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Impact of Surrender Options on Traditional Life Insurance Policies for Life Insurance Firms

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Abstract

This study explores the impact of surrender options on insurance firms under fluctuating interest rates. Non-participating life insurance policies are analyzed, which are a seldom-discussed subject of research. Additionally, utilized actuarial methods of assessment are evaluated, rather than the routinely applied option-pricing model. This study simulates the stochastic interest rate process and describes levels of policy loyalty to establish policy cash flow in our analysis. Through simulation results, this study finds that the surrender option leads to significant differences in the cost burden of insurance firms, due to trends in future interest rates and the effects of policy loyalty. Fluctuating interest rates are shown to have an asymmetrical impact on these costs.

Keywords: Surrender Option, Loyalty, Stochastic Interest Rate Process

I. Introduction

In recent years, the popularity of participating policies (or equity-linked contracts) has generated considerable interest regarding the surrender option in policy contracts. For example, when analyzing equity-linked revenue assurance (RA) contracts, Brennan and Schwartz (1976) divide policyholder benefits into two classes -- fixed benefits and benefits derived from exercising options. Grosen and Jørgensen (1997) indicate that holding financial products with RA is equivalent to holding an American (surrender) option, because the value of such products can be calculated from the principle of American options. The American option can be defined as an option that can be exercised at any time, as opposed to the European option, which can only be exercised at maturity. Grosen and Jørgensen (2000) deconstruct participating policies into three components: risk-free bonds, bonus options, and surrender options. Bonuses are associated with European options, while the surrender process is associated with American options. Bacinello (2001) analyzes participating policies with minimum guaranteed returns and explores how the value of policies is calculated in relation to gains from different asset investment portfolios. Bacinello (2003a, 2003b) uses the model developed by Cox, Ross, and Rubinstein (1979) to calculate single premiums on participating policies with RA. Chu and Kwork (2006) utilize the option pricing method to appraise participating policies, considering guaranteed interest rates, bonuses, and default risks. In addition to contemplating the above factors, Siu (2005) also incorporates the surrender option in the evaluation of policies and hypothesizes that the random process of policy-linked targets can be represented by geometric Brownian motion. Gaillardetz (2008) studies the link between participating policies and interest rates. While maintaining that the stochastic process of interest rates and variation in the force of mortality are mutually independent, Gaillardetz incorporates the random interest rate model into the evaluation of participating policies. Uzelac and Szimayer (2014) value the equity-linked life insurance contracts with surrender guarantees by using a two-state regime-switching rational expectation model.

The shared feature of the products described above, whether participating policies or equity-linked contracts, is the fact that policy payment fluctuates in direct relation to variation in gains from asset investment portfolios. However, the dismal investment environment following the global financial crisis quenched enthusiasm for this type of insurance policy and the market returned to favoring traditional life insurance policies. Compared to the previously described products, traditional non-participating policies have a fixed amount of insurance, and the policy reserve within the contractual period is calculated according to predetermined interest rates. This makes the value of the policy less susceptible to changes in the financial market. However, this feature does not mean that the insurance contract remains wholly unchanged throughout the contract period. If fluctuations in the market interest rate intensify, and the policyholder finds that the policy value is clearly less than the surrender amount, he/she may consider rescinding the contract. This means that insurance companies face the adverse selection problem of policyholders having a surrender option, the exercise of which may affect policy costs. Therefore, this study focuses on how the surrender option in non-participating life insurance policies (a less common subject of research) impacts insurance firms under interest rate fluctuations.

As outlined in previous research, most previous studies focus on the features of American options when analyzing the price of the surrender option. Assuming that the policyholder is rational and on the premise of no arbitrage, these researchers numerically analyze the price of surrender options and identify the result as the cost incurred by the insurer through the provision of a surrender option. However, this study views insurance contracts as fundamentally different from general financial products. Whether a policyholder surrenders a policy is not merely a rational behavior that can be determined from a purely financial viewpoint without considering the effects of policy loyalty. Therefore, we focus on two key elements in assessing the cost of the surrender option: one being the probability that the policyholder exercises the surrender option, and the second being the financial burden placed on the insurer at the time that the policy is surrendered. These two factors are significantly influenced by interest rate fluctuations and levels of policy loyalty. Therefore, rather than apply the option-pricing model, this study simulates the stochastic process of interest rates and considers policy loyalty in assessing policy cash flow. Based on the results, we then use traditional actuarial methods to calculate price differences associated with whether or not the policy offers a surrender option. The price difference is seen to reflect the impact of surrender options on insurance firms. Results show that the surrender option leads to significant differences in the cost burden on insurance firms, due to future interest rate trends and the extent of policy loyalty. However, we also find that the impact of the surrender option on insurance firms is not necessarily negative.

The remainder of this paper is organized as follows: Section II discusses differences in cash flow and values that result from whether a policy provides a surrender option or not. Section III explores surrender option profit patterns, hypotheses regarding surrender rates, and the interest rate stochastic process, which affects the value of surrender options. Section IV describes the simulation parameters and computational results, and Section V summarizes the conclusions of this study and the policy implications in practical applications.

