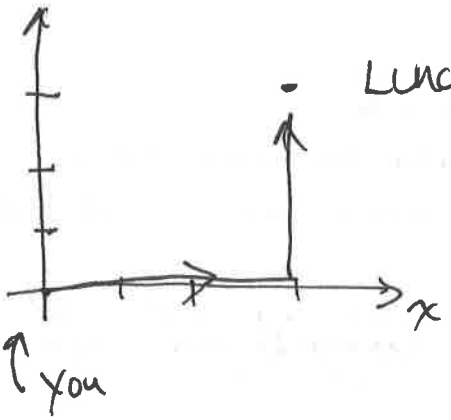
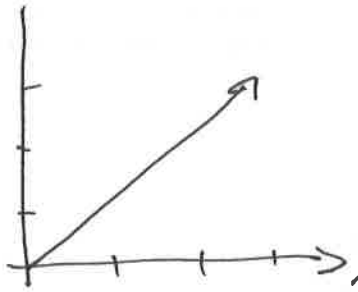
Section 10.2 Math 1497 - Calc 2



Consider

would you walks 3 blocks along x axis then 3 blocks in y direction

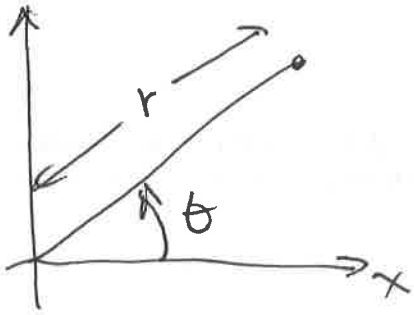
Probably not. You would walk directly



so you would rotate yourself from looking down x axis to the pt P then move a distance along this line.

So we are introducing a new way of describing pts in 2-D

we rotate an angle θ

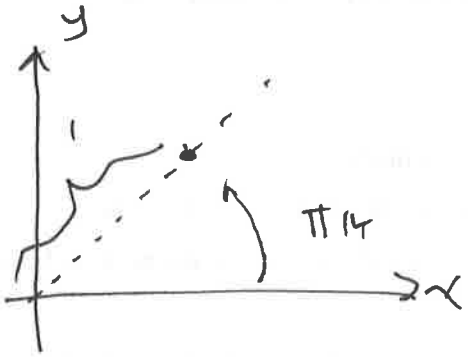


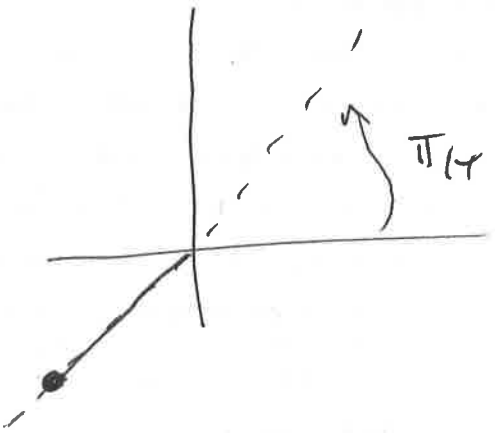
then move a distance r

Cartesian (x, y)

Polar (r, θ)

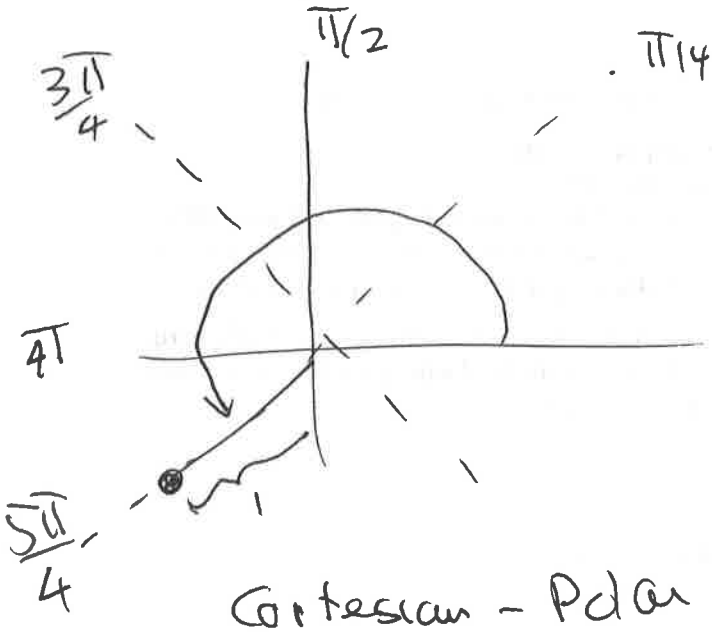
Ex $(r,\theta) = (1, \pi/4)$ $(r,\theta) = (-1, \pi/4)$





