Sparse Channel Estimation Techniques for OFDM Systems Based on Subspace Methods

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Abstract: Sparse channels are typically encountered in many communication systems like underwater acoustic channels, communications in a hilly terrain etc. Conventional channel estimation techniques like the Least Squares approach and interpolation based methods do not work well in this case, because these techniques do not exploit the sparse structure of the channel. In this paper, we introduce the subspace based methods of Multiple Signal Classification (MUSIC) and Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) algorithm when the channel is constant for a few OFDM frames within the coherence time of the channel. We propose a new method for ESPRIT based channel estimation as calculation of the ESPRIT MMSE estimator proposed in the literature is prohibitively complex and involves two matrix inversions per iteration. Simulation results show that the proposed ESPRIT estimator slightly outperforms the ESPRIT MMSE estimator in terms of bit error rate over a wide range of SNR's and that too with a reduced computational complexity. Finally the performance of these subspace based estimators have been comprehensively quantified for many different test cases.

Keywords: *Sparse channels, Subspace, MSE, BER, MUSIC, ESPRIT, MMSE, Cramer-Rao Lower bound.*

I. INTRODUCTION

If there is a wireless channel which exhibits a very large delay spread with only a few non-zero channel coefficients, then such a channel is regarded as a sparse channel. There are many communication systems which are regarded as sparse for e.g. terrestrial transmission channel of high definition television (HDTV) [1], a hilly terrain communication channel [2] and underwater acoustic channels [3] to name a few. An example of the sparse channel is shown in fig 1. The reason why these channels are sparse may be attributed as follows: In the underwater acoustic channels, few significant multipaths are relatively close to each other and have a delay spread of almost the same order as in conventional channels. But there may be one signal which is reflected from the sea bed. This signal will have a huge delay compared to the other signals, so overall delay spread of the system is now very high with the inclusion of this far away multipath. Therefore, when the receiver samples the received signal at baseband, all the channel coefficients after the closely spread significant multipaths will be zero, then finally the last coefficient (due to reflection from

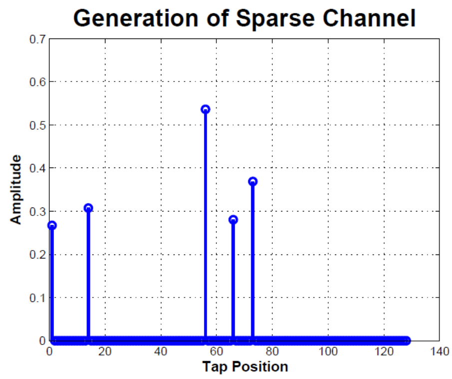

Fig. 1: A sparse channel with 5 non-zero multipaths

the sea bed) will be non-zero. This will result in a sparse channel. If the conventional channel estimation techniques are applied for sparse channel estimation, then they do not take into account the sparsity of the channel. These techniques treat the channel as if it has all non-zero coefficients and will try to estimate the channel taps in all the positions. Needless to say, the mean squared error and bit error rate performance will be highly degraded under such a scheme. Moreover, in a sparse channel the multipath delays may not be at the sampling instants. Therefore, ordinary channel estimation schemes cannot capture this delay and thus channel estimation results are more erroneous.

Sparse signal processing is in practice for quite a long time now. It was first reported in the literature for underwater acoustic channel measurements [3, 5]. Terrestrial broadcasting [1] for high definition (HD) television, communications near a hilly terrain [2] are also reported to be sparse. Besides this, sparse signal processing are predominant in many fields like spectral estimation, analysis of noisy images, coding of speech signals etc. A few application areas are listed in [1-5].

Subspace methods are markedly different from the compressive sensing methods discussed before when it comes to sparse channel estimation. If the channel is quasi-static for several OFDM frames within the coherence time of the channel, then the multipath delays for that many OFDM frames are the same and variations of the amplitudes are slow as well. Under this imposed constraint, the prevalent DoA estimation algorithms can be applied for sparse channel estimation as well [6-17]. Two of the most prominent methods for subspace based channel estimation are Multiple Signal Classification (MUSIC) [6] and Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) [11] respectively. Multiple Signal Classification or MUSIC algorithm [16, 17] was invented by Schmidt [6] in 1979. The principle on which MUSIC algorithm operates is the precise separation of signal subspace from the noise subspace. This is done after performing an eigen decomposition of channel covariance matrix [8] and applying the orthogonality principle of signal and noise subspaces [9]. In a typical DoA estimation, MUSIC furnishes the information about number of incident signals, direction of arrival (DOA) of each signal strengths and cross correlation between incident signals and noise power. The implementation of the MUSIC algorithm to sparse channel estimation is discussed in [9]. The ESPRIT algorithm was invented by R. Roy and T. Kailath and initially developed for finding oil wells [11]. Ongoing researches have found that if the pilot symbols are evenly spaced, then the signal subspace remains invariant. Under this condition ESPRIT algorithm can be applied for estimating the locations (delays) of the taps of the channel impulse response. The implementation of the ESPRIT algorithm to sparse channel estimation is discussed in [12].