II. Policy Structure and Value

In order to narrow the research topic and simplify the model, this study uses non-participating term insurance policies as case studies in our research.[1] Referring to the actuarial principles of Bowers *et al*., (1997), we price policies and evaluate their reserve and value each quarter. Regarding the key variables affecting changes in policy value and insurance benefits, we analyze policy rights and obligations based on the assumption that the fluctuation of interest rates is a stochastic process, mortality rates are a known variable (calculated according to the period life table), and these variables are mutually independent. The following section explains traditional life insurance policies in detail, particularly the surrender option problem in policy contracts.

In practice, traditional life insurance policies are calculated using predetermined assumed interest rates. Considering a non-participating policy for N years, if the insured is x years of age at the time of purchasing the policy, and the insurable amount is I, then the net single premium (P) can be calculated using Equation (1):

$$P = \sum_{n=1}^{N} \left(I \cdot {}_{n-1|1}q_x \cdot e^{-\bar{r}_{0,N} \cdot n} \right) = {}_0V_{x:\overline{N}|}(\bar{r}_{0,N}) \tag{1}$$

where $\bar{r}_{0,N}$ is the assumed interest rate of the policy. ${}_{n-1|1}q_x$ represents the probability that the policyholder passes away within n-1 to n years, and $I \cdot {}_{n-1|1}q_x$ is the expected cash flow of

[1]The unique feature of non-participating insurance is that the insured pays a one-off fee when purchasing the policy. Should the policyholder pass away within the agreed period of insurance, then the insurance firm must pay a fixed amount of compensation until the insurance contract expires. On the basis of this model, the discussion can easily be extended to traditional life insurance policies such as instalment-payout life insurance or whole life insurance (whole life insurance can be seen as a special type of term life insurance, in which the insurance period is equal to ultimate age – such as 110 years old – minus entry age).

the policy at the n^{th} quarter. ${}_0V_{x:\overline{N}|}(\overline{r}_{0,N})$ represents the present value evaluated on the basis of an assumed interest rate. As indicated, the probability of surrender is not considered in the policy cash flow of Equation (1).

Based on the concept above, the present value of this policy, assessed on the basis of fluctuations in future interest rates, are:

$$ {}_0V_{x:\overline{N}|}(\tilde{r}_{0,N}) \equiv \sum_{n=1}^{N}\left(I \cdot {}_{n-1|1}q_x \cdot \prod_{t=0}^{n-1} e^{-\tilde{r}_t} \right) \tag{2} $$

In the equation above, $\{\tilde{r}_t : t \geq 0\}$ represents the stochastic process of future interest rates, $\tilde{r}_{0,N} = (\tilde{r}_0, \tilde{r}_1, ..., \tilde{r}_N)$.

Equations (1) and (2) both show the expected insurance benefits from the policy; the difference lies in the means by which interest rates are calculated in each equation. Equation (1) uses the assumed interest rate of the policy, while Equation (2) applies the estimated value of fluctuations in future interest rates; insurance companies usually consider the type of contract and trends in the market interest rates when determining the assumed interest rate. This enables the net single premium to reflect the reasonable initial value of the policy. In other words, the relation between Equations (1) and (2) should be[2]:

$$ P = {}_0V_{x:\overline{N}|}(\overline{r}_{0,N}) \approx E[{}_0V_{x:\overline{N}|}(\tilde{r}_{0,N})] $$

In Equation (1), we use ${}_tV_{x:\overline{N}|}(\overline{r}_{0,N})$ to indicate the policy reserve at year t. This reserve is the amount that the life insurance company provides in response to contingent claims from policyholders. This amount can be calculated according to the assumed interest rate, using either: a) the future actuarial value of the net premiums paid – the future actuarial value for the insurance to age $x+t$, or b) the present actuarial value for the insurance from age $x+t$ – the present actuarial value of future net premiums payable. The former is known as the retrospective method, while the latter is termed the prospective method. The outcome tends to be consistent regardless of what method is used for calculation. The policies analyzed in this research are single-premium policies, meaning that after the single payment is made at the commencement of the policy, no subsequent payments are required. Employing the prospective method, the policy reserve at the end of year t (${}_tV_{x:\overline{N}|}(\overline{r}_{0,N})$) would be as follows:

[2] $E[{}_0V_{x:\overline{N}|}(\tilde{r}_{0,N})]$ is the expected present policy value under stochastic interest rates. That is,

$E[{}_0V_{x:\overline{N}|}(\tilde{r}_{0,N})] = \sum_{m=1}^{M} \frac{{}_0V_{x:\overline{N}|}(\tilde{r}^{(m)}_{0,N})}{M}$, where ${}_0V_{x:\overline{N}|}(\tilde{r}^{(m)}_{0,N})$ is the present value under the m^{th} stochastic interest process.

