Chapter 2
Quadratic Functions

Section 2-1
Transformations of Quadratic Functions

Essential Question

How do the constants a, h, and k affect the graph of the quadratic function $g(x) = a(x - h)^2 + k$?

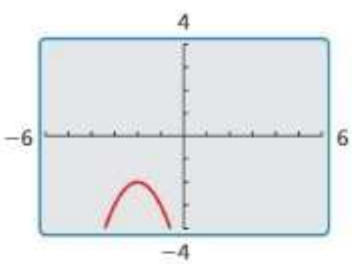
The parent function of the quadratic family is $f(x) = x^2$. A transformation of the graph of the parent function is represented by the function $g(x) = a(x - h)^2 + k$, where $a \neq 0$.

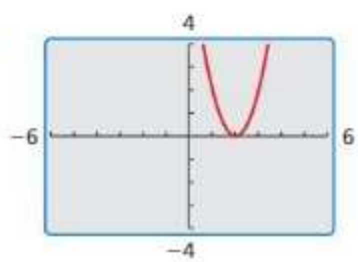
EXPLORATION 1 Identifying Graphs of Quadratic Functions

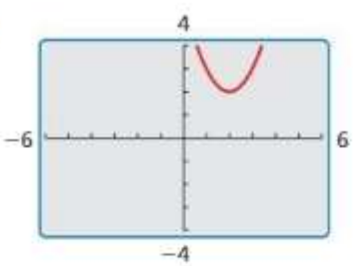
Work with a partner. Match each quadratic function with its graph. Explain your reasoning. Then use a graphing calculator to verify that your answer is correct.

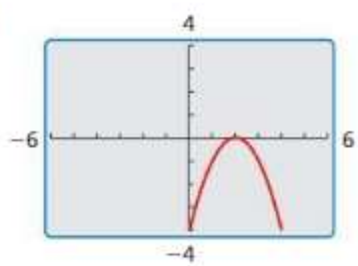
a. $g(x) = -(x - 2)^2$ **b.** $g(x) = (x - 2)^2 + 2$ **c.** $g(x) = -(x + 2)^2 - 2$

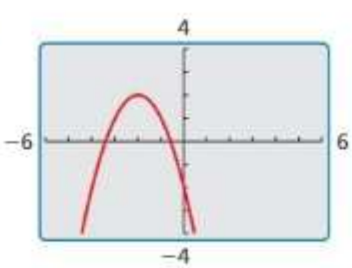
d. $g(x) = 0.5(x - 2)^2 - 2$ **e.** $g(x) = 2(x - 2)^2$ **f.** $g(x) = -(x + 2)^2 + 2$

A.


B.


C.


D.


E.


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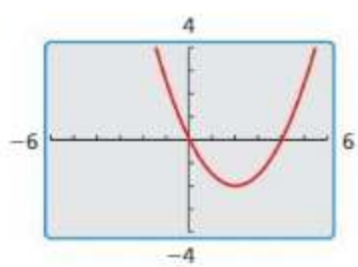
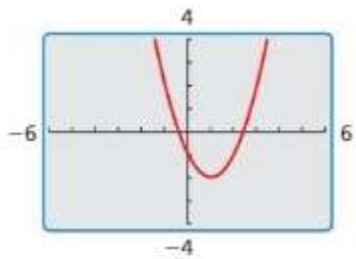


REMEMBER:

VERTEX AND INTERCEPT FORMS OF A QUADRATIC FUNCTION	
FORM OF QUADRATIC FUNCTION	**CHARACTERISTICS OF GRAPH**
Vertex form $y = a(x-h)^2 + k$	The vertex is (h,k). The axis of symmetry is $x = h$.

Communicate Your Answer

2. How do the constants a, h, and k affect the graph of the quadratic function $g(x) = a(x-h)^2 + k$?



Writing Transformations of Quadratic Functions

The lowest point on a parabola that opens up or the highest point on a parabola that opens down is the **vertex**. The **vertex form** of a quadratic function is $f(x) = a(x-h)^2 + k$, where $a \neq 0$ and the vertex is (h, k).

$$f(x) = a(x-h)^2 + k$$

- a indicates a reflection in the x-axis and/or a vertical stretch or shrink.
- h indicates a horizontal translation.
- k indicates a vertical translation.