So in the 2nd example we move backwards along ray

Ex $(r,\theta) = (1, 5\pi/4)$

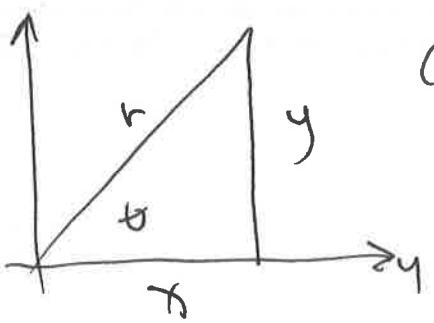


So we see that

$(-1, \pi/4)$ & $(1, \frac{5\pi}{4})$

give the same point

Cartesian - Polar Conversion



$\cos\theta = \frac{x}{r}$ or $\sin\theta = \frac{y}{r}$

$$x = r\cos\theta$$
$$y = r\sin\theta$$

$x^2+y^2 = r^2\cos^2\theta + r^2\sin^2\theta$

$= r^2(\sin^2\theta + \cos^2\theta) = r^2$

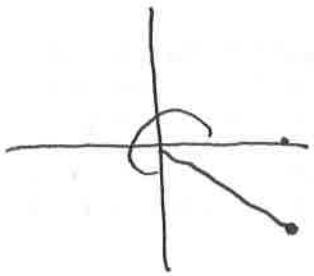
$\frac{y}{x} = \frac{r\sin\theta}{r\cos\theta}$

so $\boxed{x^2+y^2=r^2, \quad \frac{y}{x} = \tan\theta}$

if $r=1, \theta = \pi/4 \quad x = 1\cos\pi/4 = \frac{\sqrt{2}}{2}$

$y = 1\sin\pi/4 = \frac{\sqrt{2}}{2}$

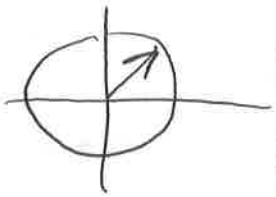
if $x=1, y=-1$



$r^2 = 1^2+1^2 \Rightarrow r = \sqrt{2}$ chosen two

$\theta = \tan^{-1}(-1) = \frac{7\pi}{4}$ or $-\pi/4$

Some Basic Curves



Circle radius a

$x^2+y^2=a^2 \quad r^2\cos^2\theta + r^2\sin^2\theta = a^2 \Rightarrow r=a$

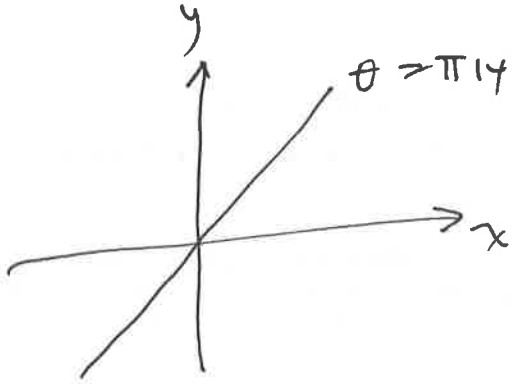
more from origin same distance for all θ

(a)

Line

$y = x$

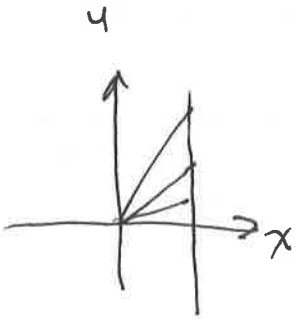
$r\sin\theta = r\cos\theta \Rightarrow \tan\theta = 1 \Rightarrow \theta = \pi/4$ for all r



