A New Approach to Predict the Reliability of Fault Tolerant Hypercube System With Faulty Nodes

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Abstract- This paper proposes a new method to identify all the maximal incomplete sub cubes present in a faulty hypercube taking maximum fault tolerance level i.e. number of faulty nodes is equal to the system dimension. The procedure is a distributed one, as every healthy node next to a failed one performs the same procedure independently and concurrently. Then the reliability expression for the incomplete hyper cube is derived. This method is well supported by an efficient algorithm which runs polynomially. The proposed method is found to be simple, general and efficient and thus is applicable to all the cube based topologies.

Keywords- *Hypercube, Maximal Sub cube, Maximal Incomplete sub cube, discarded region.*

I. INTRODUCTION

As parallel computer communication systems are very much popular and commercially widely used in real time applications, therefore considerable interest and increasing efforts have been made to develop such large communication systems. A major part of it is a parallel computer interconnection network, which is used to interconnect a large number of standalone processors. Waived in the case of incomplete cubes, which shows the emulation performance as the n/w scales up in size [5] and [6].

The probability of fault in a larger system is given due importance. Whenever a fault arises, an n-hypercube may operate in a gracefully degradable manner due to the execution of parallel algorithms in smaller fault free sub cubes[6], which are comprises of healthy nodes. In order to maintain hypercube topology in the presence of faults, researchers have proposed addition of spare nodes thereby replacing the failed components with spares. This results in a much larger system than what is attained by any conventional reconfiguration scheme which identifies only complete sub cube [7]. Also fault tolerance can be achieved by reconfiguring the larger system to smaller sized system after the occurrence of fault [4]. Unlike a complete one, an incomplete hypercube can be of any arbitrary size, i.e. can be used to interconnect systems with any numbers of processors, making it possible to finish a given batch of jobs faster than it's complete counterpart alone by supporting simultaneous execution of multiple jobs of different sizes by assigning more nodes to execute the job cooperatively. Chen et. al.[9] determine sub cubes in a faulty hypercube. Similar research work can be found in literature [6],[8]and[10].Thus reconfiguring a faulty n-hypercube in to a maximal incomplete hypercube tends to Therefore a wide variety of interconnection networks have been proposed like rectangular meshes, trees, shuffle exchange networks, omega networks and binary cubes[1]and[2].One of the widely used topology is the binary hyper cubes, also known as the Boolean n-cubes. Due to attractive properties like regularity, symmetry, small diameter, strong connectivity, recursive construction and partition ability the n-hypercube topology has enjoyed the largest popularity. These property leads to simple routing, support for wide application spectrum and fault tolerance for communication systems [3].

The n-dimensional hypercube is composed of 2^n nodes and has n- edges per node, n-bit binary addresses are assign to the nodes to the hyper cubes in such a way that an edge or link connects two nodes if and only if their binary addresses differ by a single bit [3].This Inter connection network supports large numbers of resources with small diameters. But the major drawbacks of the cube networks are the numbers of communication ports and channels per processors is the same as the logarithm of the total numbers of processors in the system. Therefore the number of communication ports and channels per processors increases by increasing the total number of processors in the system. This drawback seems to be lower potential performance degradation . This motivates our study to propose a simple, general and recursive method for finding all the incomplete sub cubes of a hypercube.

With the increase in size, the complexity of the interconnection network increases there by corresponding increase in computational power to maintain acceptable performance under reliable conditions [11] and [12]. For this the reliability prediction of a multi computer hypercube network is quite essential, to be used in critical applications [13],[14]and[15].

This paper proposes an efficient distributed procedure for locating or identifying all maximal incomplete sub cubes present in a faulty n-hypercube. There by forming the maximal incomplete sub cube. This method is illustrated through a 3-dimensinal hypercube. Then a generalized reliability expression for the maximal incomplete sub cube has been derived, which is supported by an effective algorithm.

II. METHODOLOGY FOR FINDING SUB CUBES & RELIABILITY

A. Notations:

HCn Hypercube Interconnection n/w
S Source node
D Destination node

$\otimes$ *Discarding operation*
n *System dimension*
N *numbes of nodes in hypercube*
u,v,w Adjacent nodes of source node
$\overline{v}$, $\overline{w}$ *Antipodal nodes of v, w*

B. **Assumptions:-**

1. Node failures are statistically independent of each other.
2. The hypercube interconnection networks degrade their performance exponentially with time.
3. Repair facility is not available.

