Value at Risk, 3rd Edition, Philippe Jorion
Chapter 13: Liquidity Risk

- Traditional VAR models assume that the model is “frozen” over some time horizon
- Questionable if VAR is used to measure the worst loss over a liquidation period
- Liquidity risk includes ‘funding liquidity risk’ and ‘asset liquidity risk’
- **Funding Liquidity Risk:** arises when financing cannot be maintained due to creditor/investor demands
- **Asset Liquidity Risk:** arises when a forced liquidation of assets creates unfavourable price movements

13.1 Defining Liquidity Risk

- Liquidity risk starts from the liability side, when creditors/investors demand their money back
- Usually occurs after the institution has incurred or is thought to incur losses that threaten its solvency
- Need for cash creates problems on asset side as assets will need to be sold to meet the demands of the liability side
- **Market Microstructure:** study of market clearing mechanisms
- **Optimal Trade Execution:** design of strategies to minimize trading costs or to meet some other objective function
- **Asset-liability Management:** attempts to match the values of assets and liabilities

13.1.1 Asset Liquidity Risk

- Also called market/product liquidity risk
- Risk that liquidation value of assets may differ significantly from current mark-to-market values
- Can be measured by a price-quantity function also known as the market-impact effect
- **Deep Markets:** contain highly liquid assets (major currencies, treasury bonds, etc.) where positions can be offset with very little price impact
- **Thin Markets:** where any transaction can quickly affect prices, examples are exotic over-the-counter (OTC) derivatives or some emerging market equities
- **Midprice:** average of the bid and ask quotes
- **Bid-ask spread:** difference between the bid and ask prices
- Markets with small spreads are said to have 'tightness'
- This 'tightness' is valid up to some limit in quantity (Ex: 10,000 shares)
- This limit is called the normal market size or depth
- The cost to trading is equal to half the spread and is exogenous as it does not depend on quantities transacted (as long as the quantity is below the normal market size)
- For quantities beyond this point, the sale price is a decreasing function of the quantity
- The slope of the ask price line as it changes with the quantity sold is called the market impact
- Liquidity depends on both the price-impact function and the size of the position
- Liquidity is also a function of prevailing market conditions, markets go through regular bouts of liquidity crisis
- Asset liquidity risk traditionally was controlled through position limits

13.1.2 Funding Liquidity Risk

- **Cash-flow/Funding Liquidity Risk:** the inability to meet payment obligations to creditors/investors, which causes the unwanted liquidation of a portfolio
- Arises from the liability side of the balance sheet
- Most financial institutions are leveraged and are required to post collateral (assets) in exchange for cash from a broker
- The required collateral is usually slightly more than the cash loaned, the extra amount is known as a 'haircut'
- The value of the collateral is constantly marked-to-market
- If the value of the collateral falls below a threshold, an additional payment is required called variation margin
- If the institution does not have enough cash on hand, they may be forced to liquidate assets to meet the variation margin amount
- Brokers can also change the collateral requirements, such as increase haircuts when markets are more volatile (this creates extra demands on cash)
- Cash-flow liquidity risk arises from the mismatches in the timing of payments
- Possible defences against funding liquidity risk are as follows:
    - Cash (first line of defense)
    - Line of credit (loan arrangement with a bank allowing the customer to borrow up to a prespecified amount)
    - Issue new debt or new equity (in practice, this is difficult to do when the institution is faring badly and needing it the most)
- Institutions must evaluate the likelihood of redemptions or cash requests from debt and equity holders
- Most likely to occur when the institution is or appears to be vulnerable, this turns a minor problem into a crisis
- Important to avoid debt covenants or 'triggers' that force early redemption of the borrowed funds

Box 13-1: Enron's Credit Triggers: The Bad and Stupid

- **Credit Triggers:** clauses in financial contracts that allow creditors to demand immediate payments if the credit rating of the borrower falls below some predetermined level.
- Enron was downgraded to speculative-grade after a takeover fell through, this triggered a $4B repayment in debt due to credit triggers, this forced Enron into bankruptcy
- Credit rating agencies now take credit triggers into account when setting a company's rating

Final Comments:

- Liquidity risk should be viewed in the context of both asset and liabilities
- Hedge Funds invest in illiquid assets (Ex: distressed debt)
- To minimize liquidity risk, funds impose lockup periods, or a minimum time for investors to keep their funds, as well as a loner redemption notice periods for withdrawing funds

