Comparative Analysis of Various Beamforming Techniques for Interference Cancellation in GPS receivers

N. Raj Kamala
Department of ECE
Sir C. R. Reddy College of Engineering, Eluru
rajkamalanethala476@gmail.com

K. J. Silva Lorraine
Assistant Professor, Department of ECE
Sir C. R. Reddy College of Engineering, Eluru
jacobsilvalorraine@yahoo.com

ABSTRACT

Beamforming refers to techniques for electronically focusing and steering the beam formed by the multi-element arrays in the direction of the incoming signal so as to maximize the reception of the incoming signal. It is achieved by combining antenna array in such a way that signals at particular angles experience constructive interference while others experience destructive interference. In this paper, the performance comparison of different beamforming techniques like phase shift, MVDR and LCMV has been presented for the cancellation of interference in GPS receivers. The overall analysis on phase shift, MVDR and LCMV is extended with different number of antenna elements and different Elevation angles. It can be observed from the performance comparison that LCMV beamforming technique offers better performance.

INDEX TERMS- Beamforming, GPS, phase shift, LCMV, MVDR, Direction of Arrival (DOA)

I. INTRODUCTION

The Global Positioning System (GPS) is a satellite-based navigation system that was developed by the U.S. Department of Defence (DOD) in the early 1970s.Continuous positioning and timing information anywhere in the world under any weather conditions is provided by GPS. GPS is a one-way-ranging (passive) system because it serves an unlimited number of users as well as being used for security reasons. Global Positioning System (GPS) jammers add excessive noise to the received low power GPS signals and have capability to either weaken or completely destroy the positioning performance of GPS receivers for both civilian and military users. Array radiation pattern is controlled to maximize gain in satellite directions and to create null regions toward jammers[2]. Adaptive beamforming methods provide necessary weighting coefficients to form desired radiation patterns using received data. Array processing plays a vital role in diverse application areas such as radar, sonar, communications, satellite navigation and radio astronomy. Adaptive beamforming applies spatial filtering for enhancing the desired signals while suppressing interferences and noise at the output of the sensor array [1–3]. An effective data dependent beamformer referred to as Minimum Variance Distortionless Response (MVDR), was proposed by Capon in [4]. The array configuration is a principal factor in determining the MVDR beamformer performance [5–7]. Although the nominal array configuration is uniform, it is not necessarily optimum in every sense. Increasing the number of antennas in a uniform linear array may reach the objective but at the expense of additional costly Radio Frequency (RF) chains. Sparse antenna arrays with non-uniform inter-element spacing were utilized in MIMO system as an effective solution to reduce the system's complexity and cost, yet retain multifaceted benefits [8,9]. It has been shown that sparse arrays with a given number of antennas placed at an optimum subset of grid locations, connecting with the RF front-end receivers, can preserve a large aperture while reducing system complexity [10,11]. Different design objectives have been proposed for optimum sparse arrays [12–17]. From the perspective of signal enhancement and interference reduction, the output signal-to-interference-plus-noise ratio (SINR) metric is considered an appropriate measure in optimum sparse array design and construction. It is well-known that, for a given array configuration, the MVDR beamformer is sensitive to steering vector errors, which can occur as a result of signal direction of arrival (DOA) mismatch. This causes potential degradation to the output SINR. There are several methods developed for the remedial of the DOA mismatch problem [18]. Among them, the Linearly Constrained Minimum Variance (LCMV) beamformers are most commonly used. The LCMV beamformer imposes a set of linear constraints to flatten the beampattern in the region of interest (ROI) , and as such, accounts for possible biases in the desired source DOA. An iterative second order cone programming (SOCP) algorithm was utilized in to address quadratic pattern magnitude constraints for robust beamforming. These methods, although improve robustness by formulating a new set of coefficients, they ignore the direct impact of antenna locations on the beamformer performance. Though robust sparse array design has been considered, the important issue of robust sparse array design for adaptive beamforming in terms of the output SINR has never been examined. At the core of our contribution is introducing a design method for optimizing sparse array antenna locations to achieve robustness against source arrival angle uncertainty. The optimum non robust sparse array design for MVDR beamforming, assuming exact knowledge of the DOAs of all signals incident on the array, was recently investigated in our work [13]. In this paper, we relax the above assumption, and seek the optimum intertwined array configuration and beamformer weights which maximize SINR over possible perturbations of the source DOAs. This robustness is investigated for both unconstrained and constrained designs. For LCMV, the output SINR is maximized by optimizing antenna locations under the additional linear constraints. As such, for a given number of antennas, the array can allocate its degrees of freedom (DoFs), i.e., antenna positions, towards achieving optimum robustness whether we deal with the MVDR or LCMV beamformer. The proposed approach can be applied to different environments comprising various source and interference power levels.