C. **Basic Properties**:-

Definition 1:- A maximal sub cube for a given source node S, is a fault free sub cube which involve S but not present entirely in any other fault free sub cube involving S for a given faulty node.

The maximal sub cube is defined with respect to a given node which may be of different sizes for different given nodes. Care should be taken so that no faulty nodes and antipodal nodes of given nodes present in maximal sub cube.

Lemma 1:-For an n-dimensional interconnection n/w in one of n adjacent nodes of the source node may be faulty. So the resulting maximal sub cube has a dimension of n-1.

Proof:- Consider an n-dimensional hypercube HCn in which one of the n-adjacent node to source node S is faulty. This leads to disconnection of link to faulty node. Without loss of generality and obeying the symmetry of n-hypercube. The faulty node must present in HCn. This contradicting definition of a maximal sub cube.

Definition 2:- A maximal incomplete sub cube is obtained when link is properly added to a maximal sub cube of n-1 dimension so that the destination node D is reached.

Unlike complete hypercube an incomplete hypercube can be constructed with any numbers of nodes to avoid the practical restriction of hypercube topology on the numbers of nodes in a system must be a power of 2. A proper incomplete sub cube in a faulty hypercube refers to a fault free incomplete sub cube which is not contained entirely in any of the fault free sub cube.

Definition 3:- A discarded region in an interconnection network is the smallest sub cube comprises of a faulty node and the antipodal nodes of the (n-1) fault free adjacent nodes.

For a faulty n-hypercube HCn and a given source node S. It is possible to identify systematically every fault free sub cube which involves the source node S. This is expressed by set P = {Pi/Pi is a fault free sub cube in HCn and Pi involve node S}.This can be done by determining the region which never contribute to any fault free sub cube containing the node S. Each fault results in one such regions known as discarded region which is the smallest sub cube involving both the faulty and the antipodal nodes of adjacent (n-1) nodes. A discarded region is addressed by performing $\otimes$ operation on the labels of the faulty node and the antipodal node where $\otimes$ is the bit operation defined as: it yields 0 (or 1) if the two corresponding bits are "0" (or "1"),and it is $*$ if the two corresponding bits differ.

Theorem-1:- The no. of discarded complete sub cube in an HCn of dimension 'n' is equal to the number of nodes present in a fault free maximal in complete sub cube.

Proof:- For an inter connection n/w HCn of dimension n , there are exactly n numbers of nodes adjacent to a given source node S. It can be straight forward from the properties of interconnection hypercube n/w that HCn-1 can be obtained by removing 2^{n-1} nodes from HCn. The process can be repeated such that HC_2 can be obtained which shows clearly that HCn is having a hierarchical structure .

So , for a given source node S, one out of n adjacent nodes can be faulty so that , an interconnection network of lower dimension can be obtained while preserving the hierarchical and regular properties of the n/w. Thus for a given source node(S) and destination node(D) (which can never be faulty) 2^{n-1} +1 nodes can not be faulty. Further, choosing a node N ϵ Adj(s) to be faulty. Finding the antipodal nodes Y_i' of X ϵ {Adj(s)},

i =1……(| adj(s)| -1) and carrying out $Y \otimes_i$'{Adj(s)} leads to n faulty nodes and

2^{n-1}+1 discarded regions, which proves the theorem.

Lemma 2:- In an n-dimensional incomplete hypercube there exists at most (n-1) intermediate nodes between given source node and destination node.

Proof:- From theorem 1. and properties of the interconnection n/w , the proof of the lemma is straight forward.

Theorem-2: For an n-dimensional hypercube with a given maximum tolerance level 'n', the reliability expression is recursively found to be

$$R_{s-d} = {}^{n}C_1 \, {}^{n-1}C_1 \, {}^{n-2}C_1 \ldots\ldots\ldots\ldots {}^{2}C_1 * {}^{2^{n-1}-2}C_{n-2} * P^{n+1} * (1-P)^{2^{n-1}-n}$$

Proof:- Given a source node 'S ' destination node 'D'. As the dimension of the graph is n, Adj(s)|=n. Choosing a node as faulty it can be carried out in ${}^{n}C_1$ ways. Out of the remaining n-1 Adj(s) nodes the path from source to destination can be taken in ${}^{2^{n-1}-2}C_{n-2}$ ways.