13.2 Assessing Asset Liquidity Risk

- Trading returns are measured from midmarket prices, this is adequate for daily profit and loss (P&L)
- Does not represent the actual fall in value if a large portfolio were to be liquidated
- Traditionally adjustments to VAR are done on an ad hoc basis
- Can factor Liquidity risk into VAR by ensuring the VAR forecast period is at least greater than an orderly liquidation period
- Account for longer liquidation periods by increasing the volatility assumption

13.2.1 Effect of Bid-Ask Spreads

- Focus on the various components of liquidation, the most easily measurable is the bid-ask spread:

$$S = \frac{P(ask) - P(bid)}{P(mid)}$$

- There are small spreads for assets such as major currencies, large U.S. stocks and on-the-run Treasuries
- There are larger spreads for assets such as less liquid currencies, stocks and bonds

Three types of costs that make up the spread:

- **Order processing costs:** cover the cost of providing liquidity services and reflect the cost of trading, the volume of transaction, the state of technology and competition, these decrease with transaction volume
- **Asymmetric-Information Costs:** some orders may come from informed traders, at the expense of market makers, increase the spread to provide some protection from this
- **Inventory Carrying Costs:** the cost of maintaining open positions, which increase with higher price volatility, higher interest rate carrying costs, and lower trading activity/turnover

Liquidity Adjusted VAR (LVAR 1):

- If spreads are fixed, liquidity adjusted VAR is the traditional VAR with an extra term:

$$LVAR = VAR + L_1 = (W\alpha\sigma) + 1/2(WS)$$

- Where W is the initial wealth or portfolio value

Example:

Suppose we have \$1 million invested in a stock with daily volatility of $\sigma = 0.01$ and spread of $S = 0.002$. Calculate the 1-day LVAR at the 95% confidence level.

$$LVAR_{95\%} = (1{,}000{,}000)(1.645)(0.01) + 0.5(1{,}000{,}000)(0.002) = 17{,}450$$

- Can repeat this adjustment for all assets in the portfolio, which leads to a series of add-ons that equal $1/2\Sigma_i|W_i|S_i$
- This sequence of positive terms increases linearly with the number of assets, whereas the usual VAR benefits from diversification

LVAR 2:

- Introduce uncertainty in the spread S, model the mean of the spreads as $\bar{S}$ and the standard deviation σ_S
- Considers the worst increase in the spread at some confidence level:

$$LVAR = VAR + L_2 = (W\alpha\sigma) + \frac{1}{2}[W(\bar{S} + \alpha'\sigma_S)]$$

- Assumes that the worst market loss and increase in spread will occur simultaneously
- In general, there is a positive correlation between volatility and spreads
- Typically, the value of σ_S is about half the size of the average spread $\bar{S}$

13.2.2 Incorporating Liquidity in Valuation

- Another approach is to mark the portfolio to the appropriate bid prices (for long positions) and ask prices (for short positions)
- In practice, financial institutions mark their positions to the conservative bid-offer prices
- Financial institutions often apply reserves, which are pricing changes in the valuation away from fair value to account for effects of illiquidity and model risk
- Firms deduct these reserves from the fair value positions to account for the extra time and cost required to close out positions
- Therefore, there is no need to take liquidity risk into account in VAR as it is already taken into account in the valuation of positions

13.2.3 Effect of Price Impact

- Need to account for more than just the bid-ask spread, need to account for the affect liquidation has on market prices, the market impact factor should be incorporated
- For simplicity, we assume a linear price-quantity function and ignore the spread, the price per share is:

$$P(q) = P_0(1 - kq)$$

Example:

Assume that $P_0 = \$100$ and $k = 0.5\times10^{-7}$, we start with a position of $q = 1\ million$ shares of stock. If we liquidate all at once, what is the expected price drop and total price impact? If we instead, decided to liquidate 200,000 shares over $n = 5$ days, what is the price drop and total price impact?

