

Chapter 5
Rational Exponents and Radical Functions

Section 5-5
Performing Function Operations

Operations on Functions

You have learned how to add, subtract, multiply, and divide polynomial expressions. These operations can also be defined for functions.

Core Concept

Operations on Functions

Let f and g be any two functions. A new function can be defined by performing any of the four basic operations on f and g .

Operation	Definition	Example: $f(x) = 5x$, $g(x) = x + 2$
Addition	$(f + g)(x) = f(x) + g(x)$	$(f + g)(x) = 5x + (x + 2) = 6x + 2$
Subtraction	$(f - g)(x) = f(x) - g(x)$	$(f - g)(x) = 5x - (x + 2) = 4x - 2$
Multiplication	$(fg)(x) = f(x) \cdot g(x)$	$(fg)(x) = 5x(x + 2) = 5x^2 + 10x$
Division	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$	$\left(\frac{f}{g}\right)(x) = \frac{5x}{x + 2}$

The domains of the sum, difference, product, and quotient functions consist of the x -values that are in the domains of both f and g . Additionally, the domain of the quotient does not include x -values for which $g(x) = 0$.

EXAMPLE 1 Adding Two Functions

Let $f(x) = 3\sqrt{x}$ and $g(x) = -10\sqrt{x}$. Find $(f + g)(x)$ and state the domain. Then evaluate the sum when $x = 4$.

EXAMPLE 2 Subtracting Two Functions

Let $f(x) = 3x^3 - 2x^2 + 5$ and $g(x) = x^3 - 3x^2 + 4x - 2$. Find $(f - g)(x)$ and state the domain. Then evaluate the difference when $x = -2$.

EXAMPLE 3 Multiplying Two Functions

Let $f(x) = x^2$ and $g(x) = \sqrt{x}$. Find $(fg)(x)$ and state the domain. Then evaluate the product when $x = 9$.

EXAMPLE 4 Dividing Two Functions


Let $f(x) = 6x$ and $g(x) = x^{3/4}$. Find $\left(\frac{f}{g}\right)(x)$ and state the domain. Then evaluate the quotient when $x = 16$.

SOLUTION

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{6x}{x^{3/4}} = 6x^{(1-3/4)} = 6x^{1/4}$$

The domain of f consists of all real numbers, and the domain of g consists of all nonnegative real numbers. Because $g(0) = 0$, the domain of $\frac{f}{g}$ is restricted to all *positive* real numbers. When $x = 16$, the value of the quotient is

$$\left(\frac{f}{g}\right)(16) = 6(16)^{1/4} = 6(2^4)^{1/4} = 12.$$

-  **2.** Let $f(x) = 3x$ and $g(x) = x^{1/5}$. Find $(fg)(x)$ and $\left(\frac{f}{g}\right)(x)$ and state the domain of each. Then evaluate $(fg)(32)$ and $\left(\frac{f}{g}\right)(32)$.

EXAMPLE 5 Performing Function Operations Using Technology

Let $f(x) = \sqrt{x}$ and $g(x) = \sqrt{9 - x^2}$. Use a graphing calculator to evaluate $(f + g)(x)$, $(f - g)(x)$, $(fg)(x)$, and $\left(\frac{f}{g}\right)(x)$ when $x = 2$. Round your answers to two decimal places.

SOLUTION**EXAMPLE 6** Solving a Real-Life Problem

For a white rhino, heart rate r (in beats per minute) and life span s (in minutes) are related to body mass m (in kilograms) by the functions

$$r(m) = 241m^{-0.25}$$

and

$$s(m) = (6 \times 10^6)m^{0.2}.$$

- Find $(rs)(m)$.
- Explain what $(rs)(m)$ represents.

SOLUTION