Now discarding the faulty node a maximal incomplete sub cube is obtained.

The given tolerance level is n i.e. n number of nodes can be faulty without disturbing a path from source 'S' to destination 'D'. So total numbers of working nodes= $2^n - n$. Taking P=Probability of success of a node, In sub cube I_{n-1}, choosing a node n from Adj(s) can be in ${}^{n-1}C_1$ i.e. out of n-1 nodes one will work with probability P, which contributes the term p. Now since source node and destination node are fixed, one node N ϵ I_{n-1} should be chosen working otherwise the path will be destroyed. So without loss of generality the path contains 'n' working nodes with probability p and (2^{n-1}-n) numbers of failure nodes with probability (1-p). So the Reliability expression

$$R={}^{n}C_{1} *^{2^{n-1}-2} C_{n-2} * P^{n+1} * (1-P)^{2^{n-1}-n} \quad (1)$$

Or Recursively

$$R_{s-d}={}^{n}C_{1}\,{}^{n-1}C_{1}\,{}^{n-2}C_{1}\ldots\ldots\ldots\ldots{}^{2}C_{1} *^{2^{n-1}-2} C_{n-2} * P^{n+1} * (1-P)^{2^{n-1}-n} \quad (2)$$

Lemma 3:- In an n-dimensional hypercube, for a given node, the n length independent node is present at a minimum distance of n.

Proof:- Consider an n-dimensional hypercube having 2^{n} number of nodes, keeping source and destination nodes fixed out of the 2^{n} nodes. With out loss of generality and from the properties of the hypercube topology, the disjoint path from source to destination consists of n-1 number of intermediate nodes. Hence total number of nodes in a disjoint path is n+1.As a link or an edge is defined between two nodes. Therefore the number of path required is n+1-1 i.e. 'n'. This completes the proof.

III. PROPOSED METHOD

Let HC_n denotes an n-dimensional interconnection network i.e. n-dimensional hypercube. Each node in HC_n is labeled by a n-bit string. For a given source node S, there exists a numbers of adjacent nodes, out of which at least one node is faulty. Otherwise it will destroy the regularity property of HC_n. The addresses of the adjacent nodes are differ in obtained from equation (1)exactly one bit. Assume 'u' be the faulty node, 'v' and 'w' be the non-faulty nodes. Where 'u', 'v' , 'w' are represented as binary strings [4]. $\overline{v}$ and $\overline{w}$ be the antipodal nodes of v and w. Taking bit operation $u \otimes \overline{v}$ and $u \otimes \overline{w}$ results n discarded regions. This leads to formation of an incomplete interconnection network I_{n-1}^{m}.m numbers of nodes in fault free incomplete cube with dimension of n-1.Then the reliability of incomplete hypercube is calculated by using the equation

$$R_{s-d}={}^{n}C_{1}\,{}^{n-1}C_{1}\,{}^{n-2}C_{1}\ldots\ldots\ldots\ldots{}^{2}C_{1} *^{2^{n-1}-2} C_{n-2} * P^{n+1} * (1-P)^{2^{n-1}-n}$$

A recursive algorithm for generating the reliability expression R for the maximal incomplete hypercube is provided below.

IV. ALGORITM FOR RELIABILITY EXPRESSION

Reliability (G, S, D, n)
{
If $(n \geq 2)$
{
Adjacent=Adj(S)
Choose a node N from adjacent in ${}^{n}C_{1}$ *ways.*
$N' = \{\text{Antipodal (N)}\}$
$V_i' = N' \otimes \{V \sim (S \cup D \cup N)\}$
$V' = \{V \sim (S \cup D \cup (Adj(S) \sim N)\}$
for i=1 to $|V'|$
Discard $N' \otimes V_i'$ *region*
G′= (V′,E′)
$R = R\times {}^{n}C_{1}\; p^{n+1}\, q^{2^{n-1}-n}$
Reliability (G′, S, D, n-1)
}
else
return;
}

*Efficiency:*The complexity of the proposed algorithm is found to be $O(n^2)$.

V. ILLUSTRATION

The proposed method is illustrated through a 3D-hypercube.

