

Chapter 6
Exponential and Logarithmic Functions

Section 6-4
Transformations of Exponential and Logarithmic Functions

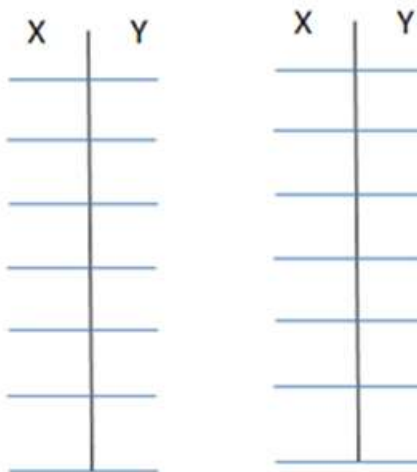
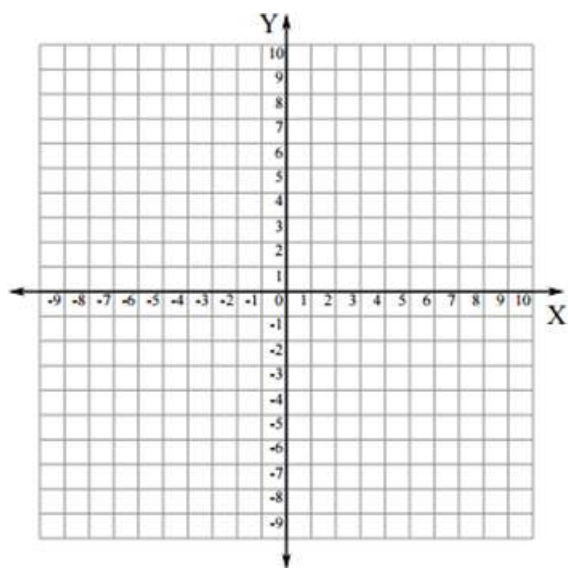
Transforming Graphs of Exponential Functions

You can transform graphs of exponential and logarithmic functions in the same way you transformed graphs of functions in previous chapters. Examples of transformations of the graph of $f(x) = 4^x$ are shown below.

EXAMPLE 1 Translating an Exponential Function

Describe the transformation of $f(x) = \left(\frac{1}{2}\right)^x$ represented by $g(x) = \left(\frac{1}{2}\right)^x - 4$.

Then graph each function.



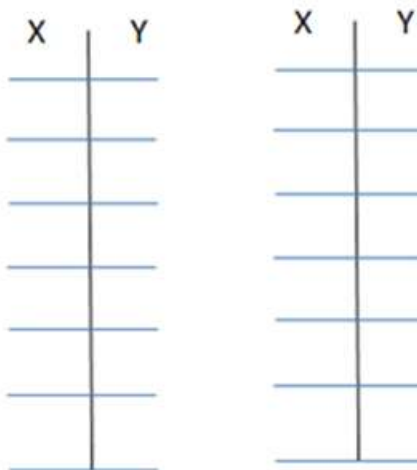
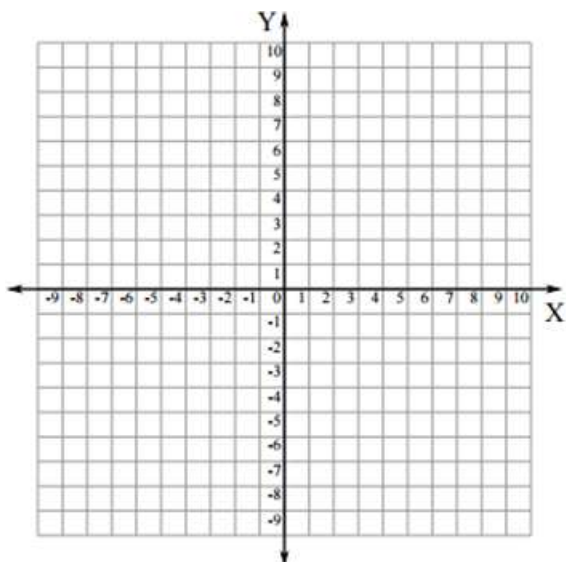
STUDY TIP

Notice in the graph that the vertical translation also shifted the asymptote 4 units down, so the range of g is $y > -4$.