II. SYSTEM MODEL

A frequency selective and Rayleigh fading sparse channel is described by

$$h(\tau) = \sum_{p=0}^{\alpha-1} c_p \, \delta(\tau - \tau_p) \quad (1)$$

where α is the number of non-zero channel coefficients (typical value is 5), c_p's are the non-zero channel coefficients and τ_p's are the respective multipath delays associated with the channel coefficients c_p's. We make the following assumptions here :

- The channel is assumed to be wide sense stationary and it changes after every "D" OFDM frames. Here "D" is calculated from the knowledge of coherence time of the channel and the useful OFDM frame duration.
- The different multipath gains are statistically uncorrelated
- The average power gain of each path is normalized to unity.

At the receiver side, the n^{th} subcarrier output during the i^{th} OFDM frame can be

$$r_i[n] = H_i[n].d_i[n] + w_i[n] \quad n = 0,1,2\ldots.(N - 1) \quad (2)$$

Here N is the number of active OFDM subcarriers.

III. ALGORITHMIC DESCRIPTION

1. ESPRIT Algorithm

A. Determination of channel order

Minimum Descriptor Length(MDL) criterion is applied first to determine the number of non-zero multipaths in the channel. The pilot symbol assisted modulation (PSAM) technique can be employed to get the least squares (LS) estimates of the channel frequency response at the pilot subcarriers. Let us denote the set of pilot symbols by p with p taking the values

$$p = \{0,1, 2,\ldots\ldots\ldots(N_p-1)\} \quad (3)$$

where N_p is the total number of pilots. Therefore at the pilot positions we must have

$$H_i[m] \approx \frac{r_i[m]}{d_i[m]}, \; m \in p \quad (4)$$

Among some of the well-established techniques for estimating the number of superimposed sinusoids the foremost is based on MDL criterion [18]. Before applying the MDL criterion, $H_{LS,i}$ is arranged in the form of a snapshot matrix

$$G(i) = \begin{bmatrix} H_{i,0} & H_{i,1} \;\ldots\ldots\ldots\ldots\ldots & H_{i,(A_s-1)} \\ H_{i,1} & H_{i,2} \;\ldots\ldots\ldots\ldots\ldots & H_{i,(A_s)} \\ & \ldots\ldots & \\ & \ldots\ldots & \\ H_{i,(N_p-A_s)} & H_{i,(N_p-A_s+1)} \;\ldots. & H_{i,(N_p-1)} \end{bmatrix} \quad (5)$$

Here N_P is the total number of pilots and A_S is the array factor. Typically A_S maybe chosen as 1.5 times to 2 times the length of the non-zero channel taps. For a 5 non-zero channel tap for instance, A_S is chosen as 8. Once we have the snapshot matrix, we can directly compute the sample autocorrelation matrix for the i^{th} frame by applying the forward-backward (FB) approach

$$\widehat{R}_{corr}\,(i) = \frac{1}{2A_s}\left(G(i)G(i)^H + J.\overline{G(i).G(i)^H}.J\right) \quad (6)$$

where $(.)^H$ represents the complex hermitian and (-) denotes complex conjugate. J is the exchange matrix which has a special structure that 1's are on its anti-diagonal and 0's elsewhere. To reduce the effect of noise, it is further averaged over Q consecutive OFDM frames and a better estimate of $\widehat{R}_{corr}$ is thus obtained.

$$\widehat{R}_{corr} = \frac{1}{Q}\sum_{i=0}^{Q-1} \widehat{R}_{corr}\,(i) \quad (7)$$

Q is typically determined from the coherence time of the channel and the useful OFDM frame duration. Then we perform an SVD on to obtain:

$$\widehat{R}_{corr} = \sum_{k=1}^{N_p-A_s+1} \hat{\lambda}_k \hat{v}_k \hat{v}_k^H \quad (8)$$