Immediate Liquidation Creates the Costs:

$$Price\ Drop = P_0 kq = (100)(0.5\times10^{-7})(1{,}000{,}000) = \$5$$

$$Price\ Impact = C_1(W) = q\times[P_0 - P(q)] = q\times(P_0 - P_0 + P_0 kq) = kq^2 P_0$$

$$C_1(W) = 0.5\times10^{-7}\times(1{,}000{,}000)^2(100) = 5{,}000{,}000$$

Uniform Liquidation Creates the Costs:

$$Price\ Drop = P_0 kq/n = (100)(0.5\times10^{-7})(1{,}000{,}000/5) = \$1$$

$$Price\ Impact = C_2(W) = q\times[P_0 - P(q/n)] = q\times(P_0 - P_0 + P_0 kq/n) = k(q^2/n)P_0$$

$$C_2(W) = 0.5\times10^{-7}\times(1{,}000{,}000^2/5)\times(100) = 1{,}000{,}000$$

- Uniform liquidation spreads the price impact over many days which leads to lower trading costs
- Drawback to liquidating more slowly is that the portfolio remains exposed to price risks over a larger period of time
- Immediate liquidation has high costs but minimum risk, uniform sale has low costs but higher volatility

Incorporating Price Risk into LVAR:

- We define σ to be the daily volatility of the share price in dollars
- We assume that the sales are executed at the close of the business day in one block, with the immediate sale, the price risk, or variance of wealth is zero, $V_1(W) = 0$
- For the uniform sale, assume that returns are independent over each day so that the total variance is the sum of the daily variances
- We have relative positions $x_0, x_1, \ldots, x_n$
- At the end of the first day, the position has decreased from $x_0 = 1$ to $x_1 = 1 - (1/n)$
- On the second day, the position has gone to $[1 - 2(1/n)]$ and so on
- With uncorrelated daily returns, the total portfolio variance over 'n' days is the sum of the daily variance

$$V_2(W) = \sigma^2 q^2 \left\{\left(1 - \frac{1}{n}\right)^2 + \left(1 - 2\frac{1}{n}\right)^2 + \cdots + \left[1 - (n-1)\frac{1}{n}\right]^2\right\} P_0^2$$

- This can be simplified to the following:

$$V_2(W) = \sigma^2 q^2 \left[n\frac{1}{3}\left(1 - \frac{1}{n}\right)\left(1 - \frac{1}{2n}\right)\right] P_0^2 = \sigma^2 q^2 T^* P_0^2$$

- Where T^* is the correlation factor, if $n = 5$, $T^* = (5)\left(\frac{1}{3}\right)\left(1 - \frac{1}{5}\right)\left(1 - \frac{1}{2(5)}\right) = 1.20$
- So uniform liquidation over 5 days is the equivalent of mark-to-market risk of a position held over 1.2 days

LVAR with Transaction Costs and Price Impact:

$$LVAR = \alpha\sqrt{V(W)} + C(W)$$

- Where α is the confidence level 'c', $V(W)$ is the variance with the associated liquidation period, and $C(W)$ is the price impact with the associated liquidation period

13.2.4 Trading Strategies

- Can choose any liquidation strategy, ideally would choose a strategy that leads to an optimal trade-off between execution costs and price risk
- The optimal trajectory is defined by a set of daily positions $x_0, x_1, \dots, x_n$
- On the first day, the optimal position drops by more than the uniform sale $x_0 - x_1 > 1/n$
- Half-life: the time required to liquidate half the portfolio
- LVAR incorporates the total execution cost and price risk components in a consistent fashion
- As the length of time for execution increases, the execution cost decreases, but the price risk component increases
- Benefit of this approach is that it draws attention to market impact effects in portfolio liquidation
- Illustrates that execution strategies should pay close attention to execution costs/price volatility

13.2.5 Example

- In practice, we need a price-quantity function for all securities in the portfolio, the portfolio position, which gives us an estimate of the price impact of a liquidation
- To estimate the total impact cost we need information about the historical bid-ask spreads, the median trading volume, and recent volatility
- The total cost of immediate liquidation is estimated at 21.5 bps, the daily mark-to-market volatility is 110 bps
- The LVAR is then: $LVAR = \$50 \times 1.645 \times 0.011 + 50 \times 0.0022 = \$0.9M + 0.1M = \$1.0M$

13.3 Assessing Funding Liquidity Risk

- Involves examining the asset-liability structure of the institution and potential demands on cash and other sources of liquidity
- The Counterparty Risk Management Policy Group (CRMPG) proposes to evaluate funding risk by comparing the amount of cash an institution has at hand with what it could need to meet payment obligations
- **Cash Liquidity:** the ratio of cash equivalent over the potential decline in the value of positions that may create cash-flow needs
- The example illustrates the difference in the Cash Liquidity Ratio with 2 different swaps, the first has a 1 day VAR of $1.1 with $5 cash on hand or a ratio of 4.5 (which is sufficient coverage), the second has a 10-day VAR of $3.5 with $5 cash on hand or a ratio of 1.4 (which is barely enough to provide protection against funding risk)