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$$ {}_{t}V_{x:\overline{N}|}(\overline{r}_{0,N}) = \sum_{n=1}^{N-t} \left(I \cdot {}_{n-1|1}q_{x+t} \cdot e^{-\overline{r}_{0-N} \cdot n} \right) \tag{3} $$

If we use the same principle for Equation (3), but recalculate using the market interest rate ($\tilde{r}_t$) at the end of year t, the policy value at the end of year t (${}_{t}V_{x:\overline{N}|}(\tilde{r}_t)$) would be as follows:

$$ {}_{t}V_{x:\overline{N}|}(\tilde{r}_t) = \sum_{n=1}^{N-t} \left(I \cdot {}_{n-1|1}q_{x+t} \cdot e^{-\tilde{r}_t \cdot n} \right) \tag{4} $$

Both Equations (3) and (4) indicate the expected value of policy renewal at the end of year *t*; the difference is that the former uses the assumed interest rate for calculation, while the latter bases its calculation on the market interest rate at year *t*.

During the valid contract period, if the insured exercises the surrender option, the insurer pays to the policyholder a surrender value, which is calculated as the policy reserve after subtracting termination fees. In practice, the longer the policy is held (as a proportion of the premium payable period), the more the termination fee is reduced. However, if the policy is a single-premium policy, the termination fee is not reduced. In this study, therefore, the surrender value of the single-premium policy at year t (${}_{t}S_{x:\overline{N}|}$) is equal to the policy reserve at the time of surrender (${}_{t}S_{x:\overline{N}|} = {}_{t}V_{x:\overline{N}|}(\overline{r}_{0,N})$). If the policy value is lower than the surrender value, this may induce the policyholder to exercise his/her surrender option. This scenario is explained in the next section.

In the circumstance addressed above, although the policyholder would be paid the surrender value if he/she surrenders the policy during the valid contract period, traditional actuarial methods generally reflect only claim costs when assessing premiums and neglect to consider surrender options. In reality, however, insurance firms that sell life insurance products also sell the right to file a death claim and the right to a surrender option. Incorporating the possibility of surrender into the discussion, we use R_t to represent the surrender rate at year t of the policy. As shown in Figure 1, the payment processes of this policy are different from those shown in Figure 1, and the calculation of the present policy value is modified as follows:

$$ {}_{0}V'_{x:\overline{N}|}(\tilde{r}_{0,N}) = \sum_{n=1}^{N} \left\{ \left[I \cdot {}_{n-1|1}q_x \cdot (1-R_{n-1}) + {}_{n}p_x \cdot \prod_{s=1}^{n-1}(1-R_s) \cdot R_n \cdot {}_{n}S_{x:\overline{N}|} \right] \cdot \prod_{t=0}^{n-1} e^{-\tilde{r}_t} \right\} \tag{5} $$

When we compare Equation (2) with Equation (5), the latter is the policy value with the possibilities of surrender. The difference between Equation (2) and Equation (5) then can be viewed as the impact on actuarial cost when considering the surrender option.

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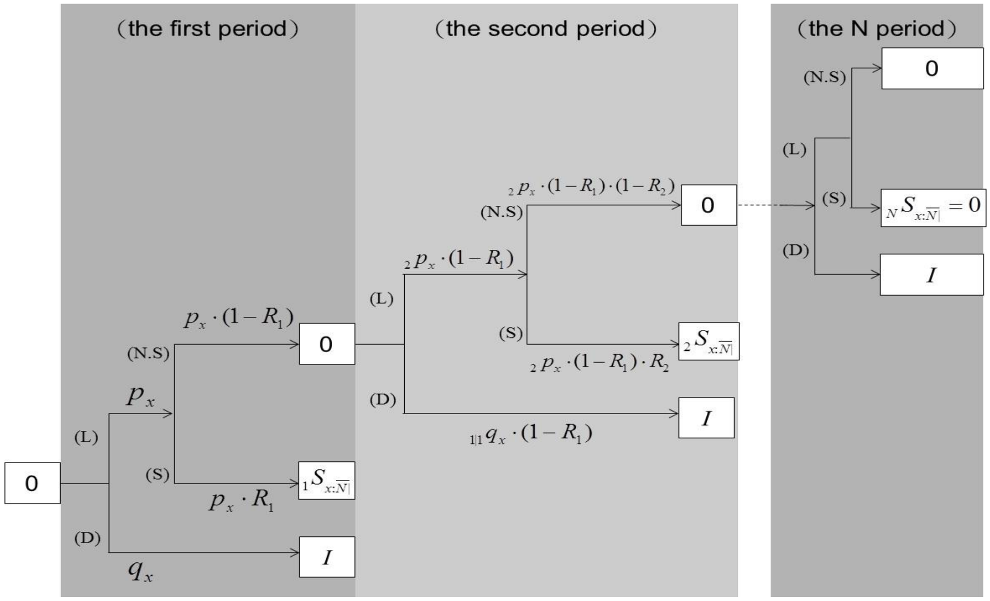

Note: (N.S.) and (S) respectively illustrate the flow of events if the policy is continued or surrendered. R_t indicates the probability of surrender at time t; other symbols are defined in Figure 1.

