Effects of Variable Viscosity and Thermal Conductivity on Heat and Mass Transfer Flow of Micropolar Fluid along a Vertical Plate in Presence of Magnetic Field

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Abstract:

The effect of temperature dependent viscosity and thermal conductivity on heat and mass transfer flow of micropolar fluid along a vertical plate under the combined buoyancy force of thermal and species diffusion in the presence of a transverse magnetic field is studied where the viscosity and thermal conductivity are assumed to be inverse linear functions of temperature. The partial differential equations governing the flow, heat and mass transfer of the problem are transformed into dimensionless form of ordinary differential equations by using similarity substitutions. The governing boundary value problems are then solved numerically using shooting method. The effects of various parameters viz. viscosity parameter, thermal conductivity parameter, mass transfer parameter, magnetic parameter, the coupling constant parameter, Prandtl number and Schimdt number on velocity, micro-rotation, temperature and concentration fields are obtained and presented graphically. The Skin-friction, Nusselt number and Sherwood number are also computed and presented in table.

Key Words and Phrases: Micropolar fluid, variable viscosity, thermal conductivity, mass transfer, MHD Flow.

INTRODUCTION: The concept of micropolar fluid derives from the need to model the flow of fluids that contain rotating micro-constituents. In micropolar fluid theories, each particle has a finite size and constitutes a micro structure, which can rotate about the centre of the volume element described by the micro-rotation vector. The effects of magneto hydrodynamics on micropolar flow has become important due to several engineering applications such as in MHD generators, designing cooling system for nuclear reactor, flow meters etc., where the micro concentration provides an important parameter for deciding the rate of heat flow. By simulating it one can obtain the desired temperature in such equipments. Several investigations have made theoretical and experimental studies of micropolar flow in the presence of a transverse magnetic field during the last few decades. Assuming fluid viscosity as a linear function of temperature the effect of variable viscosity on MHD natural convection in micropolar fluids was studied by M. Abd El-Hakiem et al. [1]. Gorla et al. [6] investigated the Magneto hydrodynamic free convection boundary layer flow of a thermo micropolar fluid over a vertical plate. Gorla [7] studied the flow of a micropolar fluid over a flat plate. Ishak et al. [8] studied the problem of steady boundary layer flow and heat transfer of a micropolar fluid on an isothermal continuously moving plane surface assuming that the micro-inertia density is variable and not constant. Muthucumaraswami et al. [10] investigated the magnetic field effects on flow past an accelerated isothermal vertical plate with heat and mass diffusion. Modather M. et al. [11] studied variable viscosity effect on heat transfer over a continuous moving surface with variable internal heat generation in micropolar fluids. Peddieson et al. [12] investigated boundary layer theory for micro polar fluid. Rajesh [13] investigated the MHD free convection flow past an accelerated vertical porous plate with variable temperature through a porous medium. Salem A.M. et al. [14] studied influence of variable viscosity and thermal conductivity on flow of micropolar fluid past a continuously moving plate with suction or injection. Hazarika and Sharma [15] studied the effects of variable viscosity and thermal conductivity on heat and mass transfer flow of
Newtonian fluid along a vertical plate in the presence magnetic field. They [16] investigated effects of variable viscosity and thermal conductivity on combined free-forced convection and mass transfer flow passed a vertical porous plate.

The main objective of our present work is to extend the work of Hazarika and Sharma [15] to study the effects of variable viscosity and thermal conductivity on heat and mass transfer flow of micropolar fluid along a vertical plate in presence of magnetic field. Viscosity and thermal conductivity are assumed to be inverse linear functions of temperature. The governing partial differential equations are reduced in to ordinary differential equations by similarity transformations. The problem is then solved numerically using Runge-kutta shooting algorithm with iteration process.

MATHEMATICAL FORMULATION OF THE PROBLEM:

We consider a steady convective flow of an incompressible micropolar electrically conducting fluid past a semi-infinite vertical plate, at constant temperature $T_0$, in the presence of uniform transverse magnetic field. The temperature of the fluid far from the plate is $T_w$. The $x$-axis is taken along the plate in upward direction and the $y$-axis is taken normal to it. The plate is parallel to the direction which is also the direction of gravity but directed vertically upward. Since the velocity of the fluid is very low, the viscous dissipative heat is assumed to be negligible. Also a magnetic field of constant intensity is assumed to be applied normal to the vertical plate and the vertically upward. The governing equations of the problem are:

Equation of continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \ldots (1)$$