- **Liquidity Risk:** the risk that a trading operation's need for cash collateral may exceed its total liquidity resources

13.4 Lessons from LTCM

- Setup a hedge fund that tried to take advantage of "relative value" or "convergence arbitrage" trades, betting on differences in prices, or spreads, among closely related securities

13.4.1 LTCM's Leverage

- These strategies generated tiny profits so leverage was used to create more attractive returns
- LTCM has a leverage ratio of 25:1, with total equity of $5B and a balance sheet of about $125B
- LTCM also had $1.25 trillion in off-balance sheet positions such as swaps, options and other derivatives
- LTCM had 2.4% of the entire swap market in 1998
- Most of the swaps were offsetting each other, what mattered was the net risk of the fund
- LTCM failed to recognize that since these positions were so large, attempts to liquidate them would provoke large market moves

13.4.2 LTCM's "Bulletproofing"

- LTCM was able to leverage its balance sheet with sale-repurchase agreements (repos) with commercial and investment banks
- **Repo Agreement:** sell assets in exchange for cash and a promise to repurchase them back at a fixed price at some future date
- Normally, there is a haircut that limits the amount of leverage, LTCM was able to obtain unusually good financing conditions with next to zero haircuts, as it was widely viewed as "safe"
- The swaps were subject to two-way marking to market
- LTCM had "bulletproofed" itself from a liquidity squeeze on the supply side
- LTCM required investors to keep their money in the fund for a minimum of 3 years, this avoided forced sales in the event of poor performance
- LTCM also secured a $900M credit line from Chase Manhattan and other banks
- LTCM had some protection against funding liquidity risk, it still was exposed to market risk and asset-liquidity risk

13.4.3 LTCM's Downfall

- LTCM profited handsomely with the narrowing of credit spreads during early years (had returns above 40 percent after fees)
- Trouble began in May/June of 1998, with the downturn in mortgage-backed securities led to a 16% loss in LTCM's capital
- On August 17, Russia announced that it was "restructuring" its bond payments (defaulting on its debt)
- This led to a reassessment of credit and sovereign risks across financial markets
- Credit spreads, risk premiums and liquidity spreads jumped up sharply, stock markets dived
- LTCM lost $550 million on August 21 alone

- By August it had lost 52% of its Dec 31 value, but still had $126B, and the leverage ratio had increased from 28:1 to 55:1
- In September, the portfolios losses accelerated and Bear Stearns required a large margin call, it also increased collateral requirements, this depleted the funds liquid resources
- LTCM was now caught in a squeeze between funding risk (as its reserves decreased) and asset risk (as the size of its positions made it impractical to liquidate assets)
- Lenders had low current exposure but significant potential exposure
- The New York Federal Reserve arranged a bailout of the firm to avoid a potential meltdown, the investors of LTCM lost 92% of their year to date investment

13.4.4 LTCM's Liquidity

- LTCM failed due to its inability to manage risk
- LTCM had undiversified trades
- LTCM was exposed to increases in volatility on its long interest rate swap positions, its short equity options, its long mortgage backed securities (hedged) and its long sovereign debt positions
- LTCM was exposed to increases in default on its long interest rate swap positions and its long sovereign debt positions
- LTCM was exposed to increases in Illiquidity on its long interest rate swap positions, its long off-the-run/short on-the-run treasury positions, its long mortgage backed security positions and its long sovereign debt positions
- LTCM was exposed to both asset and funding liquidity risk

13.5 Conclusions

- Introduced liquidity adjusted VAR or LVAR
- Bid-ask spreads are less important than traditional VAR measures
- What matters are the large price drops from liquidating large positions
- Alternative approach is to hold a reserve for liquidity risk in the valuation of positions and not in the VAR
- Funding liquidity risk arises when financing cannot be obtained
- Liquidity risk involves both the asset side and liability side of the balance sheet
- Bid-ask spreads are positively correlated with volatility
- Illiquid assets will incur greater execution costs as volatility increases
- Liquidity risk can be mitigated by taking offsetting positions in assets, businesses that benefit from increased volatility (long positions in options, customer trading)
- This analysis provides a systematic framework for thinking about the interactions among market risk, asset liquidity risk and funding liquidity risk