfolios' "terminal wealth means" are above the initial investment of $1 million. The "terminal wealth means" are similar to each other across all simulated portfolios within each investment horizon.

Figure 2. Simulated Portfolios' Terminal Wealth Standard Deviations Expressed as % of Single-Stock Terminal Wealth Standard Deviation in each of the Holding Periods: January 2010-December 2010, January 2010-December 2011, January 2010-December 2012, January 2010-December 2013, and January 2010-December 2014.



The "terminal wealth standard deviations" are reduced sharply, ranging from 65% to 71% (84% to 87%), when the portfolio size is increased from 1 to 10 (50) stocks across all investment periods. The risk reduction benefits level off when the portfolio size exceeds 50 stocks across all investment periods.

Figure 3. Simulated Portfolios' Terminal Wealth Coefficients of Variation Expressed as % of Single-Stock Terminal Wealth Coefficient of Variation in each of the Holding Periods: January 2010-December 2010, January 2010-December 2011, January 2010-December 2012, January 2010-December 2013, and January 2010-December 2014.



The "terminal wealth coefficients of variation" drop rapidly, ranging from 65% to 71%, across the five investment horizons when moving from 1-stock to 10-stock portfolios. Moreover, the "terminal wealth coefficients of variation" continue to drop ranging from 74% to 80% (84% to 87%) across the investment periods when the number of stocks in the portfolios is increased from 1 to 20 (50). The risk reduction benefit exhausts after holding more than 50 randomly selected stocks in the portfolios.

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