Figure 1: Payment on a Non-Participating Term Policy with Consideration of a Surrender Probability

III. Model Settings

III.1 The assessment of the Surrender Ratio

Generally speaking, unless the policy has a lock-up clause, the policyholder is entitled to a surrender option during the valid contract period. If the insured surrenders the policy during this period, he/she forfeits the value of that policy; however, the insurance company pays the surrender value to the policyholder. This is equivalent to the policyholder selling the policy value back to the insurance company in the form of the surrender price. In other words, the policyholder has executed the American option, wherein the underlying asset is the policy value and the strike price is the surrender value. Therefore, the benefits that the policyholder really receives if they surrender at year t are the difference between the policy value and the surrender value (${}_tS_{x:\overline{N}|} - {}_tV_{x:\overline{N}|}(\%)$). The value of this gain, which varies due to fluctuation in market interest rates, affects the willingness of the policyholder to surrender the policy. For example, when market interest rates are high, if the policyholder feels that the policy premiums are somewhat high (assuming that the interest rate is significantly lower than the market interest rate), he/she may wish to surrender the option to recover cash. On the other hand, the policyholder may surrender the policy because he/she is looking to replace the policy with a new insurance product offering lower premiums (supposing that the assumed interest rate of the new product fully reflects market interest rates and exceeds the assumed interest rate of the original policy). However, life insurance policies differ from regular financial products, and the attitude of the policyholder towards the surrender option is more easily influenced by factors such as how satisfied he/she is with the customer service provided by the insurer and how easily he/she can obtain an alternative policy after surrender. We combine these elements into a single factor termed "loyalty." In other words, the policy surrender rate is largely determined by changes in policy value and loyalty (*L*). However, because statistics providing this type of information are not readily available, this study combines the factors above and uses Equation (6) to describe the surrender rate at time *t*:

$$R_t = \left[1-\left(\frac{{}_tV_{x:\overline{N|}}(\tilde{r}_t)}{{}_tS_{x:\overline{N|}}}\right)^{1/L}\right]\cdot B_t = \left[1-\left(\frac{{}_tV_{x:\overline{N|}}(\tilde{r}_t)}{{}_tV_{x:\overline{N|}}(\overline{r}_{0,N})}\right)^{1/L}\right]\cdot B_t \qquad (6)$$

In Equation (6), B_t is the indicator variable. If the surrender value exceeds the policy value, $B_t = 1$; if not, $B_t = 0$.

The variation for surrender probabilities as reflected in Equation (6) is illustrated in Figure 2. That is, if the policy value falls below the surrender value, the policyholder is motivated to exercise its surrender option. The gain from surrendering the policy increases as the policy value declines, which in turn raises the probability of surrendering. For example, if $L = 1$, the probability of the policy being surrendered increases linearly as the policy value declines. If *L* is less than 1, the curve of the surrender probability shows a concave-to-origin shape, and the lower the value of *L*, the greater the curvature. When *L* approaches 0 (extreme disloyalty), the surrender probability curve approaches a right angle, indicating that as soon as the policy value falls below the surrender value, the policyholder must surrender the policy. In contrast, if L exceeds 1, the surrender probability curve demonstrates a convex-to-origin shape, and the greater the value of *L*, the greater the curvature. When *L* approaches ∞ (extreme loyalty), the surrender probability curve approaches a right-angle, indicating that although the policy value has been dramatically reduced, the policyholder does not exercise his/her surrender option lightly unless the policy value is reduced to 0. Although the policyholder may still surrender the policy for other reasons, even when the surrender value is lower than the policy value, this study focuses on policy surrender induced by interest rate fluctuation and the resulting changes in policy value. Therefore, when the surrender value is lower than the policy value, this study does not consider that the policyholder still insists on surrendering the policy.

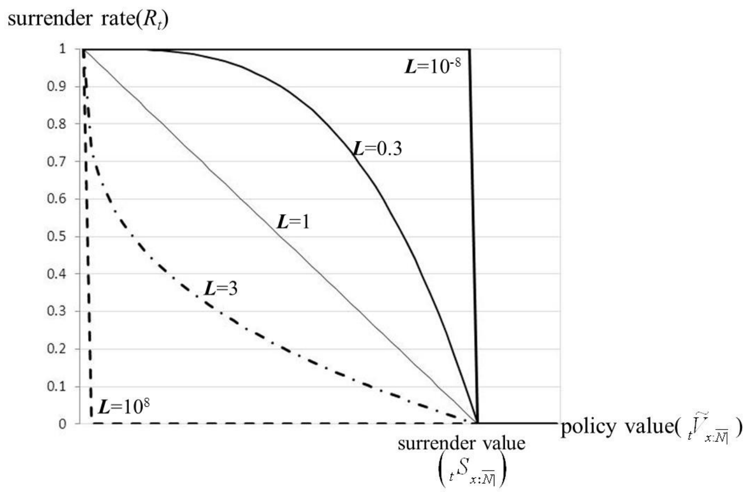

Figure 2: Fluctuation of Surrender Probability with Varying Degrees of Loyalty

We incorporate the assumed surrender rate formula of Equation (6) into Equation (5) to re-evaluate policy value under increased probability of surrender. This value reflects the re-estimated cost of the policy. The difference (${}_0V'_{x:\overline{N}|}(\tilde{r}_{0,N}) - {}_0V_{x:\overline{N}|}(\tilde{r}_{0,N})$) evidenced from comparing Equation (5) and Equation (2) highlights the increased burden of cost incurred by an insurance company when offering a surrender option.