$x = 1$

$r\cos\theta = 1$

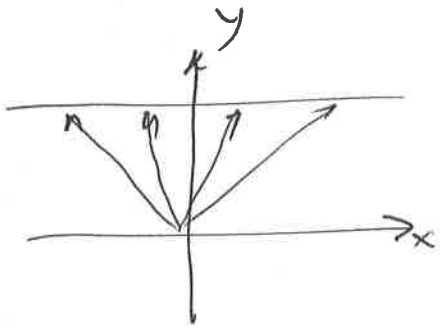
$r = \sec\theta$



$y = 2$

$r\sin\theta = 2$

$r = 2\csc\theta$



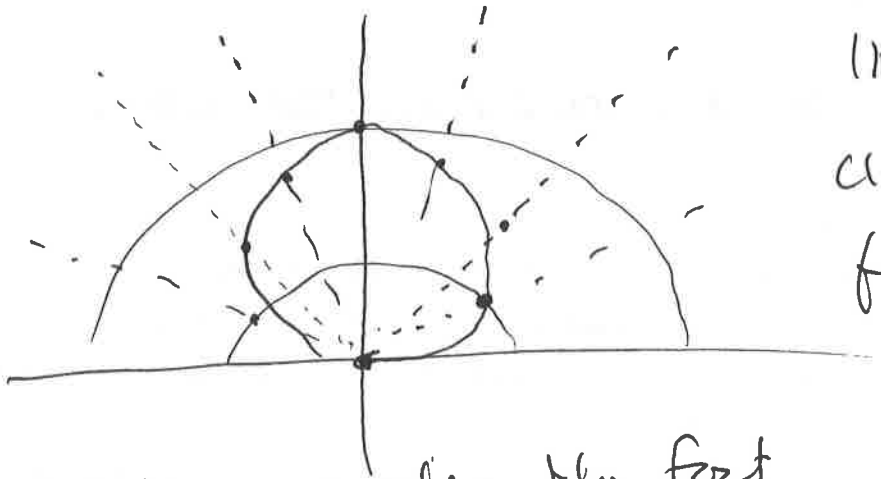
so we consider Polar eq^n $r = f(\theta)$

Ex $r = 2\sin\theta$

lets create a table of value

θ	r
0	0
$\frac{\pi}{6}$	$2\cdot\frac{1}{2} = 1$
$\frac{\pi}{4}$	$2\cdot\frac{\sqrt{2}}{2} > 1.4$
$\frac{\pi}{3}$	$2\cdot\frac{\sqrt{3}}{2} > 1.7$
$\pi/2$	$2\cdot 1 = 2$

θ	r
$\frac{2\pi}{3}$	1.7
$\frac{3\pi}{4}$	1.4
$\frac{5\pi}{6}$	1
π	0



lines and circles are for reference

looks like a circle. Un fact

$r = 2\sin\theta \Rightarrow r^2 = 2r\sin\theta$

$\Rightarrow x^2+y^2 = 2y$ (complete the □)

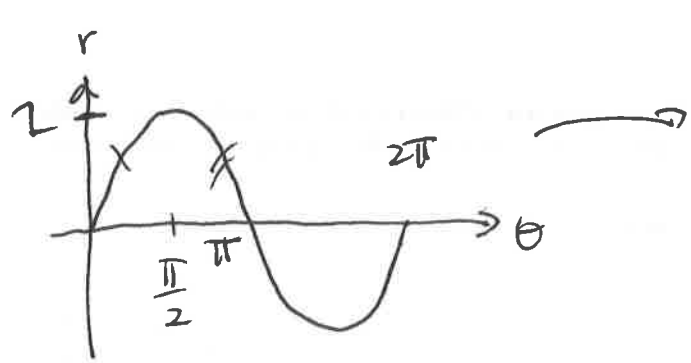
$x^2+y^2-2y+1 = 1 \Rightarrow x^2+(y-1)^2 = 1$

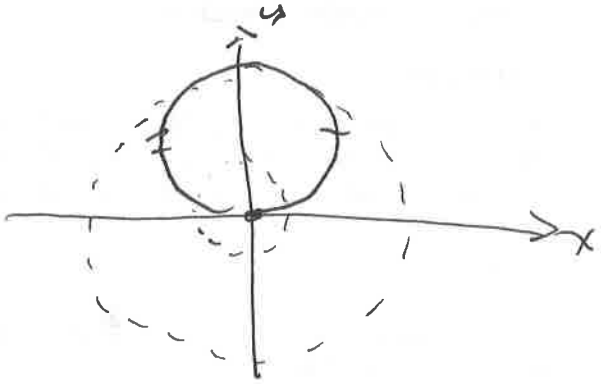
a circle with center $(0,1)$ & radius 1.