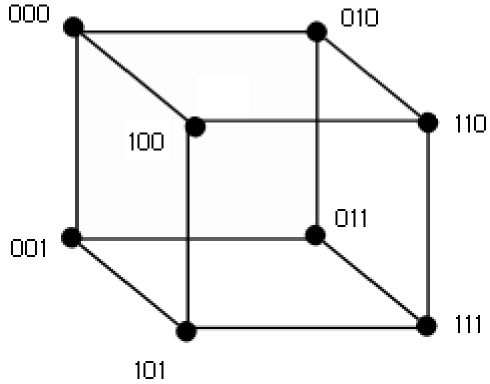
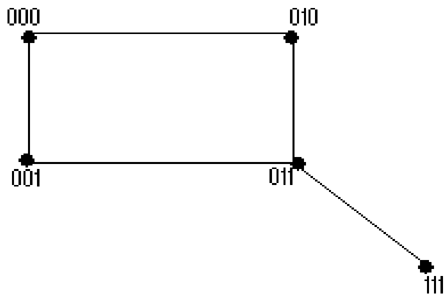
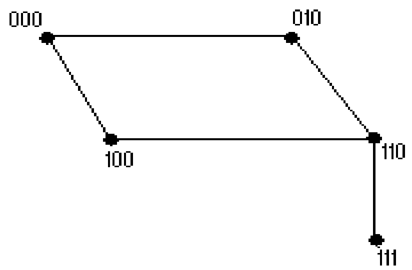


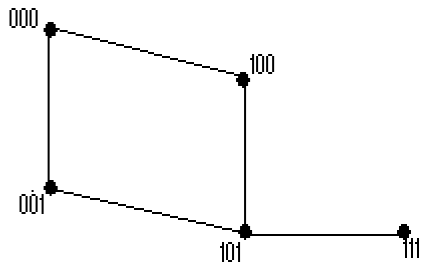
Fig.1: Hypercube (*N=8, n=3)*



2(a) 100 taken as faulty



2(b) 001 taken as faulty



2(c) 010 taken as faulty

Fig.2: (a),(b) and (c) represents incomplete maximal sub cubes of a 3-hypercube.

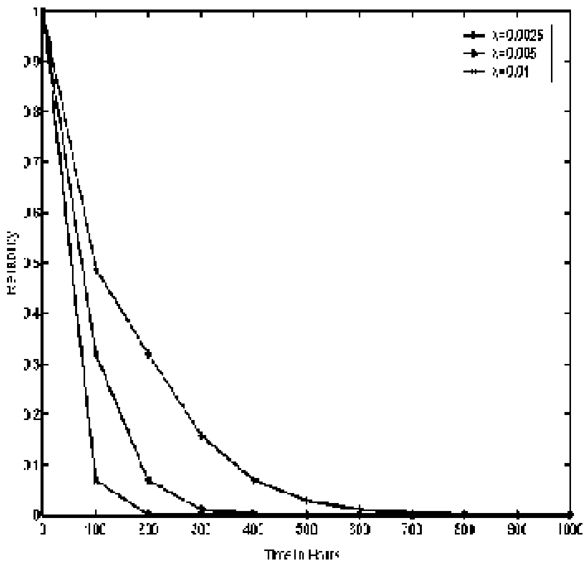


Fig.3: Reliability of hypercube of n=3

Consider a 3-D Hyper cube HC_3 as shown in fig.1 having the source and destination nodes labeled as 000 and 111 respectively. Out of the three adjacent nodes of source node let cube node 001 is faulty, Then the antipodal nodes of the two other adjacent nodes are 011 and 101.A discarded region is addressed simply by performing operation $\otimes$ on the labels of the faulty node and the antipodal nodes. When faulty node is 001,antipodal node of 100 is 011 and 001 $\otimes$ 011 = 0∗1 and 001 $\otimes$ 101 = ∗01.After removing this two discarded regions a maximal incomplete sub cube results which is shown in fig.2(b).The same operation can be performed by taking 100 and 010 cube nodes as faulty nodes. This results in the two other maximal incomplete sub cube as shown in fig.2(a) & 2(c).

VI. CONCLUSIONS

A fast, simple and effective procedure has been introduced for identifying all maximal fault free incomplete sub cubes from a faulty hypercube, taking maximum fault tolerance capacity is equal to the system dimension. In this process every fault free node is required to participate in the identification process. An efficient algorithm has been incorporated to support the derived reliability expression of the incomplete hypercube and the complexity of the proposed algorithm is found to be $O(n^2)$. This procedure is suitable for a hypercube with arbitrary node failure and can be used for all types of cube based topologies which are operating in a gracefully degradable manner after fault occur.

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