Now the minimum descriptor length criterion can be readily applied [58] for the forward backward method by

$$\mathrm{MDL}(\xi) = -Q(N_p - A_s + 1 - \xi) * \log \frac{\prod_{k=\xi+1}^{N_p-A_s+1} \hat{\lambda}_k^{\;1/(N_p-A_s+1-\xi)}}{\frac{1}{N_p - A_s + 1 - \xi}\sum_{k=\xi+1}^{N_p-A_s+1} \hat{\lambda}_k} + \frac{1}{4}\left[2(N_p - A_s + 1) - \xi + 1\right]\log(Q) \quad (9)$$

with ζ taking the values $0,1,\ldots\ldots(N_p-A_s)$. When the operation have been performed over all such ζ, we can estimate the number of paths by

$$\hat{\alpha} = \underset{\xi\,\in\{0,1,\ldots..(N_p-A_s)\}}{\mathrm{argmin}} \mathrm{MDL}(\xi) \quad (10)$$

B. Acquisition of multipath channel delays

The MMSE estimator of the channel $\hat{h}_{MMSE}$ is a significant improvement from the Least Squares Estimator. Following the standard compressive sensing model, we know

$$r = (G_sW)c + w \quad (11)$$

where r is the received symbol vector after FFT of dimension N -1, (G_sW) is the maintained dictionary of dimension N-N_t, c is the channel impulse response of dimension N_t - 1 and w is the additive white gaussian noise vector of dimension N -1. Since we have used pilot symbol assisted modulation, and have extracted the pilot positions only, so the above system model for pilot positions may be restated as $r_p = (G_sW)_p c + w_p$ where the subscript p denotes pilot positions only. We define a vector h as one which constitutes of non-zero channel coefficients only with a span of $\hat{\alpha}$ where $\hat{\alpha} << N_t$. Thus h now is of dimension $\hat{\alpha}$*1. Then we can define W_p as follows:

$$W_p = \begin{bmatrix} \exp\left(-j.2\pi.\frac{p(0)}{N}\frac{\hat{\tau}_0}{T_s}\right) \;\ldots.. \; \exp\left(-j.2\pi.\frac{p(0)}{N}\frac{\hat{\tau}_{\alpha-1}}{T_s}\right) \\ \ldots\ldots. \\ \ldots\ldots. \\ \ldots\ldots.. \\ \exp\left(-j.2\pi.\frac{p(N_p-1)}{N}\frac{\hat{\tau}_0}{T_s}\right) \;\ldots\; \exp\left(-j.2\pi.\frac{p(N_p-1)}{N}\frac{\hat{\tau}_{\alpha-1}}{T_s}\right) \end{bmatrix} \quad (12)$$

Equation 12 can be put as

$$r_p = W_p.h + w_p \quad (13)$$

The cross-correlation matrix of h and r_p related by above equation is given by

$$R_{h,r_p} = E\left[h.r_p^H\right]$$

$$= K_h W_p^H \quad (14)$$

Where $K_h = diag([\sigma_{h1}^2\ ,\sigma_{h2}^2\ \ldots\ldots.\sigma_{h\alpha}^2\ ]^T)$ is the covariance matrix of h. We also note that

$$R_{r_p,r_p} = E\left[r_p r_p^H\right]$$

$$= W_p K_h W_p^H + \frac{\sigma^2}{P} I_{N_p} \quad (15)$$

Then the MMSE estimator for $\hat{h}$ is

$$\hat{h} = R_{h,r_p}\left(R_{r_p,r_p}\right)^{-1} r_p$$

$$= \left(\frac{\beta}{SNR}K_h^{-1} + W_p^H W_p\right)^{-1} W_p^H r_p \quad (16)$$

Where $\beta = {E[|d_i[n]|^2]}/{P}$ is the ratio of average signal power to pilot power and $SNR = {E[|d_i[n]|^2]}/{\sigma^2}$ is the average signal-to-noise ratio.