Antenna Measurement Co-ordinate system

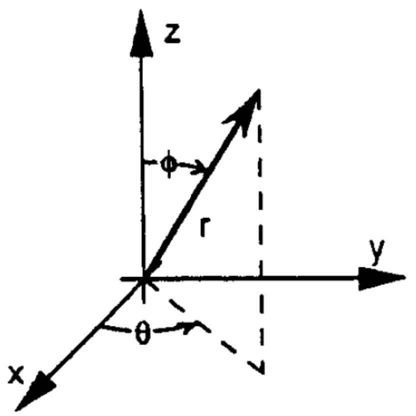

Figure 1. Orthogonal Coordinate Systems with θ-Azimuth Angle, φ-Elevation Angle.

The electric field of the received signal is

$$E_m(\theta,\varphi) = \sum_{n=0}^{N-1} e^{jmkdsin\theta\ sin\varphi} \quad (1)$$

A. ***Beamforming***

Beam forming is a method used to create the radiation pattern of an array antenna by adding constructively the weights of the signals in the direction of SOI and nulling the pattern in the direction of SNOI (interference). It can be used at both the transmitting and receiving ends in order to achieve spatial selectivity. The improvement compared with omnidirectional reception/transmission is known as the directivity of the array. Beamforming can be used for radio or sound waves. It has found numerous applications in radar, sonar, seismology, wireless communications, radio astronomy, acoustics and biomedicine. Adaptive beamforming is used to detect and estimate the signal of interest at the output of a sensor array by means of optimal (e.g. least-squares) spatial filtering and interference rejection.

In transmission mode, the majority of signal energy transmitted from a group of antenna array can be directed in a chosen angular direction.

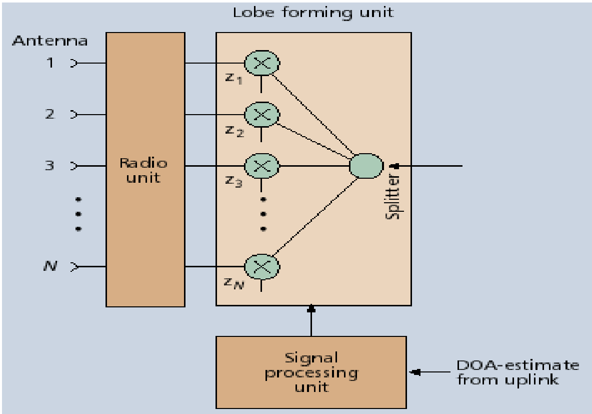

Figure 2. Signal Transmission Mode

In reception mode, you can calibrate your group of antenna elements when receiving signals such that you predominantly receive from a chosen angular direction.

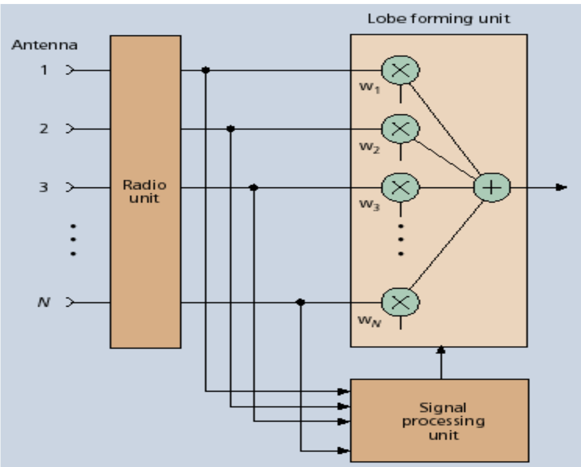

Figure 3. Signal Reception Mode

II THEORETICAL ANALYSIS

A. Weight Vector

Weight vector is a vector of complex weights w, each element consists of real and imaginary components, or alternatively, amplitude and phase components

$$w_m = \alpha_m e^{j\beta m}$$

w_m is the complex weight of the n_{th} element, α_m is the amplitude weight of the n_{th} element and β_m is the phase weight of the n_{th} element. Amplitude components control the side lobe level and main beam width. Phase components control the angle of the main beam and nulls Phase weights for narrowband arrays are applied by a phase shifter.