EXAMPLE 2**Translating a Natural Base Exponential Function**

Describe the transformation of $f(x) = e^x$ represented by $g(x) = e^{x+3} + 2$. Then graph each function.

**STUDY TIP**

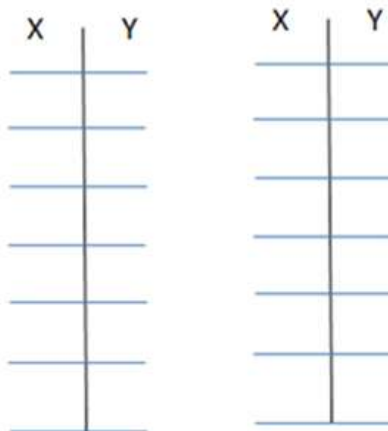
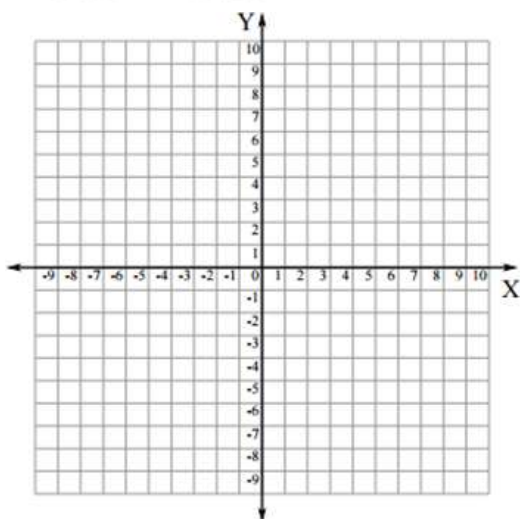
Notice in the graph that the vertical translation also shifted the asymptote 2 units up, so the range of g is $y > 2$.

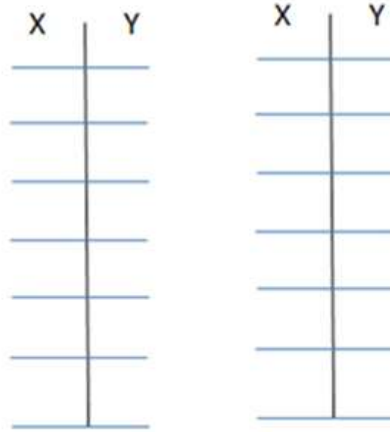
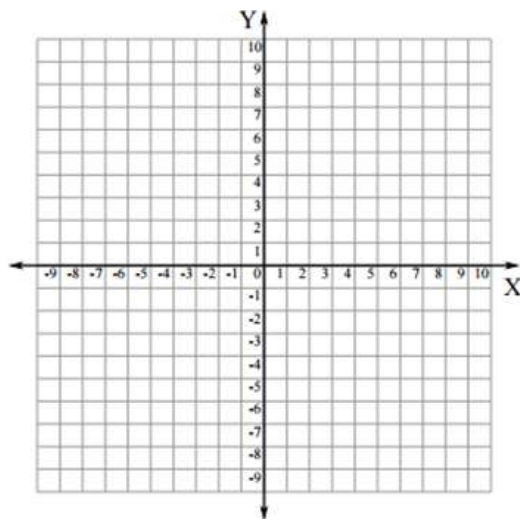
**EXAMPLE 3****Transforming Exponential Functions**

Describe the transformation of f represented by g . Then graph each function.

a. $f(x) = 3^x, g(x) = 3^{3x-5}$

b. $f(x) = e^{-x}, g(x) = -\frac{1}{8}e^{-x}$





Transforming Graphs of Logarithmic Functions

Examples of transformations of the graph of $f(x) = \log x$ are shown below.