Hence the MMSE estimator of channel frequency response is $\hat{H} = W_H \hat{h}$

$$= W_H\left(\frac{\beta}{SNR}K_h^{-1} + W_p^H W_p\right)^{-1} W_p^H r_p \quad (17)$$

We note that W_H is nothing but the N *$\hat{\alpha}$ discrete Fourier transform matrix.

$$W_H = \begin{bmatrix} \exp\left(-j.2\pi.\frac{0}{N}\frac{\hat{\tau}_0}{T_s}\right) \;\ldots.. \; \exp\left(-j.2\pi.\frac{0}{N}\frac{\hat{\tau}_{\alpha-1}}{T_s}\right) \\ \ldots\ldots. \\ \ldots\ldots. \\ \ldots\ldots.. \\ \exp\left(-j.2\pi.\frac{N-1}{N}\frac{\hat{\tau}_0}{T_s}\right) \;\ldots\; \exp\left(-j.2\pi.\frac{N-1}{N}\frac{\hat{\tau}_{\alpha-1}}{T_s}\right) \end{bmatrix} \quad (18)$$

2. Proposed ESPRIT algorithm

A. Acquisition of multipath delays

The multipath delays are captured in exactly the same way as described in the ESPRIT MMSE estimation. These detected multipath delays are denoted by τ_l

$$\tau_l = \{\tau_0, \tau_1, \ldots.\tau_{(N-1)}\} \quad (19)$$

B. Formulation of a reduced Least Squares Problem

We had maintained the dictionary of vectors (G_sW) which is given by the equation

$$G_s.W = \begin{bmatrix} \exp\left(-j.2\pi.\frac{0}{N}\frac{\hat{\tau}_0}{T_s}\right) \;\ldots.. \; \exp\left(-j.2\pi.\frac{0}{N}\frac{\hat{\tau}_{\alpha-1}}{T_s}\right) \\ \ldots\ldots. \\ \ldots\ldots. \\ \ldots\ldots.. \\ \exp\left(-j.2\pi.\frac{N-1}{N}\frac{\hat{\tau}_0}{T_s}\right) \;\ldots\; \exp\left(-j.2\pi.\frac{N-1}{N}\frac{\hat{\tau}_{\alpha-1}}{T_s}\right) \end{bmatrix} \quad (20)$$

where N_t is estimated from the maximum possible delay spread. The dictionary can be computed offline beforehand and prior to channel estimation. Therefore the proposed method is more suitable for real-time applications. We choose those columns from the dictionary that correspond to the multipath delays τ_l. obtained from the previous step. So, $(G_sW)_{p,Red}$ at the pilot locations is given by

$$(G_s.W)_{p,Red} = \begin{bmatrix} \exp\left(-j.2\pi.\frac{p(0)}{N}\frac{\hat{\tau}_0}{T_s}\right) \;\ldots.. \; \exp\left(-j.2\pi.\frac{p(0)}{N}\frac{\hat{\tau}_{\alpha-1}}{T_s}\right) \\ \ldots\ldots. \\ \ldots\ldots. \\ \ldots\ldots.. \\ \exp\left(-j.2\pi.\frac{p(N_p-1)}{N}\frac{\hat{\tau}_0}{T_s}\right) \;\ldots\; \exp\left(-j.2\pi.\frac{p(N_p-1)}{N}\frac{\hat{\tau}_{\alpha-1}}{T_s}\right) \end{bmatrix} \quad (21)$$

Then we solve a reduced Least Square problem by

$$\hat{h}_{ESPRIT} = \arg\min_{h}\left\|r_p - (G_sW)_{p,Red}h\right\|^2 \quad (22)$$

Then the estimated channel coefficients are placed in the channel impulse response vector c at the respective locations. The frequency response is determined by

$$\hat{H}_{ESPRIT} = \sum_{p=0}^{\hat{\alpha}-1} h(p)\,(G_s.W)_{Red(p)} \quad (23)$$

where $(G_sW)_{Red}(p)$ is the p^{th} column of the matrix $(G_sW)_{Red}$ and $\hat{\alpha}$ is the estimated number of non-zero multipaths. It is evident that the proposed ESPRIT estimator has a reduced complexity by a factor of $O(n^3)$ than the ESPRIT MMSE estimator proposed in literature. As evident from the simulation results below, it slightly outperforms the conventional MMSE estimator over a wide range of SNR's.

3. MUSIC algorithm

It is claimed in earlier researches [39] that the process of channel estimation by using interleaved pilots in a comb fashion presents itself as a linear array. Furthur more, if pilot positions are hardcoded or fixed, then it defines an invariant signal subspace. With this marvelous advantage at hand, we are in a position to use the prominent Direction-Of-Arrival estimation algorithm (MUSIC algorithm) for our pertinent problem of capturing the multipath delays at precise locations for channel estimation as well. As was the case in ESPRIT algorithm, we insert N_p pilot subcarriers uniformly interleaved among N data subcarriers in a comb pattern. The detailed detection procedure is described is described in [9].