B. Steering vector:

If there are K transmitters and K received signal vectors in a wireless communications system where multiple users are active, the received signal vector of the d_{th} signal is frequently referred to as the steering vector $a(\theta_d)$ which is the array factor of any array, depends on the angle of arrival of each incident signal.

C. The array outputs y(t) at any instant snapshot time t (or n):

The array output can be determined for any instant of an incoming signal plus noise once the weight vector is calculated. Let us start with a description of the array, the received signal, and the additive noise. Consider D signals arriving from D directions. They are received by an array of M elements with M potential weights.

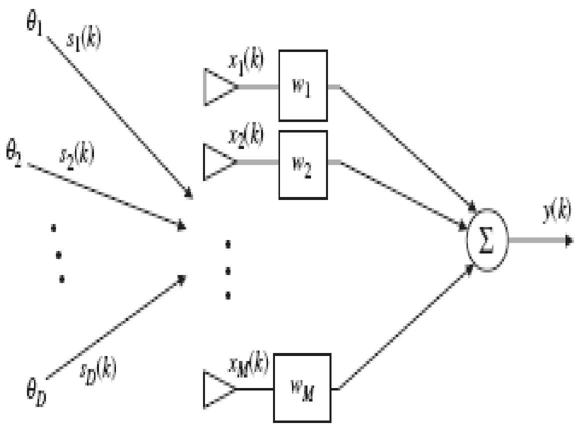

Figure 4. Array Output y(t) structure

Each incoming signal sd(k) has a correspondence additive zero mean Gaussian noise.-Each array element will get a snapshot at a time t; x(t); an element of the vector X(t) as

$$X(t) = [a(\theta_1), \quad a(\theta_1), \ldots, \quad a(\theta_D)] \begin{bmatrix} S_1(t) \\ S_2(t) \\ \vdots \\ S_n(t) \end{bmatrix} + n(t)$$

$$X(t)=AS(t)+n(t)$$

Where, S(t) is vector of incident complex monochromatic signals at a time, n(t) is noise vector at each array element m, zero mean, variance σ_n^2 , $a(\theta)$ is M-element array steering vector for the θ direction of arrival, A is an M× D matrix of steering vectors.

$$W=[w_1 \ \ w_2 \ldots . \ w_M]^T$$

The array output y can be given in the following form

$$y(t)=W^T X(t) \qquad (2)$$

D. Three impacts of complex weight function on the array pattern

i. Main beam steering

A simple beamformer steers the main beam in a particular direction (θ,φ) .In case of an environment consisting only of noise, i.e., no interfering signals, this beamformer provides maximum SNR because of the antenna gain. The weight vector for steering the main beam is

$$W=a_0/M \qquad (3)$$

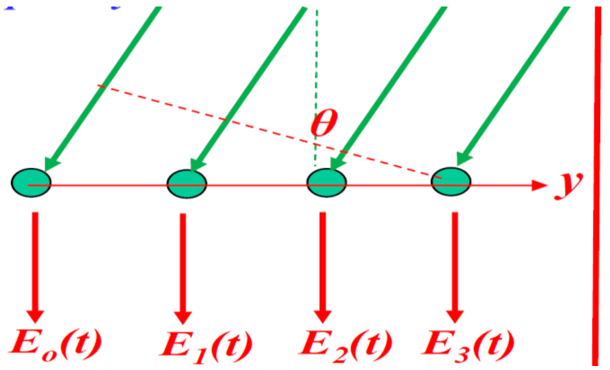

Figure 5. Arrival of signal and Electric Field representation.

ii. Null steering

The previous case will not produce a maximum SNR in the presence of directional interference. The null steering is useful when it is necessary to attenuate unwanted signals arriving at angles other than that of the main beam. Let a_o be the main beam steering vector, $a_1, \ldots, a_K$ are k steering vectors for the K nulls.