III.2 The Stochastic Interest Rate Process

The general stochastic process of interest rates ($\tilde{r}_t$), risk-adjusted can be expressed as follows:

$$d\tilde{r}_t = k\left(\theta - \tilde{r}_t\right)dt + \sigma_{\tilde{r}}\tilde{r}_t^{\beta}\, dZ_t$$

The parameter $\sigma_{\tilde{r}}$ is the diffusion volatility coefficient; k is the reverting rate and θ is long-term equilibrium of the interest rates. The order under a volatility-term interest rate presents different kinds of stochastic processes models. For example,

Vasicek[3] model $\beta = 0$

CIR[4] model $\beta = 1/2$

When using the Vasicek model, one might come up with a negative interest rate, which cannot happen in the real world. In this paper, the CIR model is used instead. Following Baz and Das (1996), the stochastic interest rate with a jump-diffusion under the CIR framework is calculated as:

$$d\tilde{r}_t = k\left(\theta - \tilde{r}_t\right)dt + \sigma_r\sqrt{\tilde{r}_t}\, dZ_t + JdN_t$$

where J is a random jump amplitude, which follows a normal distribution, and the jump arrivals follow a Poisson process dN_t with λ, which is the mean number of jumps per year.

III.3 Least Squares Monte Carlo Approach

The problem of American-type options is the existence of several possible exercise dates

[3] See Vasicek (1977).
[4] See Cox *et al*. (1985).

during the life of the option. Therefore, the option holdler must determine whether to exercise the option early. The decision depends on which is larger: the value of exercising the option immediately or holding.

Longstaff and Schwartz[5] (2001) derived a least-square regression function comprising several basis functions. All of the arguments of orthogonal basis functions must follow the Markov process. In their article, the elements of the regression function are based mainly on underlying asset prices with the regression function used to obtain the conditional continuation value. Thus, the optimal exercise rule involves comparing the values of conditional continuation with those of immediate exercise and exercising if advantageous to do so. These procedures are simply repeated recursively and then all of the cash flow is discounted to time zero to estimate the value of the American option. The Least Squares Monte Carlo provides a fast and easy approach to determine appropriate exercise strategies at each decision date.

IV. Numerical Results

The model in the previous section shows that in traditional life insurance policies, an assumed interest rate is used to calculate premiums and reserve. When fluctuation in market interest rates drives change in policy value, this may cause policyholders to exercise their surrender option, which in turn, alters the policy cash flow. Therefore, the cost incurred by an insurance company when issuing a policy is closely related to future interest rate trends. Using the CIR model, this study simulates the stochastic interest rate process to estimate and observe changes in policy value, which are then compared to the surrender value in each period. Using these results, we analyze the potential behavior of policyholders in exercising their surrender options and proceed to quantify the cost of surrender options to insurance companies.

To analyze and evaluate the impact of surrender, we use the following actuarial assumptions as the basis of simulation. First, we assume that the policyholder is a 30-year-old male who has purchased a 20-year term life insurance in the amount of $1 million with a single premium. We use the Taiwan standard ordinary experience mortality table (TSO) to calculate the annual mortality probability of the policyholder and apply the results to calculate the premium, policy reserve, policy value, and surrender value. Referring to Das parameter settings (1997), we enter the following into the CIR model to stimulate the stochastic interest rate process: an initial interest rate (r_0) of 6 percent, a mean value (θ) of 10 percent, a standard deviation (σ_r) of 5 percent, and a mean-reverting rate (κ) of 0.5.

IV.1 Present Value and Reserve at Time 0 (Without Surrender)

In accordance with the parameters described above, we evaluate present policy value in relation to fluctuation in market interest rates. For a contract without a surrender option, we incorporate the simulated interest rate as the discount rate into Equation (2) to compute expected present policy value ($E[{}_0V_{x:\overline{N}|}(\tilde{r}_{0,N})]$). By using the parameters above, the expected

[5] Please refer to Longstaff and Schwartz (2001) for more details

present policy value is 16,584. To achieve actuarial estimation results, this study performs 200,000 path simulations when solving for subsequent values. If the net single premium reflects the expected present policy value, which is $P={}_0V_{x:\overline{N|}}(\bar{r}_{0,N})=E[{}_0V_{x:\overline{N|}}(\tilde{r}_{0,N})]=16{,}584$, then the discount rate is 9.54 percent. This is because we can use Equation (2) to inversely calculate the interest rate during the contract period. Ideally, $\bar{r}_{0,N}$ is viewed as a weighted interest rate reflecting future cash flow and potential changes in market interest rates. However, it is seen as the internal rate of return on the policy. In accordance with the assumed interest rate, we calculate policy reserve at time 0 using Equation (3). This reserve is also the surrender value that the policyholder can claim when exercising his/her surrender option. Because the policy in question has a high assumed interest rate (9.54 percent) and the policyholder is young (30 years old), the accrued interest on the reserve exceeds the expected death payout, which incrementally increases the reserve. As the expected death payout cumulatively increases, the reserve begins to decrease in the 10th year of the policy until it reaches 0 at the end of the policy.