<u>Quadratic Equations in different forms (Identify the form and state the vertex).</u>

$f(x) = x^2$

$f(x) = x^2 + 3x - 2$

$f(x) = -(x-2)^2 + 3$

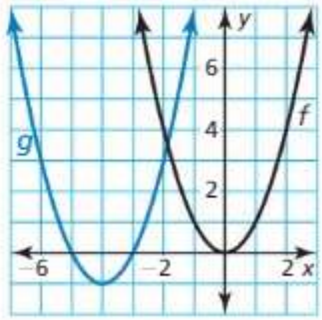
$f(x) = (x-3)(x+2)$

Describing Transformations of Quadratic Functions

A **quadratic function** is a function that can be written in the form $f(x) = a(x - h)^2 + k$, where $a \neq 0$. The U-shaped graph of a quadratic function is called a **parabola**.

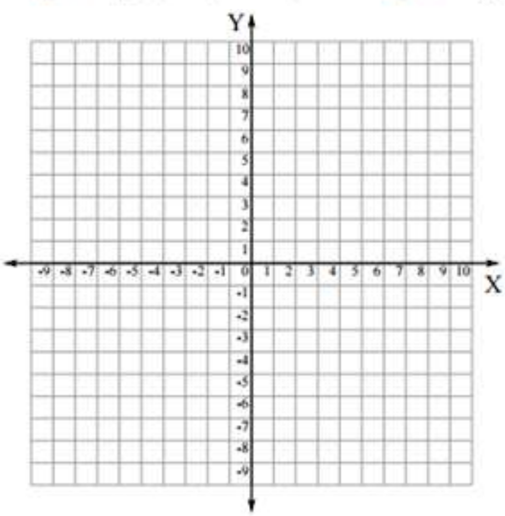
EXAMPLE 1 **Translations of a Quadratic Function**

Describe the transformation of $f(x) = x^2$ represented by $g(x) = (x + 4)^2 - 1$. Then graph each function.

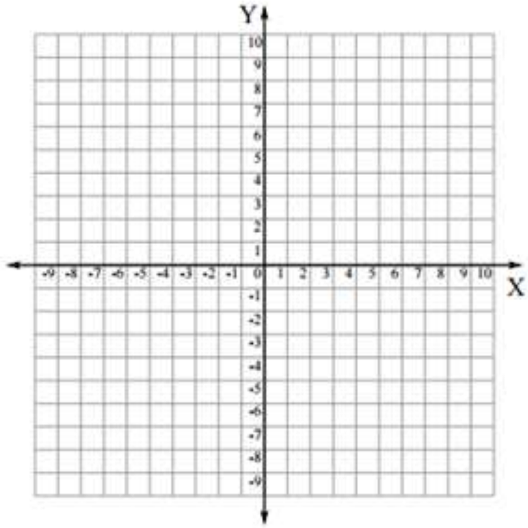


Describe the transformation of $f(x) = x^2$ represented by g. Then graph each function.

1. $g(x) = (x - 3)^2$ **2.** $g(x) = (x - 2)^2 - 2$ **3.** $g(x) = (x + 5)^2 + 1$



X	Y



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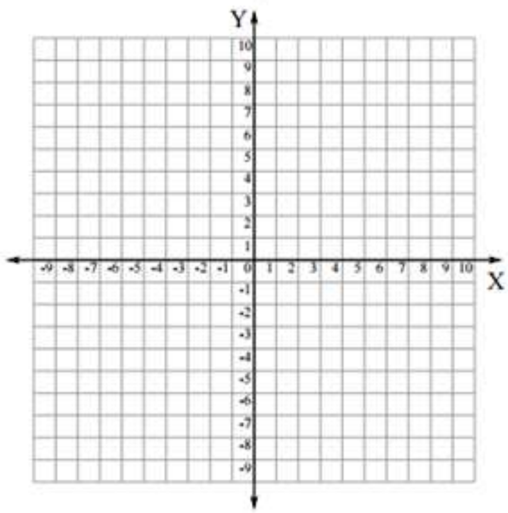