Momentum equation:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \rho_0 \beta (T - T_w) + \rho_0 \beta' (C - C_w) + \frac{\partial}{\partial y} \left( \frac{\sigma B^2}{\rho} \right) - \frac{\partial P}{\partial y} \quad \ldots (2)$$

Angular momentum equation:

$$u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} = \frac{1}{\rho} \frac{\partial}{\partial y} \left( \frac{\sigma B^2}{\rho} \right) \quad \ldots (3)$$

Energy equation:

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{\rho c_p} \frac{\partial}{\partial y} \left( \frac{\lambda}{\rho} \frac{\partial T}{\partial y} \right) \quad \ldots (4)$$

Equation of mass transfer:

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = \frac{1}{s_c} \frac{\partial}{\partial y} (\nu \frac{\partial C}{\partial y}) \quad \ldots (5)$$

The equation of continuity being identically satisfied by velocity profile $(u(y), 0)$. It should be mentioned that $N = 0$ indicates strong concentration by Gurum and Smith [7] and represents concentrated particles flows in which the microelements close to the wall surface are unable to translate or rotate. It is known (Gorla [6]) that $N$ is the total spin of microstructure and fluid media in the flow field. In some cases, the
microstructure effects become negligible and the fluid behaves like an ordinary (Newtonian) viscous flow. Therefore, if we state that $N$ is angular velocity is a valid solution, then this is possible only if

$$Y = \left( \mu_c + \frac{x}{2} \right) j = \mu_c \left( 1 + \frac{K}{2} \right) j,$$

where $K = \frac{x}{\mu_c}$, coupling constant parameter.

Following Lai and Kulacki [9] we assume that the viscosity and thermal conductivity are inverse linear functions of temperature, i.e.

$$\frac{1}{\mu} = \frac{1}{\mu_c} \left[ 1 + \delta(T - T_c) \right], \text{ or } \frac{1}{\lambda} = \alpha(T - T_c) \quad \text{where } \alpha = \frac{\delta}{\mu_c} \text{ and } T_c = T_c - \frac{1}{\delta}$$

$$\frac{1}{\lambda} = \frac{1}{\lambda_c} \left[ 1 + \xi(T - T_c) \right], \text{ or } \frac{1}{\lambda} = \beta(T - T_c) \quad \text{where } \beta = \frac{\xi}{\lambda_c} \text{ and } T_c = T_c - \frac{1}{\xi}$$

Where $\alpha, \beta, T_c, \text{ and } T_c$ are constants and their values depend on the reference state and thermal properties of the fluid i.e., $\alpha$ and $\beta$. In general $\alpha > 0$ for liquids and $\alpha, \beta < 0$ for gases.

The appropriate boundary conditions are

$$\begin{align*}
Y = 0: & \quad u = \nu = 0, N = 0, T = T_{w,e}, C = C_{w,e} = 0 \\
Y = \infty: & \quad u = 0, T = T_{w,e}, C = C_{w,e}, \quad N = 0
\end{align*} \quad \text{(7)}$$

We introduce the following non-dimensional variables

$$\psi = 4c \nu \alpha \Delta \delta f(\eta) \quad \eta = \frac{4}{x} \delta \nu \alpha \theta(\eta), C = \frac{C-C_{w,e}}{\theta_{w,e}} \theta(\eta), N = 4 \nu \alpha \Delta \delta \frac{x^2}{\nu_{w,e}} \theta(\eta), \quad \text{and }$$

$$\theta_c = \frac{T_{w,e} - T_c}{\nu_{w,e}}, \quad \theta_\nu = \frac{T_{w,e} - T_c}{\nu_{w,e}}, \quad \nu = -\nu_c \frac{\theta_c}{\theta_\nu}, \quad \lambda = -\lambda_c \frac{\theta_c}{\theta_\nu}, \quad G_T = \frac{\Delta \delta (T_{w,e} - T_c)}{\nu_{w,e}^2}, \quad \text{and} \quad U = \sqrt{\frac{\nu \alpha \Delta \delta (T_{w,e} - T_c)}{}}$$

is the quantity with the dimension of the speed.