If the long-term equilibrium rate (θ) of the CIR model (θ) is changed to 6 percent (while other parameters remain unchanged), the present policy value increases to $23,322, and the internal rate of return will be 5.92 percent. Fluctuation in reserve follows a pattern of increase followed by decrease (although the reserve begins to progressively decrease from the seventh year of the policy). If the long-term equilibrium rate (θ) is changed to 2 percent (while other parameters remain unchanged), the present policy value is $35,750, and the internal rate of return is 2.28 percent. This low assumed interest rate (2.28 percent) causes the reserve interest to remain below the expected death payout; therefore, during the policy contract period, the reserve is in a state of monotone decrease (Figure 3).

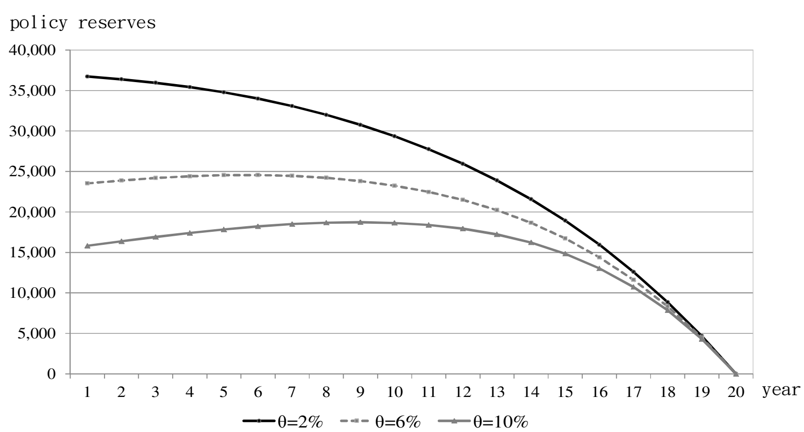

Figure 3: Change of Policy Reserves during the Period of Insurance

IV.2 Present Value (Considering the Possibility of Surrender)

Incorporating the possibility of surrender (Figure 1), we use the interest rate simulated from the original parameters [long-term equilibrium rate (θ) of CIR model = 10 percent] as the discount rate and input it into Equation (6). From Table 1, we can see that when policy loyalty is at an extreme low (L=0.00001), the re-estimated expected present value ($E[{}_0V'_{x:\overline{N|}}(\tilde{r}_{0,N})]$) increases to $17,473 (an increase of 5.36 percent). The difference of $889 reflects the cost of the surrender option that must be borne by the insurance company. This cost, however, decreases as loyalty increases. When loyalty (L) increases to 1, the cost of the surrender option is reduced to $450 (=$17,034-$16,584). When loyalty is at an extreme high (L=10000), this implies that the policyholder is extremely unlikely to be in favor of

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surrendering the policy. At this point, the policy value approaches a state of no surrender, which means that the cost of the surrender option nears 0.

In the simulated analysis above, if we change the long-term equilibrium rate to 2 percent (while all other parameters remain unchanged), the fluctuation of the expected present value shows reverse variation ($E[{}_0V'_{x:\overline{N|}}(\tilde{r}_{0,N})] < E[{}_0V_{x:\overline{N|}}(\tilde{r}_{0,N})]$). When the long-term equilibrium rate is changed to 6 percent (while all other parameters remain unchanged), it is uncertain as to whether the expected present value of a policy with the surrender option is higher or lower than a policy without a surrender option ($E[{}_0V'_{x:\overline{N|}}(\tilde{r}_{0,N})] \gtrless E[{}_0V_{x:\overline{N|}}(\tilde{r}_{0,N})]$).

Table 1: Evaluation of Present Value (with/without Considering Surrender)

Average Interest Rate	Without considering policy surrender, the policy expected present value ($E[{}_0V_{x:\overline{N|}}(\tilde{r}_{0,N})]$)	Consideration of policy surrender, the policy expected present value ($E[{}_0V'_{x:\overline{N|}}(\tilde{r}_{0,N})]$)		
		L=10000	L=1	L =0.00001
θ=2%	34,261	34,261(0%)	33,682(-0.017%)	33,123(-3.32%)
θ=6%	23,294	23,294(0%)	23,463(+0.73%)	3,270(-0.01%)
θ=10%	16,584	16,584(0%)	17,034(+2.71%)	17,473(+5.36%)

Note: The figures in brackets "[ ]" indicate the differences in the expected present value that results from considering surrender compared to not considering surrender: $E[{}_0V'_{x:\overline{N|}}(\tilde{r}_{0,N})] - E[{}_0V_{x:\overline{N|}}(\tilde{r}_{0,N})]$ ∘The figures in parentheses "( )" indicate the differences in the fluctuation rate of the present value that results from considering surrender compared to not considering surrender: $\left[E[{}_0V'_{x:\overline{N|}}(\tilde{r}_{0,N})] - E[{}_0V_{x:\overline{N|}}(\tilde{r}_{0,N})]\right] / E[{}_0V_{x:\overline{N|}}(\tilde{r}_{0,N})]$. We assume that the policyholder is a 30-year-old male who has purchased a 20-year term life insurance in the amount of $1 million with a single premium.