In general, a table of values takes too long and there's a better way

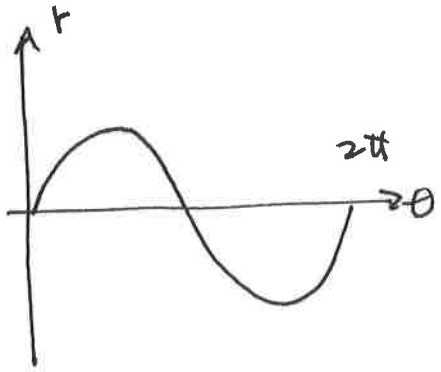
Start with Cartesian → Polar Method

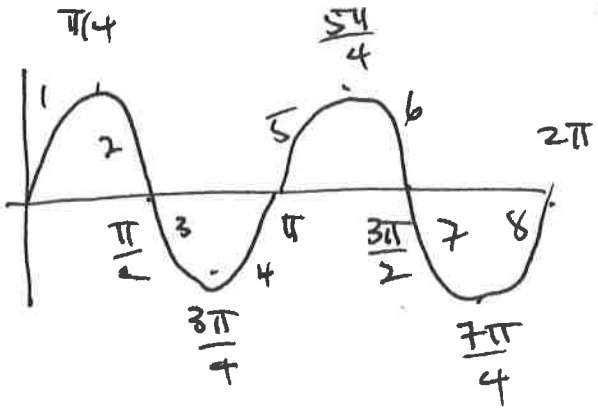