Substituting above transformations in equations (2) — (6), we get

$$\left(1 + K_1 \frac{\theta_c - \theta_\nu}{\theta_c} \right) f'''' = \left[ \theta_c - 2 \theta_c + K_1 f'' + \left( 3 f f'' - 2 f'' + 2 f'''' \right) \right] \frac{\theta_c - \theta_\nu}{\theta_c}, \quad \text{and} \quad$$

$$\left(1 + \frac{1}{2} K_1 \right) f h'' = \left( h f'' - 3 f f'' \right) t - K_1 (2 f + f'') \quad \text{(9)}$$

$$\theta'' = 3 R \left( \frac{\theta_c}{\theta_\nu} \right)^{\theta_c - \theta_\nu} \frac{\theta_c - \theta_\nu}{\theta_\nu}, \quad \text{and} \quad$$

$$\frac{\theta''}{\theta_c} = 3 R \left( \frac{\theta_c}{\theta_\nu} \right)^{\theta_c - \theta_\nu} \frac{\theta_c - \theta_\nu}{\theta_c}, \quad \text{and} \quad$$

$$\Delta = \frac{\theta_c (C_C - C_{w,e})}{\theta_\nu (T_{w,e} - T_c)}$$

represents the relative effect of chemical diffusion on thermal diffusion. When $\Delta = 0$, there is no mass diffusion and the buoyancy force arises solely from the temperature difference.

The corresponding boundary conditions are
The physical quantities of interest in this problem are the skin-friction coefficient $C_f$, Nusselt number $Nu$ and Sherwood number $S_h$ which indicate physically wall shear stress, rate of heat transfer and rate of mass transfer respectively. For micropolar boundary layer flow, the wall shear stress $\tau_w$ is given by

$$\tau_w = \left( \mu + k \frac{\partial^2 \theta}{\partial y^2} + kN \right)_{y=0}$$

(14)

The variations in dimensionless velocity distribution, micro-rotation distribution, species concentration distribution, temperature distribution with thermal conductivity parameter $\theta_\tau$, viscosity parameter $\theta_\nu$, magnetic parameter, coupling constant parameter $K_1$, chemical - thermal diffusion ratio parameter $\Delta$ and Schmidt number $Sc$.

The heat transfer from the plate is given by

$$q_w = -\frac{\lambda}{\theta_\gamma} \left[ \frac{\partial T}{\partial y} \right]_{y=0} = \lambda \frac{\theta_\tau}{\theta_\tau - \theta_\nu} \left[ \frac{\partial T}{\partial y} \right]_{y=0}$$

(15)

$$Nu = \frac{q_w}{\lambda c (T_w - T_0)} = \frac{1}{\sqrt{2}} \frac{\theta_\tau}{1 - \theta_\nu} Gr^{1/2} \theta' (0)$$

(16)

The mass flux at the wall is given by

$$M_w = -D \left[ \frac{\partial C}{\partial y} \right]_{y=0} = -\frac{\nu}{Sc} (C_w - C_\infty) \theta_\nu \frac{1}{\sqrt{3}} Gr^{1/2} \theta' (0)$$

(17)

$$S_h = \frac{Sc M_w h}{nu (C_w - C_\infty)} = \frac{1}{\sqrt{3}} Gr^{1/2} \frac{\theta_\nu}{1 - \theta_\nu} g'' (0)$$

(18)

RESULTS AND DISCUSSION:

The equations (9) — (13) together with the boundary conditions (14) are solved for various combination of the parameters involved in the equations using an algorithm based on the shooting method and presented results for the dimensionless velocity distribution, micro-rotation distribution, species concentration distribution, temperature distribution with the variation of different parameters.

Initially solution was taken for constant values of $\Delta=0.10, M=1.00, Pr=0.70, Sc=1.00, K_1 =0.10$ with the viscosity parameter $\theta_\nu$ ranging from -10.00 to -1.00 at certain value of $\theta_\tau = -10.00$. Similarly solutions have been found with varying the thermal conductivity parameter $\theta_\tau$ ranging from -10.00 to -1.00 at certain value of $\theta_\nu = -10.00$ keeping the other values remaining same. Solutions have also been found for different values of Magnetic parameter ($M$), Prandtl number ($Pr$), the coupling constant parameter ($K_1$), Chemical - thermal diffusion ratio parameter $\Delta$ and Schmidt number $Sc$.

The variations in velocity distribution, micro-rotation distribution, species concentration distribution and temperature distribution are illustrated in figures (1) — (14) with the variation of different parameters.
and velocity decreases. From figures (2) it is clear that velocity increases with the increasing values \( \frac{\partial}{\partial z} \) because when the temperature increases viscosity decreases and therefore velocity increases. The figures (3) — (5) represent the variations in dimensionless velocity distribution with the variation of \( M, K_1 \) and \( \Delta \). From figure (3) it is clear that velocity decreases with increase of magnetic parameter \( M \). It is because that the application of transverse magnetic field will result a resistive force (Lorentz force) similar to drag force, which tends to resist the fluid flow and thus reducing its velocity. From figure (4) it is clear that velocity decreases with increasing value of coupling constant parameter \( K_1 \). For small values of this parameter viscous force is pre-dominant and as result viscosity increases and therefore velocity decreases. As the viscosity increases with the increasing values of Schmidt number we have observed from figure (5) that velocity decreases with the increasing values of Schmidt number. It is due to the fact that increases in values of the \( Sc \) viscosity increases and therefore velocity decreases.