IV.3 Analysis of the Influence of Surrender Possibility on Present Value

The simulation results above highlight two important points:

1. The attitude of the policyholder towards surrendering the policy is determined by comparing the surrender value with the policy value (evaluation is done using the current interest rate). Apart from degree of loyalty, the inclination of the policyholder to surrender the policy during the contract period (assumed to be time t) is also affected by comparison of the current interest rate ($\tilde{r}_t$) and the assumed interest rate ($\overline{r}_{0,N}$). When the market interest rate at time t exceeds the assumed interest rate ($\tilde{r}_t > \overline{r}_{0,N}$), the policyholder feels that the policy value (${}_tV_{x:\overline{N|}}(\tilde{r}_t)$) is lower than the surrender value (${}_tS_{x:\overline{N|}} = {}_tV_{x:\overline{N|}}(\overline{r}_{0,N})$), which provides motivation to surrender the policy and triggers some policyholders to choose to exercise their surrender options (R_t >0).

2. The impact of policy surrender on an insurance company is determined by comparing the surrender value with the policy value (evaluation is done using the continued interest rate). If policyholders surrender their policies during the valid contract period (assumed to be time t) because the market interest rate exceeds the assumed interest rate, but then the post-surrender market interest rate falls below the assumed interest rate ($\overline{r}_{t,N} < \overline{r}_{0,N}$), we must consider that the present value as reflected by the subsequent interest rate ($\tilde{r}_{t,N}$)

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actually exceeds the surrender value (${}_tV_{x:\overline{N}|}(\tilde{r}_{t\sim N}) = {}_tV_{x:\overline{N}|}(\overline{r}_{t\sim N}) > {}_tV_{x:\overline{N}|}(\overline{r}_{0\sim N})$)[6]. The policy surrender in this case actually benefits the insurance company (for the policyholder, the surrender option is a negative value). Although the downturn in the interest rate initially causes spread loss to the company, the interim high interest rate may cause some policyholders to surrender their policies, which lightens the impact of spread loss on the insurance company. In contrast, if the interest rate subsequently increases, then surrender does not benefit the insurance company. Subsequent high interest rates initially result in spread benefit for the company, but they can potentially reduce company profit from this spread benefit by inducing some policyholders to surrender their policies.

The above analysis allows us to more clearly understand how policy value fluctuates in different circumstances (Table 1). When the long-term equilibrium rate falls below its initial value (θ =2%< r_0 =6%), the interest rate initially experiences a downturn during the contract period, but the mean value of subsequent fluctuation approaches the θ level. This type of interest rate fluctuation easily causes the long-term equilibrium rate in the latter half of the contract period to fall below that of the overall contract duration ($\overline{r}_{t,N} < \overline{r}_{0,N}$).

As a result, policyholders surrendering their policies mid-term (at time t) benefit the insurance company (${}_tV_{x:\overline{N}|}(\overline{r}_{t,N}) > {}_tV_{x:\overline{N}|}(\overline{r}_{0,N})$) and reduce policy value (the lower the policy value, the lower the payout cost of the policy). The lower the degree of loyalty, the higher the proportion of policy surrender, which is reflected in lower policy value.
($E[{}_0V'_{x:\overline{N}|}(\tilde{r}_{0,N})]\Big|_{L=0.00001} = 35{,}359 < E[{}_0V'_{x:\overline{N}|}(\tilde{r}_{0,N})]\Big|_{L=1} = 35{,}711 < E[{}_0V'_{x:\overline{N}|}(\tilde{r}_{0,N})]\Big|_{L=100000} = 35{,}750$).

In contrast, when the long-term equilibrium rate exceeds its initial value (θ =10%< r_0 =6%), lower degrees of loyalty reflect higher policy value ($E[{}_0V'_{x:\overline{N}|}(\tilde{r}_{0,N})]\Big|_{L=0.00001} = 16{,}505 <$ $E[{}_0V'_{x:\overline{N}|}(\tilde{r}_{0,N})]\Big|_{L=1} = 16{,}257 < E[{}_0V'_{x:\overline{N}|}(\tilde{r}_{0,N})]\Big|_{L=100000} = 16{,}052$).

IV.4 Changes in the Stochastic Process (Jump Process) of Interest Rates and the Influence of these Changes on Present Policy Value

The analysis above allows us to understand how the surrender option affects present value under different interest rate trends. Due to the volatility of the financial environment in recent years, market interest rates often change significantly and unexpectedly. This study further discusses how the effect of the surrender option on present policy value changes if there is a jump model in the fluctuation process of interest rates. We extend the initial interest rate parameters (r_0=6%、θ=10%、σ_r=5% and κ=0.5) and employ the model of Baz and Das (1996) to simulate the jump process of interest rates. We follow the procedures described above to calculate and observe the influence of the surrender option on present policy value. Results show that when the mean value of the interest rate jump is 0 or a negative value, the interest rate jump does not significantly influence present policy value. When the value of the interest rate jump is positive, however, it noticeably increases present policy value (Table 2). This is