IV. SIMULATION RESULTS

The following parameters have been fixed for simulations

Simulation parameters of OFDM	Values with respective units
Number of OFDM Subcarriers	1024
Sampling time period	0.1 μs
Length of Cyclic Prefix	128
Modulation Type	4 QAM
Physical Channel Length	128
Number of OFDM frames for which the channel is constant	20
Range of SNR	5 to 25 dB in steps of 4
PDP of the Channel	Uniform

Table (i) : Input parameters for simulation

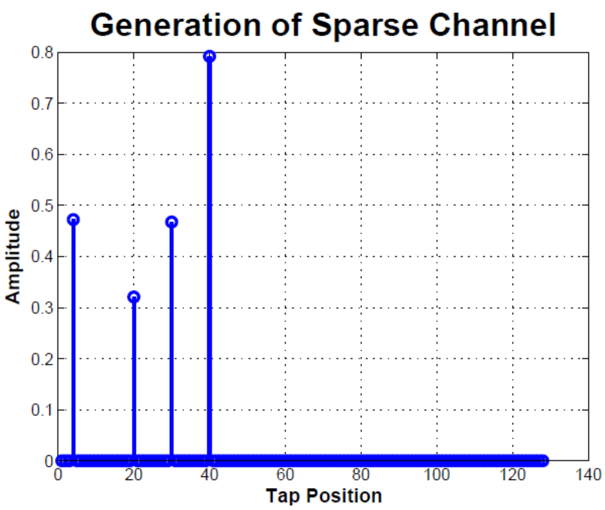

Fig. 2 Generation of a 4 tap sparse channel

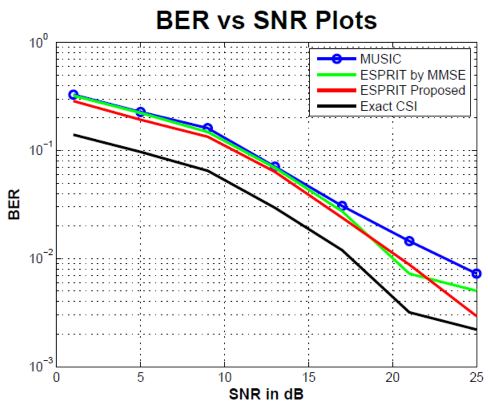

Fig. 3: BER vs SNR plot for subspace methods with 4 multipaths and uniform pdp.

The plots shown above correspond to four different non-zero multipaths. Since the subspace based algorithms are very sensitive to SNR, so at low SNR all of them give a poor performance. This is because the computation of autocorrelation matrix R_{HH} is far from perfect at low SNR. At high SNR's around 20 dB, the ESPRIT algorithms present a bit error level of about 10^{-2}. This is because of the precise calculation of R_{HH} at high SNR's and subsequent exact capture of the multipath delays. It is evident that the proposed ESPRIT estimator slightly outperforms the ESPRIT MMSE estimator in all SNR range. In addition, the proposed ESPRIT estimator has a reduced computational complexity. The estimator based on MUSIC algorithm is inferior in bit error rate performance to that of both types of ESPRIT algorithm.