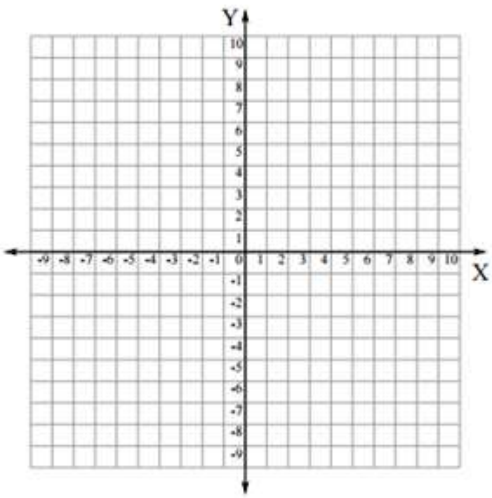
X	Y

EXAMPLE 2 Transformations of Quadratic Functions

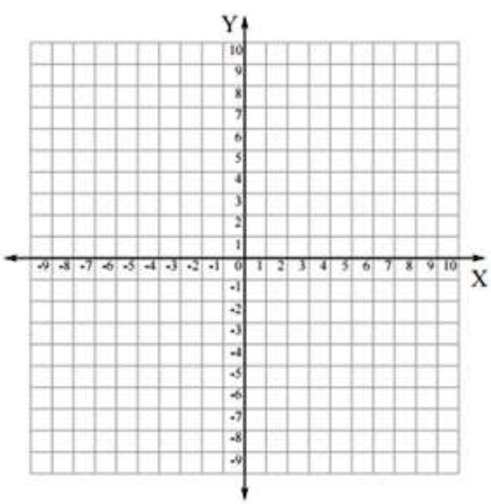
Describe the transformation of $f(x) = x^2$ represented by g. Then graph each function.

a. $g(x) = -\frac{1}{2}x^2$

b. $g(x) = (2x)^2 + 1$



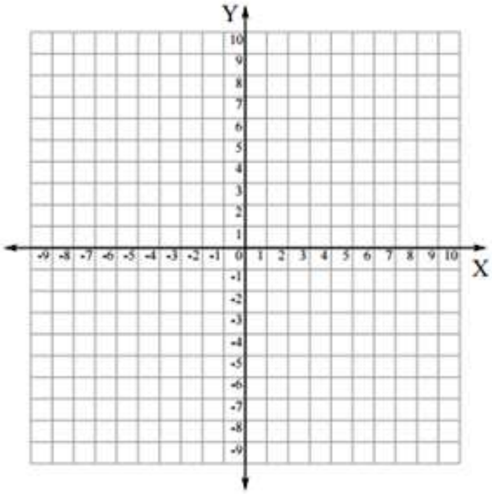
X	Y



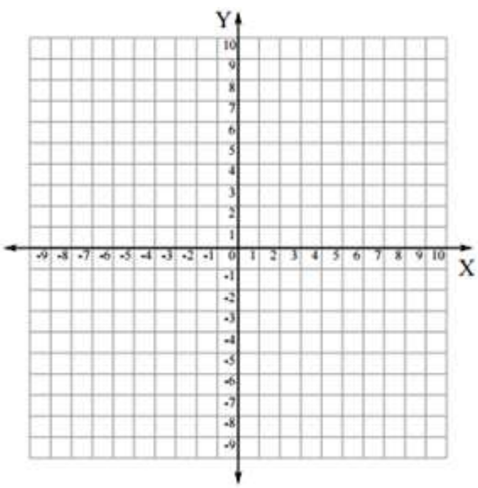
X	Y

Describe the transformation of $f(x) = x^2$ represented by g. Then graph each function.