$$W^H= [w_1 \ w_2 \ w_3 \ldots \ w_M]$$

Which could be written in matrix form as

$$W^H A = C$$

$$WH\ [a_o\ a_1 \ldots a_K]= [1\ 0 \ldots 0\ ]=C$$

$$W^H = CA^{-1} \quad (4)$$

The null steering technique described here jointly steers the main beam and nulls to the desired angles. Modifying the vector C enables the existence of nulls and beams (or signal minima and maxima) to be specified according to the prevailing requirements. When the number of required nulls is less than K, K= M – 1, M is the number of array elements. Matrix A is not a square matrix (the matrix inversion will be singular and cannot be inverted). Under such conditions suitable weights may be given by:
For a solution does not minimize the uncorrelated noie at the array output,

$$W^H=CA^H(AA^H)^{-1} \quad (5)$$

For a formulation requires noise with variance σ_n be added in the system

$$W^H=CA^H(AA^H + \sigma_n^2 I)^{-1} \quad (6)$$

Where I is the identity matrix, A^H is the transpose of A.

III PROPOSED ALGORITHMS

A. MVDR Algorithm:

When a beamformer has a constant response in the direction of a useful signal, the LCMV algorithm becomes an MVDR algorithm [25]. The MVDR algorithm is capable of suppressing the interference, but with high value in SNR and low noise. At the same time, the MVDR algorithm depends on the steering vectors, which in turn depend on the incident angle of the received signal from the element of the array antenna. The direction of useful signal must be known and the output power subject to a unity gain constraint in the direction of desired signal must be minimized. The array output is given by

y

$$= w^H x \quad (7)$$

The output power is as follows

$$P= \{E\,|y|^2\} = E\{w^H x x^H w\} = w^H E\{x x^H w\}= w^H R \quad (8)$$

where the R covariance matrix should be (M, 1) for the received signal x and H is the hermitian transpose.
The optimum weights are selected to minimize the array output power P_{MVDR} while maintaining unity gain in the look direction $a(\theta)$, which is the steering vector of the desired signal. The MVDR adaptive algorithm can be written as follows:

$$W_{min}\{w^H R_W\} \text{ subject to } w^H a(\theta)=1$$

The steering vector a(θ) is given by

$$a(\theta) = \begin{bmatrix} 1 \\ \exp\{j\frac{2\pi}{\lambda}(sin\theta_i)d \\ \exp\{j\frac{2\pi}{\lambda}(sin\theta_i)(m-1)d \\ \vdots \\ \exp[j\frac{2\pi}{\lambda}(sin\theta_i)(m-1)d \end{bmatrix}$$

where , d is the space between the elements of the antenna, θi is the desired angle, and m is the number of elements.

The optimization weight vector is given as,

$$W_{MVDR}=\frac{R^{-1}a(\theta)}{a^H(\theta)R^{-1}a(\theta)} \quad (9)$$

B. LCMV Algorithm:

The uniform linear array (ULA) consisting of M sensor elements as an example, let us assume that there is one desired signal d(t) and J narrowband interferences $i_j(t)$, j=1,2,......J in the far field, with the direction of arrival (DOA) θ_d and θ_j respectively. Additive white noise on each array element is $n_k(t)$, and noise variance is σ_n^2. Therefore, the received signal on the array element k can be modeled as

$$x_k(t) = x_k(\theta_d)d(t) + \sum_{j=1}^{J} a_k(\theta_{ij})i_j(t) + n_k(t) \quad \textbf{(10)}$$

Where $a_k(\theta_d)$, $a_k(\theta_{ij})$ are steering vectors for desired signal and interference signals, and the three terms on the right side of the equation represents signal, interference and noise respectively.
The covariance matrix of the received signal array is given as

$$R_x = E(X(t)X^H(t)) \quad \textbf{(11)}$$

Where $X(t)=[x_1(t),x_2(t)\ldots\ldots x_M(t)]^T$ is the sampling matrix of the received signals.

