Math 3331 - ODE's

Newton's Law of Cooling

An objects rate of heat loss (or gain) is directly proportional to the difference in temp between the object & its surroundings

If $T = T(t)$ temp of body
T_∞ - ambient temp

$$\frac{dT}{dt} = k(T - T_\infty)$$

Sol'n $\frac{dT}{T - T_\infty} = k\,dt$ (if T_∞ is constant)

$$\ln|T - T_\infty| = kt + \ln C$$

$$T - T_\infty = Ce^{kt} \quad \text{or} \quad T = T_\infty + Ce^{kt}$$

if $T(0) = T_0$ (const) $\Rightarrow$ $T_0 = T_\infty + C$ $\quad C = T_0 - T_\infty$

$$\Rightarrow \quad T = T_\infty + (T_0 - T_\infty)e^{kt}$$ just need k

Ex 1 A cup of coffee is 200°F when it is poured and cools to 190°F after 1 min. If the room temp is 70°F (fixed) find the temp at any t.

Sol": Assuming Newton's law of cooling

$$T = T_\infty + (T_0 - T_\infty) e^{kt}$$

Here T_∞ - 70°F room temp

T_0 - 200°F initial temp

$$\text{So} \quad T = 70 + (200 - 70) e^{kt}$$

Now $T(1) = 190$

$$\text{So} \quad 190 = 70 + 140 e^{k(1)}$$

$$e^k = \frac{190-70}{130} \quad \text{so} \quad k = \ln \frac{12}{13} = -.08$$

$$T = 70 + 130 e^{-.08t}$$

$$T(30) = 70 + 130 e^{-.08(30)} = 81.8°F$$

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Ex2 You show up at midnight 12am to a murder scene. You take the body temp (90°F) then inspect the area. At 1:30am the body temp has dropped to 87°F. The room temp was 82°F. Find the time of death.

Assuming Newton's Law:

$$T = T_\infty + (T_0 - T_\infty)\, e^{kt}$$

$$= 82 + (90-82)\, e^{kt}$$

$$= 82 + 8e^{kt}$$

Now at 1:30 so $t=90$ $T(90) = 87$

$$\text{so } 87 = 82 + 8e^{90k}$$

$$\Rightarrow e^{90k} = \frac{87-82}{8} = 5/8 \quad \text{so } k = \frac{1}{90}\ln 5/8$$

$$= -.0052$$

$$T = 82 + 8e^{-.0052t}$$

when is $T = 98.6$ so $82 + 8e^{-.0052t} = 98.6$

$$e^{-.0052t} = \frac{98.6 - 82}{8}$$

$$t = \frac{1}{-.0052} \ln\left(\frac{98.6 - 82}{8}\right)$$

$$= -140.37 \text{ minutes} \approx 2\text{hr } 20\text{min}$$

so 12 midnight - 2:20

so 9:40 pm