Core Concept

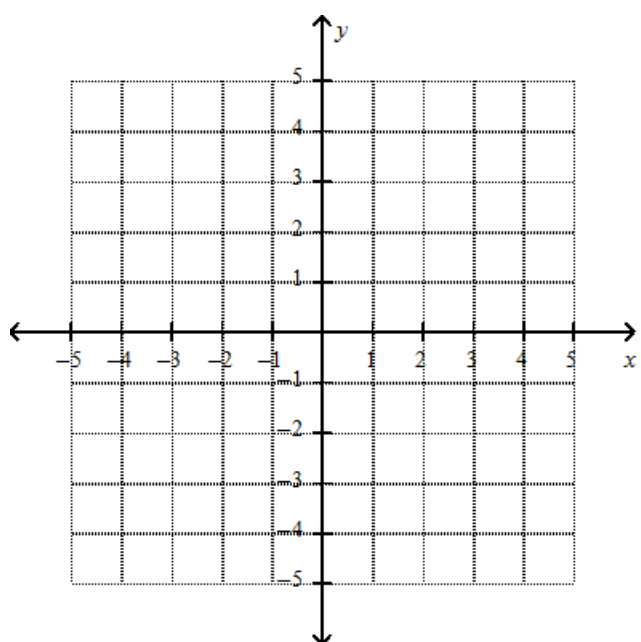
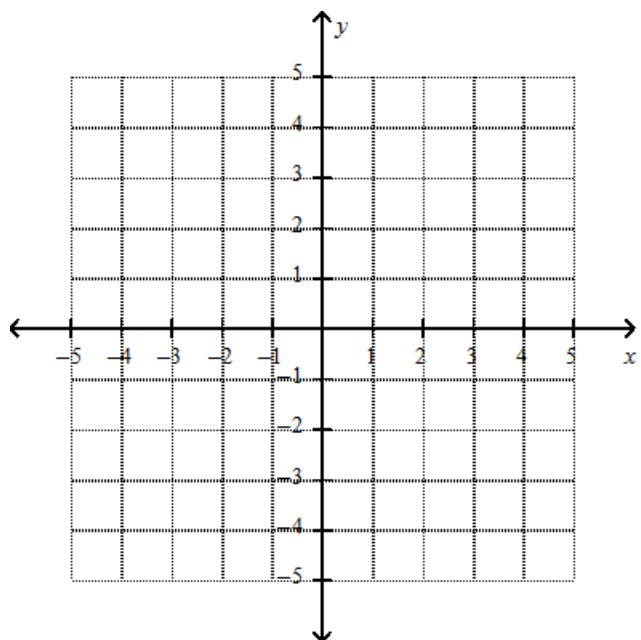
Transformation	$f(x)$ Notation	Examples
Horizontal Translation Graph shifts left or right.	$f(x - h)$	$g(x) = \log(x - 4)$ 4 units right $g(x) = \log(x + 7)$ 7 units left
Vertical Translation Graph shifts up or down.	$f(x) + k$	$g(x) = \log x + 3$ 3 units up $g(x) = \log x - 1$ 1 unit down
Reflection Graph flips over x - or y -axis.	$f(-x)$ $-f(x)$	$g(x) = \log(-x)$ in the y -axis $g(x) = -\log x$ in the x -axis
Horizontal Stretch or Shrink Graph stretches away from or shrinks toward y -axis.	$f(ax)$	$g(x) = \log(4x)$ shrink by $\frac{1}{4}$ a factor of $\frac{1}{4}$ $g(x) = \log\left(\frac{1}{3}x\right)$ stretch by a factor of 3
Vertical Stretch or Shrink Graph stretches away from or shrinks toward x -axis.	$a \cdot f(x)$	$g(x) = 5 \log x$ stretch by a factor of 5 $g(x) = \frac{2}{3} \log x$ shrink by a factor of $\frac{2}{3}$

EXAMPLE 4 Transforming Logarithmic Functions

Describe the transformation of f represented by g . Then graph each function.

a. $f(x) = \log x$, $g(x) = \log\left(-\frac{1}{2}x\right)$

b. $f(x) = \log_{1/2} x$, $g(x) = 2 \log_{1/2}(x + 4)$



Writing Transformations of Graphs of Functions

EXAMPLE 5 Writing a Transformed Exponential Function

Let the graph of g be a reflection in the x -axis followed by a translation 4 units right of the graph of $f(x) = 2^x$. Write a rule for g .

EXAMPLE 6 Writing a Transformed Logarithmic Function

Let the graph of g be a translation 2 units up followed by a vertical stretch by a factor of 2 of the graph of $f(x) = \log_{1/3} x$. Write a rule for g .