According to linear constrained criteria, LCMV algorithm should meet the following linear constrained conditions.

$$w^H a(\theta_d) = 1 \quad \text{and}$$

$$w^H a(\theta_{ij}) = 0 (j = 1, 2, \dots J) \tag{12}$$

Where, w is the complex weight vector, and the matrix representation can be expressed as

$$C^H W = f \tag{13}$$

Where C=[a(θ_d),a(θ_{i1}),….,a(θ_{iJ})] represents the constrained matrix, f is constrained vector, and W(θ)=[w1(θ),w2(θ)…w_M (θ)]^T is the complex weight vector. since θ_d is the direction of desired signal, and i_j(t),j=1,…J are interference signals, we can set the constrained vector f to be f=[1,0,..0]^T, which ensures the desired signal is received free of distortion and interference signals are inhibited.

The optimal weight vector is given as,

$$w_{opt} = R_x^{-1} C (C^H R_x^{-1} C)^{-1} f \tag{14}$$

C. Phaseshift Algorithm:

The Phase shift beamformer is a specialize case of the discrete Fourier transform. Beamformer and is only applicable to narrow band signal. Since method is frequency domain concepts, steering delays are realize by phase shift are not dependent on the sampling frequency [26].

$$X_m(k) = \sum_{m=0}^{N-1} x_m(n) e^{\frac{j2\pi k}{N}} \; where 0 \le k \le N-1 \tag{15}$$

$$\omega_k t_{mb} = \frac{2\pi k}{N} m \delta_k \sin\psi_k \tag{16}$$

$$\omega_k t_{mb} = \phi_b m k \tag{17}$$

$$where \; \phi_b = 2\pi \delta_S \sin\frac{\psi_b}{N}$$

Finally , we have

$$Y(k, \psi_b) = \sum_{m=0}^{M-1} a_m X_m(k) e^{-j\omega_0 mk} \tag{18}$$

The phaseshift beamformer makes the approximation that the phase shift in the DFT beamformer are replaced by a constant phase shift,

$$2\pi f t_m = 2\pi f_0 t_{mb}$$

Where f_0 is the centre frequency of the narrow band signal. This is only valid for narrow band signals, otherwise errors occur in the beampatterns. Note that the phase shift beamformer is related to DFT beamformer if only single frequency in the fourier transformed is used.

$$Y(f, \phi_b) = \sum_{m=1}^{M} a_m X_m(f) e^{-j\omega_0 t_{mb}} \tag{19}$$

And in time domain equivalent is obtained by taking the inverse fourier transform. As a result, an implementation of phase shift beamformer is relatively simple.

IV. RESULTS AND DISCUSSIONS

The following assumptions have been considered to compare the performance of MVDR, Phaseshift and LCMV algorithms

- Frequency of the desired GPS signal (L1 Frequency) : 1575.42GHz
- No. of interference signals : 2
- Direction of Arrival (DOA) for interference signals: 30^0 and 50^0
- Direction of Arrival (DOA) for desired GPS signal: $\mathbf{0^0}$
- Type of array antenna considered: Uniform Linear Array Antenna (ULA)
- Antenna array elements : wavelength/2

Case I: Considering 4 Antenna Array elements

From the Figure 6, it can be observed that MVDR seems to perform better in creating a null in the direction of arrival of interference signals compared to LCMV and Phase shift. For an elevation angle of $\boldsymbol{\varphi}=0^0$, Deep nulling is formed in the direction of 30^0, 50^0 using MVDR. However, for higher elevation angles, the performance of MVDR has been degraded as nulling is formed in the direction of 35^0,55^0 for Elevation angle $\boldsymbol{\varphi}=20^0$ and for an Elevation angle $\boldsymbol{\varphi}=40^0$, null has been shifted to 40^0.