$r = 2\sin\theta$

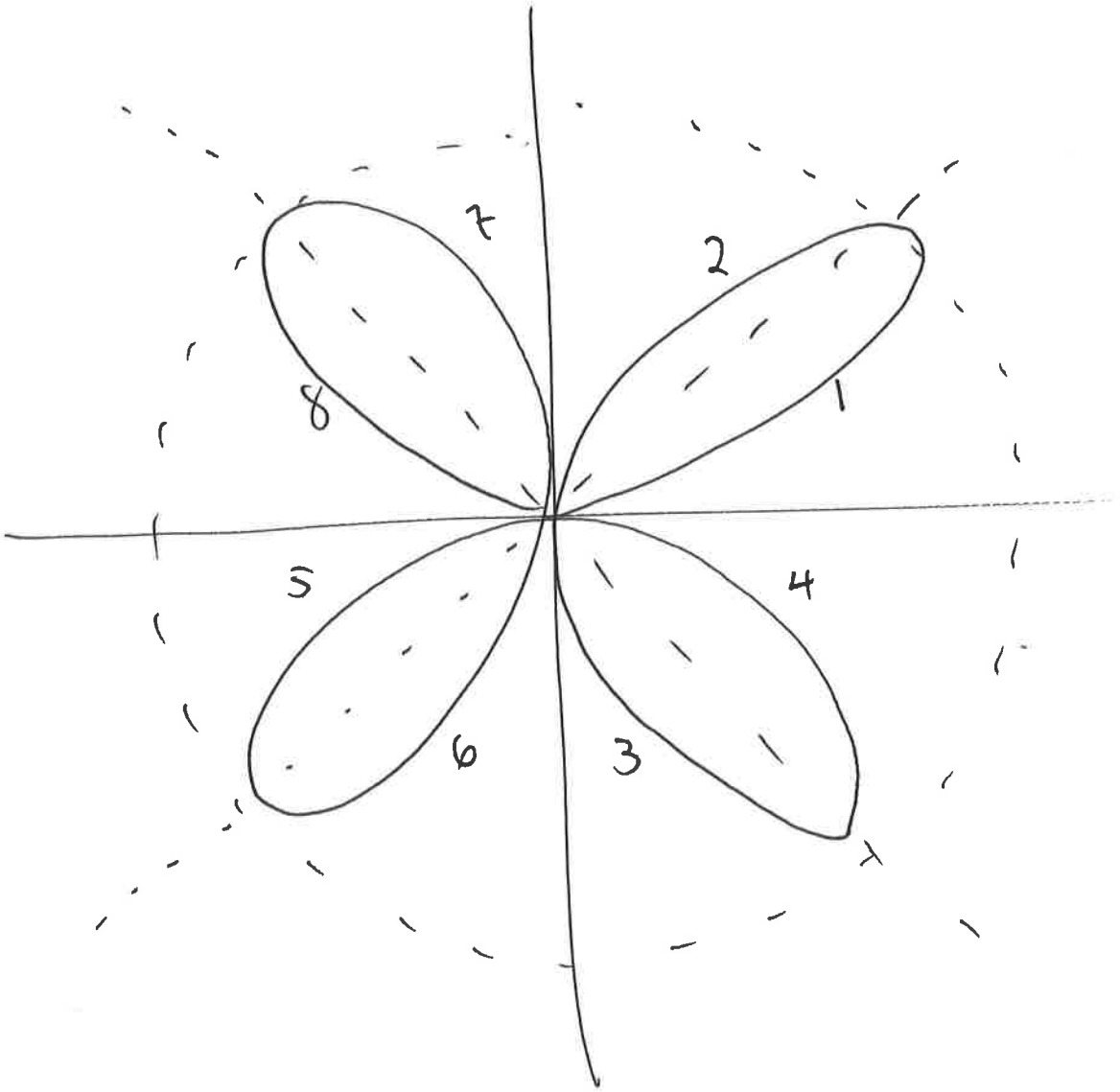




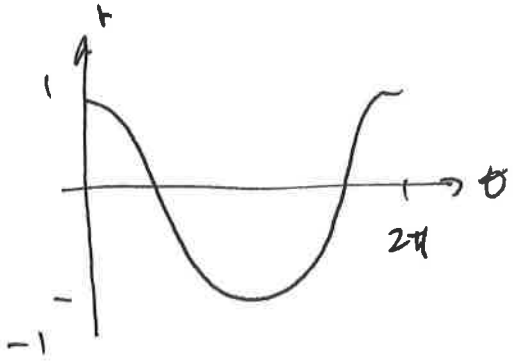
Graph $r = \sin 2\theta$

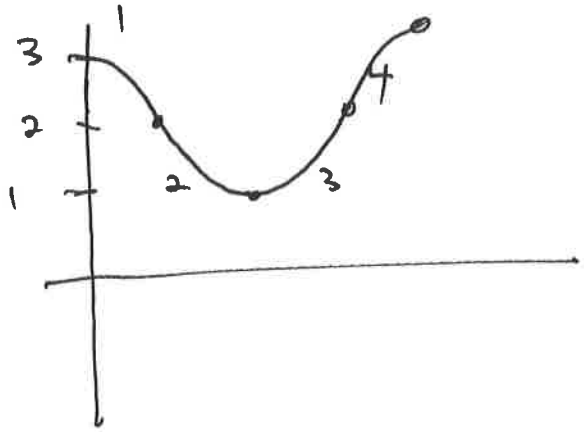