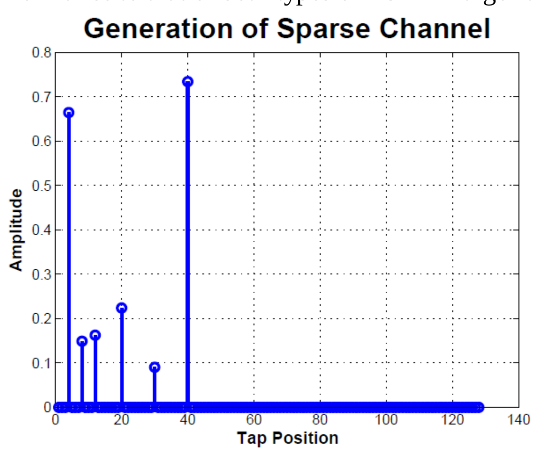

Fig. 4: Generation of a 6 tap sparse channel

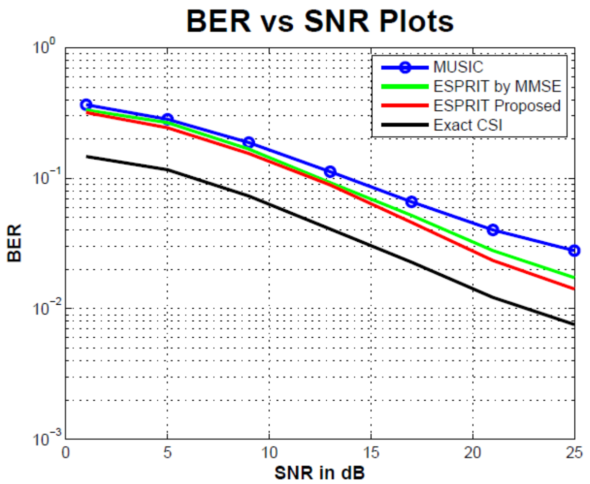

Fig. 5: BER vs SNR plot for subspace methods with 6 multipaths and uniform pdp.

As we increase the number of non-zero channel taps, (from α = 4 to α = 6) then the probability of correct detection of the number of farthest multipaths goes on decreasing. The effect is most pronounced for the sixth multipath in this case. Once there is slight error in detection of multipaths, this reflects itself in bit error rate performance, because the wrong columns of the dictionary are chosen as the one corresponding to signal in case of our proposed ESPRIT and MUSIC algorithm. For ESPRIT estimate by MMSE method, the covariance of the channel coefficient matrix is in error when the number of multipaths increase, and that also results in a degraded bit error rate performance. From the above figure, it is evident that even with increased non-zero channel order the proposed ESPRIT estimator slightly outperforms the ESPRIT MMSE estimator. Performance of MUSIC estimator slightly lags that of the ESPRIT estimators.

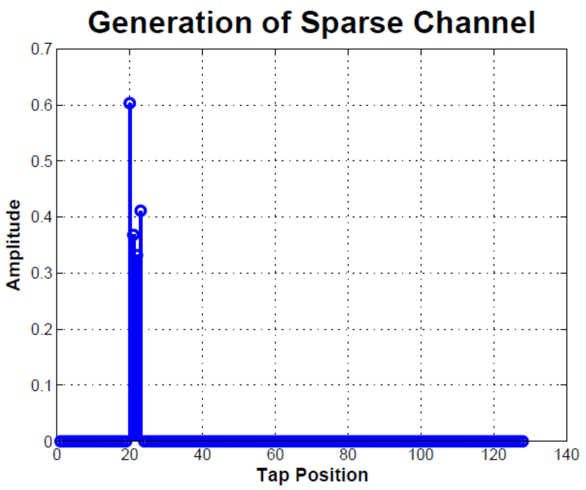

Fig. 6: Sparse channel with 4 non-zero adjacent multi paths.

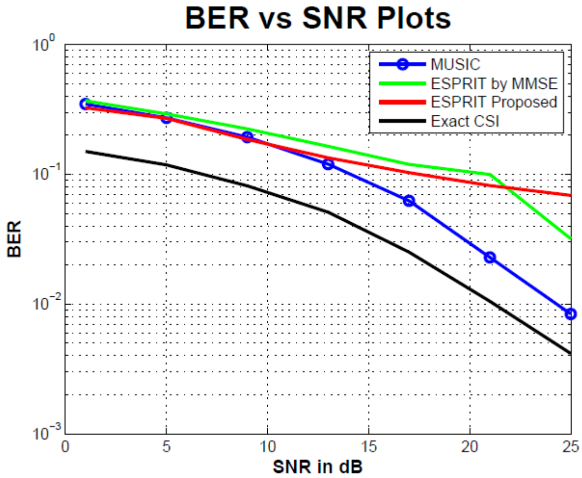

Fig. 7: BER vs SNR plot for subspace methods with 4 adjacent multipaths

V. CONCLUSION

In this paper, we have proved that evaluation of the MMSE Estimate by ESPRIT algorithm proposed in literature is prohibitively complex and involves 2 matrix inversions of the order of α*α in each iteration (where α is the actual number of non-zero multipaths). So the MMSE estimate can pose a serious problem of computational complexity if the order of non-zero channel coefficients increase. On the other hand, our proposed ESPRIT estimator involves only one matrix inversion per iteration. Hence this proposed method reduces the computational complexity by a factor of $O(n^3)$ and outperforms the conventional ESPRIT MMSE estimate over a wide range of SNR's. Finally, the performances of these subspace based estimators (MUSIC and ESPRIT) have been comprehensively quantified for many different test cases. The performance of the proposed estimator is also presented alongside the conventional techniques and demonstrated to be superior.

VI. ACKNOWLEDGEMENT

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