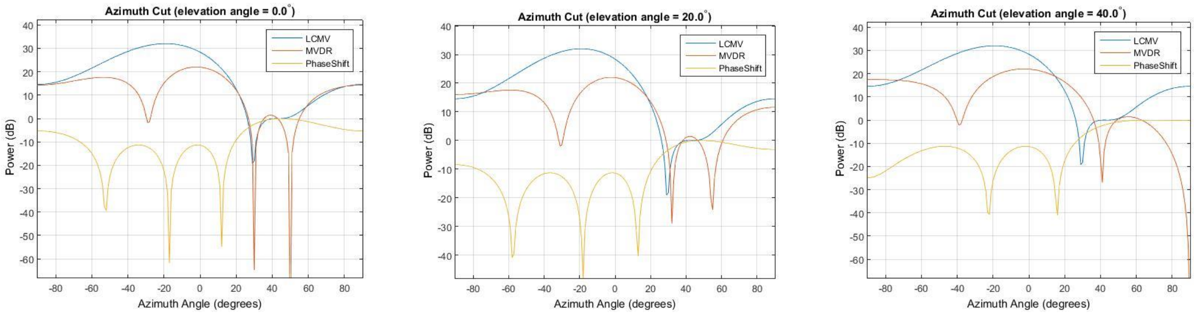

Figure6: Simulation results when 4 antenna array elements has been considered

Case II: Considering 6 Antenna Array elements

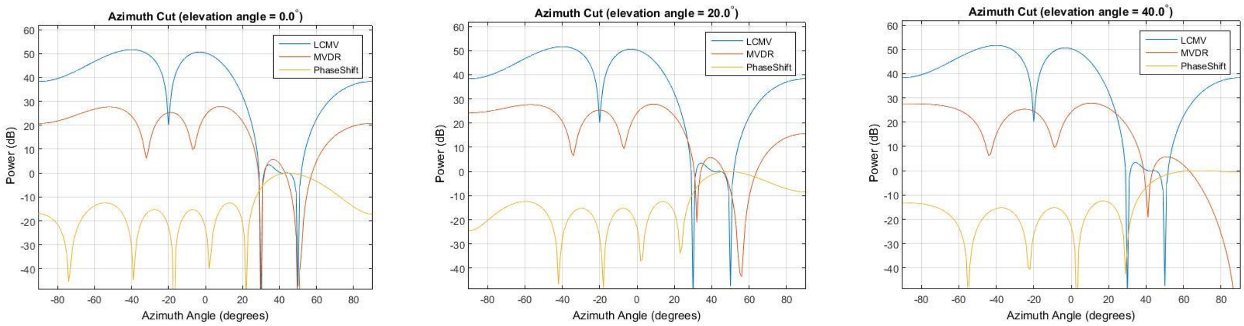

Figure 7: Simulation results when 6 antenna array elements has been considered

From the Figure 7, it can be observed that LCMV seems to perform much better in creating a null in the direction of arrival of interference signals compared to MVDR and Phase shift. For elevation angles of $\boldsymbol{\varphi}=0^0$, 20^0 ,40^0, it can be observed that Deep nulling is formed perfectly in the direction of 30^0, 50^0 using LCMV.

Case III: Considering 8 Antenna Array elements

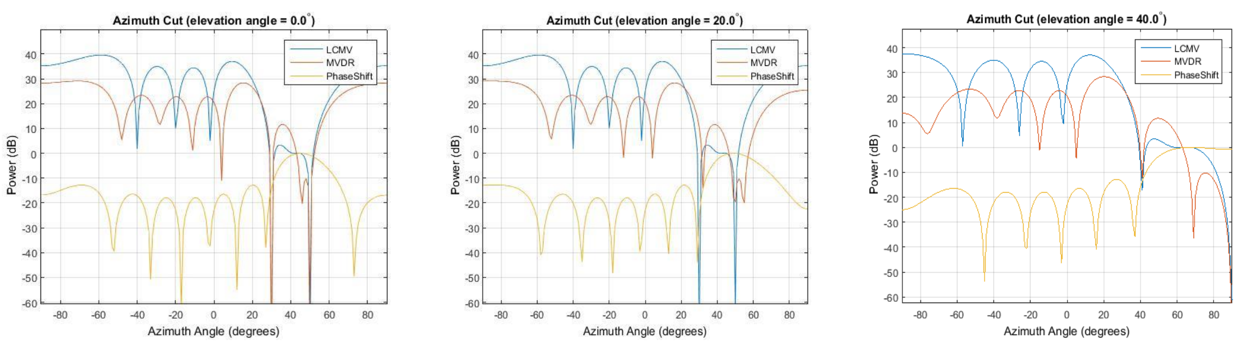

Figure 8: Simulation results when 8 antenna array elements has been considered

From the figure 8, Considering 8 Antenna Array Elements, Deep nulling is formed in the direction of 30^0,50^0 using MVDR at Elevation angle $\boldsymbol{\varphi}=0^0$, while Deep nulling is formed in the direction of 30^0,50^0 using LCMV for an Elevation angle $\boldsymbol{\varphi}=20^0$. And or Elevation angle $\boldsymbol{\varphi}=40^0$, the performance of all the techniques has been degraded.