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[6] The present value calculated using interest rate subsequent to time t ($\tilde{r}_{t,N}$) is as follows:

$${}_tV_{x:\overline{N}|}(\tilde{r}_{t,N}) = \sum_{n=t+1}^{N}\left(I \cdot {}_{n-1|1}q_x \cdot \prod_{s=t}^{n-1} e^{-\tilde{r}_s}\right) \text{ ; if } {}_tV_{x:\overline{N}|}(\tilde{r}_{t,N}) = \sum_{n=t+1}^{N}\left(I \cdot {}_{n-1|1}q_x \cdot e^{-\overline{r}_{t,N}\cdot n}\right) = {}_tV_{x:\overline{N}|}(\overline{r}_{t,N}) \text{ , } \overline{r}_{t,N} \text{ is}$$

seen as the weighted mean interest rate of the policy subsequent to time t.

because the interest rate jump induces more policyholders to surrender their policies. Comparatively speaking, a dip in the interest rate is not nearly as effective in reducing surrender rate (this is because as soon as the interest rate falls below the assumed interest rate, the policy value exceeds the surrender value; in such a case, the interest rate is not an incentive to surrender a policy). Therefore, an interest rate jump causes an increase in surrender rate. When the long-term equilibrium rate exceeds the initial rate, surrender does not benefit the insurance company. An increase in surrender rate increases present policy value.

Table 2: Valuation of Policy Present Value Under a Stochastic Interest Rate with a Jump Component

Mean number of jumps per year	Standard deviation of jump amplitude	Policy expected present value		
		L=10000	*L*=1	*L* =0.00001
Panel A: *Mean of jump amplitude*(μ) *equal to* -15%				
λ=20	γ=5%	16,448	16,632(+1.12%)	17,285(+5.09%)
	γ=10%	16,448	16,633(+1.12%)	17,285(+5.09%)
	γ=15%	16,449	16,636(+1.14%)	17,286(+5.09%)
	γ=20%	16,449	16,640(+1.16%)	17,288(+5.10%)
λ=30	γ=5%	16,447	16,629(+1.11%)	17,277(+5.05%)
	γ=10%	16,447	16,629(+1.11%)	17,278(+5.05%)
	γ=15%	16,448	16,630(+1.11%)	17,278(+5.05%)
	γ=20%	16,447	16,630(+1.11%)	17,279(+5.06%)
Panel B: *Mean of jump amplitude*(μ) *equal to* 0				
λ=20	γ=5%	16,630	17,112 (+2.90%)	17,508(+5.28%)
	γ=10%	16,634	17,118(+2.91%)	17,515(+5.30%)
	γ=15%	16,592	17,157(+3.41%)	17,532(+5.67%)
	γ=20%	16,411	17,124(+4.34%)	17,522(+6.77%)
λ=30	γ=5%	16,602	17,061(+2.76%)	17,483(+5.31%)
	γ=10%	16,616	17,133(+3.11%)	17,520(+5.44%)
	γ=15%	16,480	17,133(+3.96%)	17,527(+6.35%)
	γ=20%	16,132	17,032(+5.58%)	17,485(+8.39%)
Panel C: *Mean of jump amplitude*(μ) *equal to* +15%				
λ=20	γ=5%	9,191	13,795(+50.09%)	14,959(+62.76%)
	γ=10%	9,073	13,631(+50.24%)	14,831(+63.46%)
	γ=15%	8,911	13,392(+50.29%)	14,663(+64.55%)
	γ=20%	8,715	13,106(+50.38%)	14,450(+65.81%)
λ=30	γ=5%	5,378	8,936(+66.16%)	9,739(+81.09%)
	γ=10%	5,404	8,947(+65.56%)	9,786(+81.09%)
	γ=15%	5,452	8,954(+64.23%)	9,873(+81.09%)
	γ=20%	5,503	8,960(+62.82%)	9,966(+81.10%)

Note: The figures in parentheses "( )" indicate the rate of fluctuation in present policy value (as compared to present value of policy loyalty (*L*)=10000). We assume that the policyholder is a 30-year-old male who purchases a 20-year term life insurance in the amount of $1 million with a single premium.

V. Conclusions

Following a global financial crisis, sales of investment-linked insurance policies sharply decline and traditional life insurance products again are favored by the market. The main benefit of such products is guaranteed stability. However, because such products use a fixed assumed interest rate, insurance companies bear heavy spread loss when impacted by low interest rates. By contrast, when market interest rates increase, insurance companies of course avoid spread loss. As traditional life insurance products tend to have low assumed interest rates, a wave of policy surrenders results. This condition puts pressure on the company to realize assets, thus placing it at liquidity risk and reducing profit from spread benefit.

To observe how the surrender option of non-participating life insurance policies affects insurance firms, this study proposes a new viewpoint on evaluating the surrender option. From this perspective, we conduct quantitative analysis of the value of surrender options under different scenarios involving interest rate fluctuation. We find that the surrender option affects insurance firms differently due to varying future interest rate trends – some influences are positive and others negative.

The current method of charging a termination fee for the costs incurred by the company when a policy is surrendered transfers some of the surrender cost to the policyholder, and thus reduces the benefit of the surrender option. This benefit reduction, in turn, diminishes the willingness of policyholders to exercise this option. However, charging this fee does not completely offset the potential costs of surrender to an insurance company. A termination fee is often not charged in single-premium policies. If insurance companies record real-life data during market fluctuation, policyholder loyalty is calculated. With such data, the model of this study aids such businesses in evaluating and responding to the potential impact of the surrender option, especially when facing future volatility in financial markets.

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