Ex 2 $\quad \dfrac{dx}{dt} = -x \qquad \dfrac{dy}{dt} = 1-x^2-y^2$

cp $\quad x=0 \quad 1-x^2-y^2=0 \Rightarrow y = \pm 1$

$$D_x f = \begin{pmatrix} -1 & 0 \\ -2x & -2y \end{pmatrix}$$

at $(0,-1)$ $\quad \dot{\bar{x}} = \begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix} \bar{x}$

Eigenvalue $\quad \begin{vmatrix} \lambda+1 & 0 \\ 0 & \lambda-2 \end{vmatrix} = 0 \quad (\lambda+1)(\lambda-2)=0$

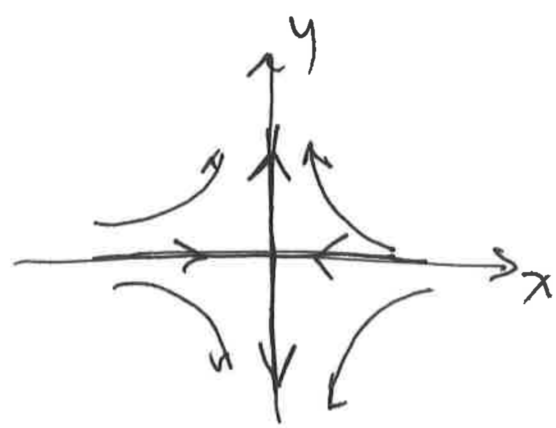
$\lambda = -1, 2$

$\underline{\underline{\lambda=-1}}$ $\quad \begin{pmatrix} 0 & 0 \\ 0 & -3 \end{pmatrix} \bar{u} = \bar{0} \Rightarrow \bar{u} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\underline{\underline{\lambda=2}}$ $\quad \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix} \bar{u} = \bar{0} \Rightarrow \bar{u} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Sol^n

$$\bar{x} = c_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{t}$$



at $(0,1)$ $\quad \dot{\bar{x}} = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} \bar{x}$

Eigenvalues $\quad \begin{vmatrix} \lambda+1 & 0 \\ 0 & \lambda+2 \end{vmatrix} = 0 \quad (\lambda+1)(\lambda+2) = 0$

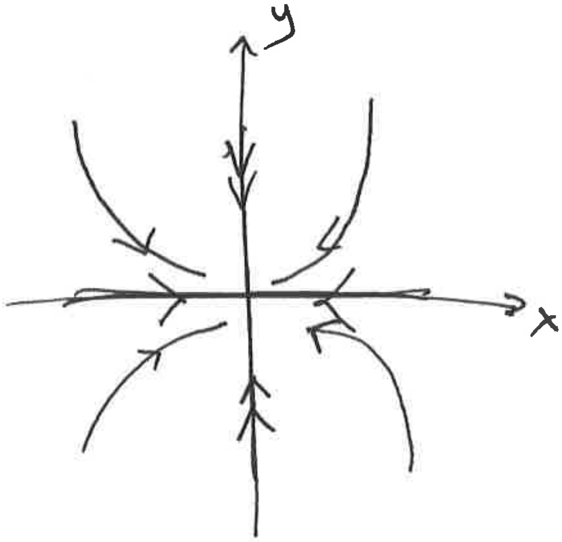
$\lambda = -1, -2$

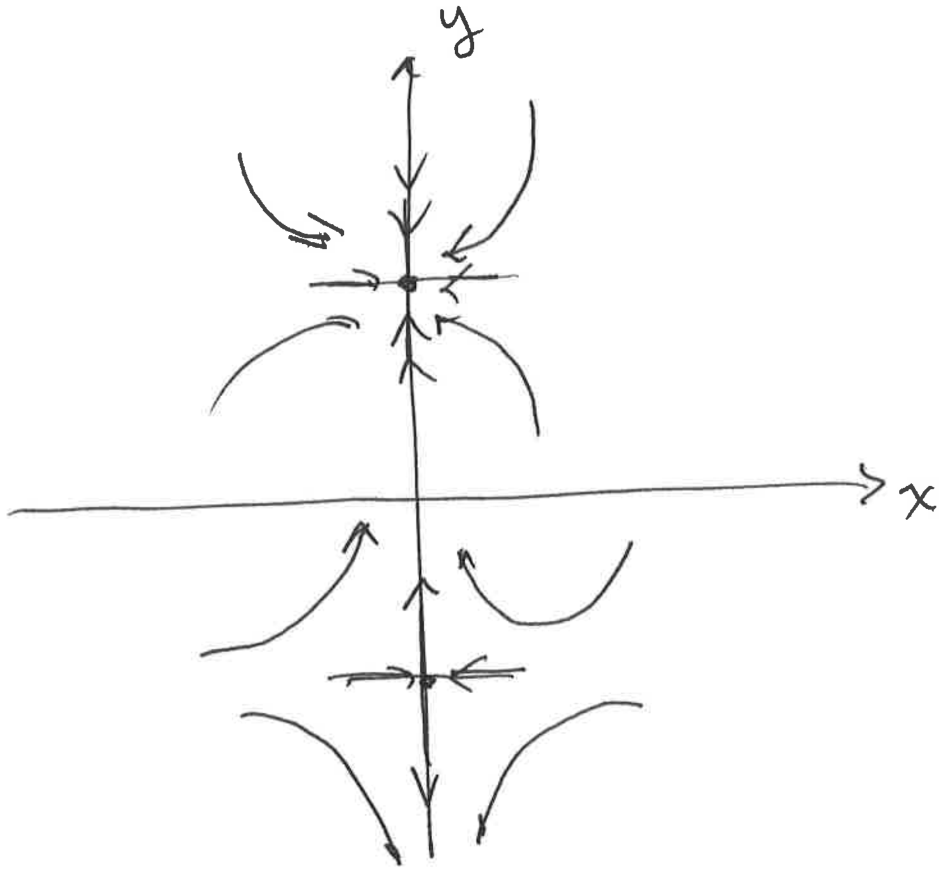
$\underline{\underline{\lambda = -1}}$ $\quad \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \bar{u} = \bar{0} \Rightarrow \bar{u} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\underline{\underline{\lambda = -2}}$ $\quad \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix} \bar{u} = \bar{0} \Rightarrow \bar{u} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\underline{\text{Sol}^n}$

$\bar{x} = c_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{-2t}$





To find the actual manifold, we need to

solve $\dfrac{dy}{dx} = \dfrac{1-x^2-y^2}{-x}$

If we let $y = -x\dfrac{u'}{u}$ then

$$y' = -x\frac{u''}{u} + \cancel{x\frac{u'^2}{u^2}} - \frac{u'}{u} = -\frac{1}{x} + x + \cancel{\frac{x^2 u'^2}{x u^2}}$$

$$\Rightarrow \quad x^2 u'' + x u' + (x^2-1)u = 0$$

Sol^n is : $u = c_1 J_1(x) + c_2 Y_1(x)$

where J_1 & Y_1 are Bessel functions

The sol^n for y is

$$y = -\frac{c_1\left(xJ_0(x) - J_1(x)\right) + c_2\left(xY_0(x) - Y_1(x)\right)}{c_1 J_1(x) + c_2 Y_1(x)}$$

if $c_1 \neq 0$ let $c = c_2 / c_1$ so

$$y = -\frac{xJ_0(x) - J_1(x) + c\left(xY_0(x) - Y_1(x)\right)}{J_1(x) + cY_1(x)}$$

The unstable manifold through the CP $(0,-1)$ is when $c = 0$ so

$$y = \frac{J_1(x) - xJ_0(x)}{J_1(x)}$$