Case IV: Considering 10 Antenna Array elements

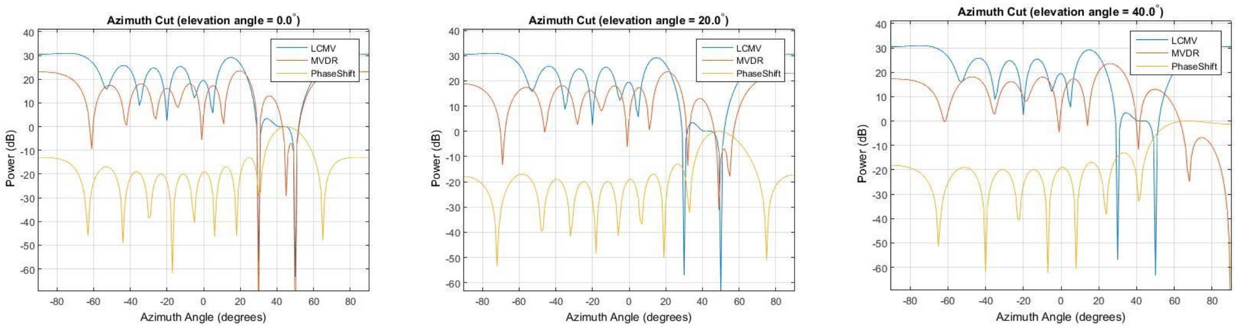

Figure 9: Simulation results when 10 antenna array elements has been considered

From figure 9, when 10 Antenna Array Elements has been considered, LCMV seems to perform much better for different elevation angles.

Case V: Considering 50 and 100 Antenna Array elements

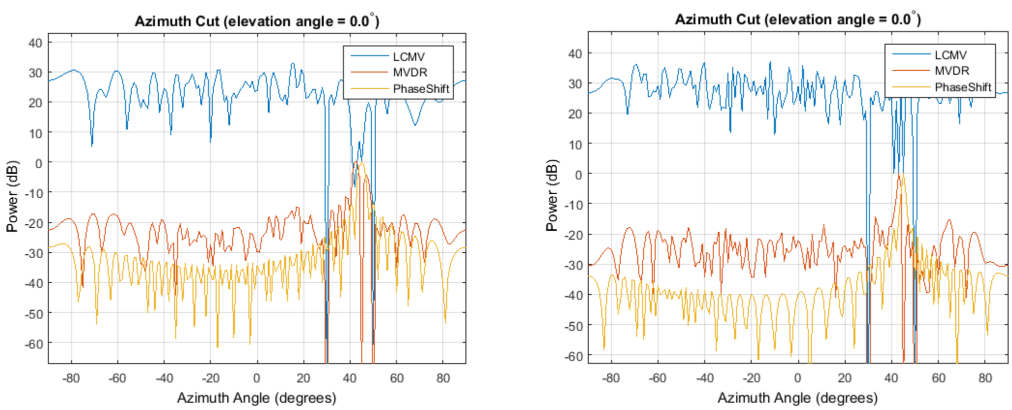

Figure 10: Simulation results of LCMV, MVDR and Phaseshift algorithms considering 50 and 100 Antenna Array Elements

It can be observed from fig 6, fig7,fig8,fig9 that as the number of array elements increases, better null steering is done towards the interference angles using LCMV.

