Math 2471 Calculus III − Sample Final Questions

1. Find the unit tangent and unit normal vector for $\vec{r}(t) = \, <t, \frac{1}{2}t^2>$.

2. (i) Prove the limits either exist or doesn't exist

$$(i) \quad \lim_{(x,y)->(0,0)} \frac{x^2 + 2y^2}{x^2 + y^2} \qquad (ii) \quad \lim_{(x,y)->(0,0)} \frac{x^2 y^4}{x^2 + y^2}$$

3. Find the equation of the tangent plane to the given surface at the specified point

$$x^2 y + xz + yz^2 = 3, \quad P(1, 2, -1)$$

4. Find the directional derivative of $z = x^2 + 3xy + y^2$ at $(1, 1)$ in the direction of $<-3, 4>$

5. Classify the critical points for $z = x^2 y - x^2 + y^2 - 18y$.

6. Set up and evaluate an integral to calculate

   (i) the volume bound by $z = 0, \quad z = 1 - x^2 - y^2$
   (ii) the volume inside both $x^2 + y^2 + z^2 = 2, \quad x^2 + y^2 = 1$
   (ii) the surface area of $x + 2y + 3z = 6, \quad x, y, z \geq 0$

7. Set of the triple integral $\iiint f(x, y, z) dV$ in both cylindrical and spherical coordinates for the volume inside the cone $z = \sqrt{x^2 + y^2}$ bound by $z = 1$

8. Is the following vector field conservative?

$$\vec{F} = \, <y^2 + 3yz, 2xy + 3xz, 3xy>.$$

If so, determine the potential function f such that $\vec{F} = \vec{\nabla} f$. Use this to evaluate

$$\int_c (y^2 + 3yz) \, dx + (2xy + 3xz) \, dy + 3xy \, dz > .$$

where c is a curve that connects the points $(0, 0, 0)$ and $(1, 2, 3)$.

9. Evaluate the following line integrals:

   (i) $\int_c 2xy \, dx + (x + 1) \, dy$ where c is the ccw direction around the square with vertices $(0, 0), (1, 0), (1, 1)$ and $(0, 1)$

   (ii) $\int_c (x - y) \, dx + (x + y) \, dy$ where c is clockwise direction around a circle of radius 2.

10. Verify Green's Theorem

$$\oint_c P\,dx + Q\,dy = \iint_R (Q_x - P_y)\,dA$$

where $\vec{F} = <3x^2y, x^3 + x>$ and c the curve enclosing the area given by $y = x$ and $y = x^2$.

11. Evaluate the following surface integrals

(i) $\iint_S xy\,dS$ where S is the portion of the plane $2x + y + z = 6$ in the first octant.

(ii) $\iint_S (x+z)\,dS$ where S is the portion of $y^2 + z^2 = 9$ in the first octant between

    $x = 0$ and $x = 4$.

12. Verify the divergence theorem

$$\iint_S \vec{F} \cdot \vec{N}\,dS = \iiint_V \nabla \cdot \vec{F}\,dV$$

where $\vec{F} = <x + yz, y + xz, z + xy>$ and V is the volume of the tetrahedron bound by $x + y + z = 1$ and the planes $x = 0$, $y = 0$ and $z = 0$.