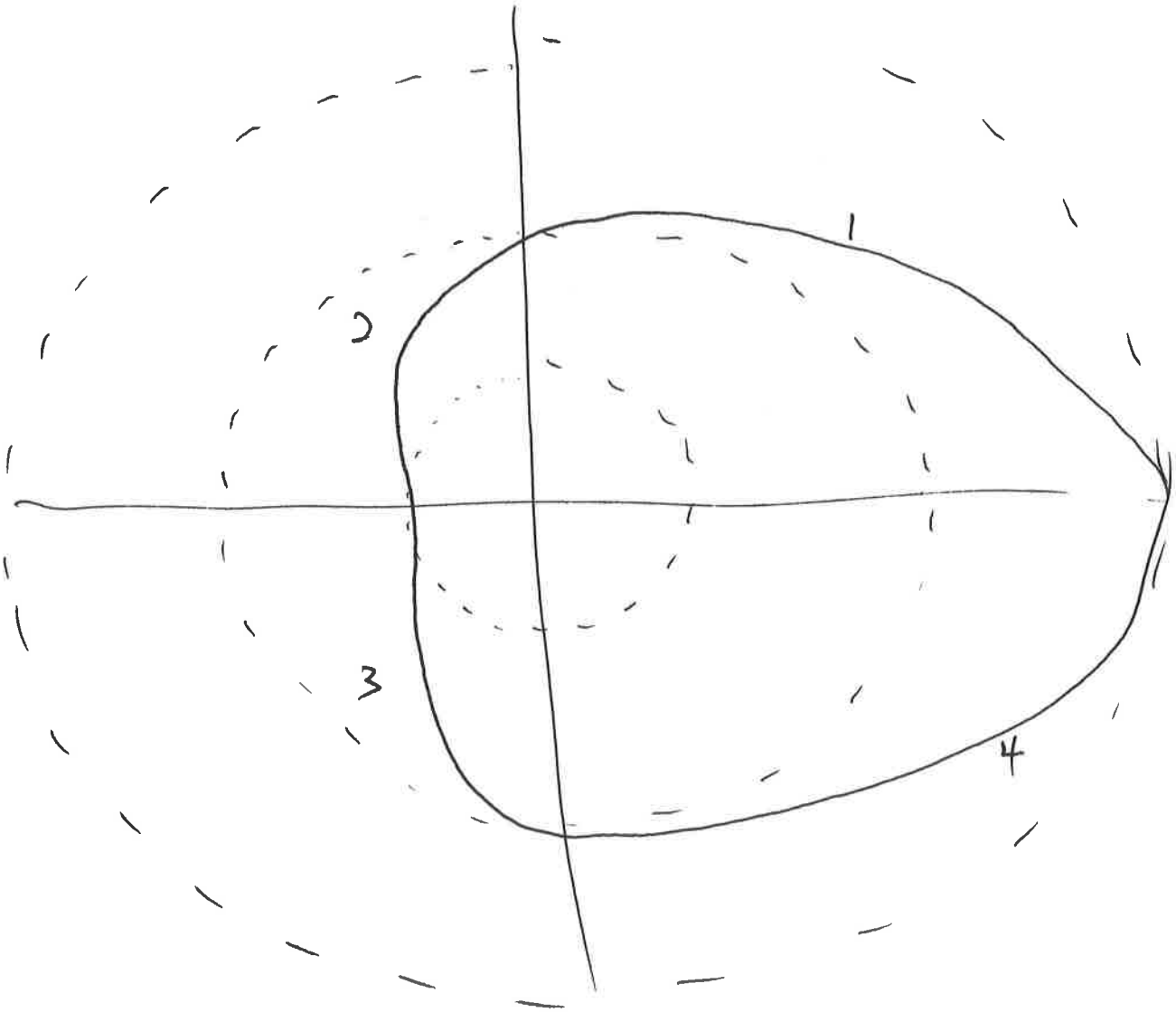




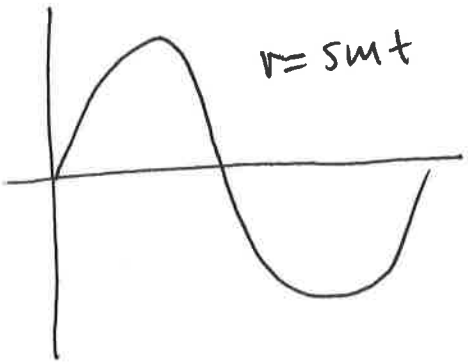
Graph $r = 2 + \cos t$

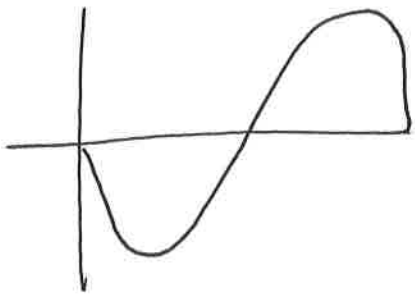






$r = 1 - \sin t$





$r = 1 - \sin t$