Table:1 Comparison between LCMV,MVDR and Phase shift Algorithm

No of Array Elements	Phase Shift Beamforming Weights	MVDR Beamforming Weights	LCMV Beamforming Weights	Remarks
M=4	-0.2455 + 0.0474i 0.1110 - 0.2240i 0.1110 + 0.2240i -0.2455 - 0.0474i	-3.1787 - 1.1256i -2.8117 + 1.1930i -3.1287 - 1.7051i -3.3916 + 1.1433i	-0.2960 - 4.9855i -12.8566 - 7.6487i -12.8439 + 7.6720i -0.2872 + 4.9861i	MVDR performed better at Elevation angle **φ**=0^0 where as the performance got degraded at other elevation angles

M=6	0.1242 + 0.1111i -0.1636 + 0.0316i 0.0740 - 0.1493i 0.0740 + 0.1493i -0.1636 - 0.0316i 0.1242 - 0.1111i	-5.1055 + 3.8016i 0.1744 + 7.9370i -2.8550 - 1.5127i -3.5939 + 2.4655i 0.1199 - 6.1662i -4.6199 - 3.3406i	-0.1456 - 0.3828i -1.1918 - 0.0792i -0.3247 + 0.7358i -0.3003 - 0.6829i -1.1610 + 0.0874i -0.1397 + 0.3783i	LCMV performance is satisfactory compared to Phase shift and MVDR
M=8	0.0099 - 0.1246i 0.0932 + 0.0833i -0.1227 + 0.0237i 0.0555 - 0.1120i 0.0555 + 0.1120i -0.1227 - 0.0237i 0.0932 - 0.0833i 0.0099 + 0.1246i	5.5928 + 3.5215i 2.0715 - 1.2462i -0.0513 + 6.2109i -1.7186 - 2.7678i -2.8843 + 1.5022i -0.8749 - 5.3149i 2.7087 + 1.6193i 5.7218 - 3.7422i	-8.5441 +10.3272i 11.5531 +20.6056i -5.7143 - 4.0206i -5.8372 +14.5376i -5.9527 -15.0938i -6.3088 + 3.7420i 11.2705 -20.3310i -8.5037 -10.2430i	Deep nulling is formed in the direction of 30^0,50^0 using MVDR at Elevation angle $\boldsymbol{\varphi}=0^0$, Deep nulling is formed in the direction of 30^0,50^0 using LCMV at Elevation angle $\boldsymbol{\varphi}=20^0$, Deep nulling is formed in the direction of 40^0 using LCMV at Elevation angle $\boldsymbol{\varphi}=40^0$.
M=10	-0.0841 + 0.0541i 0.0079 - 0.0997i 0.0745 + 0.0667i -0.0982 + 0.0189i 0.0444 - 0.0896i 0.0444 + 0.0896i -0.0982 - 0.0189i 0.0745 - 0.0667i 0.0079 + 0.0997i -0.0841 - 0.0541i	0.5629 - 3.6546i 0.1040 + 0.0797i 2.2948 - 0.4895i -0.3414 + 1.9237i -0.9679 - 0.9282i -0.8488 + 0.9633i -0.6061 - 2.3354i 1.2997 + 0.5733i -0.4304 + 0.6430i 0.5881 + 3.9781i	5.5930 + 1.2668i 3.2652 - 5.3767i 1.5781 + 5.2968i 0.2466 - 1.9596i -4.1676 + 4.6674i -2.5997 - 4.3510i -2.0629 - 0.0892i -0.1708 - 4.4310i 2.6689 + 4.8607i 4.9986 - 1.2412i	Deep nulling is formed in the direction of 30^0,50^0 using LCMV Elevation angle $\boldsymbol{\varphi}=0^0$, Deep nulling is formed in the direction of 30^0,50^0 using LCMV at Elevation angle $\boldsymbol{\varphi}=20^0$, Deep nulling is formed in the direction of 30^0, 50^0 using LCMV at Elevation angle $\boldsymbol{\varphi}=40^0$.

Table 1 shows the performance comparison of all the three beam forming techniques with respect to different DOA of interference signals, different elevation angles considering different antenna array elements. It also shows the various weights considered for the creation of deep null in the direction of interference signals.

V. CONCLUSION

This paper presents the performance comparison of various beamforming techniques for the cancellation of interference in GPS receivers. It can be observed from the simulation results that LCMV shows better performance at different elevation angles. Deep nulling is formed in the interference direction 30^o and 50^o using LCMV at Elevation angle $\varphi=0^o$, 20^o, 40^o. Also, it can be observed that increasing the number of array elements has steered the null towards the interference direction more effectively than less number of array elements.

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