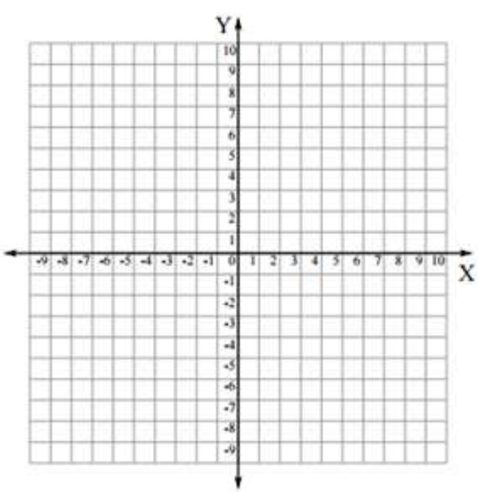
4. $g(x) = \left(\frac{1}{3}x\right)^2$ **5.** $g(x) = 3(x - 1)^2$ **6.** $g(x) = -(x + 3)^2 + 2$



X	Y



X	Y



X	Y

EXAMPLE 3 Writing a Transformed Quadratic Function

Let the graph of g be a vertical stretch by a factor of 2 and a reflection in the x-axis, followed by a translation 3 units down of the graph of $f(x) = x^2$. Write a rule for g and identify the vertex.

EXAMPLE 4 Writing a Transformed Quadratic Function

Let the graph of g be a translation 3 units right and 2 units up, followed by a reflection in the y-axis of the graph of $f(x) = x^2 - 5x$. Write a rule for g.

REMEMBER

To multiply two binomials, use the FOIL Method.

First Inner

$(x + 1)(x + 2) = x^2 + 2x + x + 2$

Outer Last

7. Let the graph of g be a vertical shrink by a factor of $\frac{1}{2}$ followed by a translation 2 units up of the graph of $f(x) = x^2$. Write a rule for g and identify the vertex.
8. Let the graph of g be a translation 4 units left followed by a horizontal shrink by a factor of $\frac{1}{3}$ of the graph of $f(x) = x^2 + x$. Write a rule for g.

Tutorial

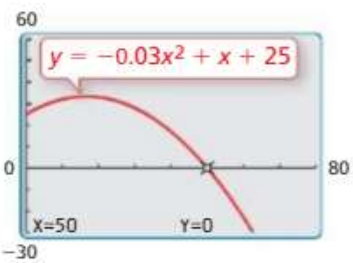
EXAMPLE 5 Modeling with Mathematics

The height h (in feet) of water spraying from a fire hose can be modeled by $h(x) = -0.03x^2 + x + 25$, where x is the horizontal distance (in feet) from the fire truck. The crew raises the ladder so that the water hits the ground 10 feet farther from the fire truck. Write a function that models the new path of the water.

SOLUTION

1. **Understand the Problem** You are given a function that represents the path of water spraying from a fire hose. You are asked to write a function that represents the path of the water after the crew raises the ladder.
2. **Make a Plan** Analyze the graph of the function to determine the translation of the ladder that causes water to travel 10 feet farther. Then write the function.
3. **Solve the Problem** Use a graphing calculator to graph the original function.

   Because $h(50) = 0$, the water originally hits the ground 50 feet from the fire truck. The range of the function in this context does not include negative values. However, by observing that $h(60) = -23$, you can determine that a translation 23 units (feet) up causes the water to travel 10 feet farther from the fire truck.



$g(x) = h(x) + 23$ — Add 23 to the output.

$= -0.03x^2 + x + 48$ — Substitute for $h(x)$ and simplify.

▶ The new path of the water can be modeled by $g(x) = -0.03x^2 + x + 48$.

4. **Look Back** To check that your solution is correct, verify that $g(60) = 0$.

   $g(60) = -0.03(60)^2 + 60 + 48 = -108 + 60 + 48 = 0$ ✓

Section 2-1 Homework #3,5,11,13,15,17,19,23,25,27,29,31,33,35,37,39,41,51