Figures (6) — (8) display the distribution representing micro-rotation within the boundary layer with the variation of \( \frac{\partial}{\partial x}, M \) and \( Sc \). From the figure (6) and (8) we have observed that the micro-rotation near the surface increases for increasing values of the parameter \( \frac{\partial}{\partial x} \) and \( Sc \) respectively and then decreases gradually. After \( \eta \approx 12 \) the micro-rotation again increases for increasing values of the parameter \( \frac{\partial}{\partial x} \) and \( Sc \) respectively. It is to be observed that at certain point the Parameters have no effect on the micro-rotation distribution. The effect of the Hartmann number \( M \) on micro-rotation is shown in the figure (7). The values of micro-rotation are negative in the first half whereas in the second half, these are positive, thus showing a reverse rotation near the boundary. An increase in magnetic field leads to a decrease in micro-rotation. Figure (9) displays the variations of micro-inertia density with variation of the Hartmann number \( M \). It is observed that micro-inertia density increases for increasing values of \( M \).

Figures (10) and (11) display the distribution representing concentration profile within the boundary layer with the variation of \( K_1 \) and \( Sc \). It is observed that concentration decreases with the increasing values of \( K_1 \) and \( Sc \). The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. With the increasing value of coupling constant parameter \( K_1 \) the concentration of the fluid decreases due to the decrease of viscosity. As \( Sc \) is the connecting link between velocity and concentration profiles, therefore with the increasing value of \( Sc \) molecular mass diffusivity decreases and as a result concentration decreases.

Figures (12) — (14) display the variations of dimensionless temperature profile \( \frac{\partial}{\partial \eta} \) with the variation of coupling constant parameter \( K_1 \), Prandtl number \( Pr \) and dimensionless reference temperature corresponding to thermal conductivity parameter \( \frac{\partial}{\partial \tau} \). From figure (12) we have observed that temperature decreases with the increasing values of \( K_1 \). If \( K_1 \) is small the viscous forces will be predominant inside the boundary layer and hence temperature is decreasing with the increasing values of viscosity. It is observed from the figure (13) that temperature decreases with the increasing values of \( Pr \). It is due to the reason that with the increasing values of the Prandtl number the viscosity increases and as a result temperature decreases. From figure (14) we have observed that temperature decreases when \( \frac{\partial}{\partial \tau} \) increases. It is due to the fact that the kinematic viscosity of the fluid increases with the increase of \( \frac{\partial}{\partial \tau} \) and as a result temperature decreases.

CONCLUSION:

In this study, the effects of variable viscosity and thermal conductivity on the flow, heat and mass transfer of an incompressible micropolar fluid past an accelerated infinite vertical plate are examined. The results demonstrate clearly that the viscosity and thermal conductivity parameters along with the other parameters such as \( K_1, \Delta, Sc, M \) and \( Pr \) have significant effects on velocity, temperature, concentration and micro-rotation distributions within the boundary layer. Thus assumption on constant properties may cause a significant error in flow problem.
Finally, the effect of the above-mentioned parameters on the values of $f'(0)$, $g'(0)$, $h'(0)$, $\gamma'(0)$, $C_p$, $N_u$, and $S_k$ are shown in the tables (1) — (5). The behaviour of these parameters is self-evident from the tables and hence any further discussions about them seem to be redundant.

Fig. 1 Velocity profile with the variation of $\theta_r$.

Fig. 2 Velocity profile with the variation of $\theta_c$.

Fig. 3 Velocity profile with the variation of Magnetic parameter $M$. 

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Fig. 4 Velocity profile with the variation of coupling constant parameter $K_1$

Fig. 5 Velocity profile with the variation of $\Delta$

Fig. 6 Velocity profile with the variation of $Pr$
Fig. 6 Micro-rotation profile with the variation of $\theta_r$.

Fig. 7 Micro-rotation profile with the variation of $\theta_c$.

Fig. 8 Micro-rotation profile with the variation of Magnetic parameter $M$. 
Fig. 9 Micro-rotation profile with the variation of $K_1$

Fig. 10 Micro-rotation profile with the variation of $\Delta$

Fig. 11 Micro-rotation profile with the variation of Schmidt number
Fig. 12 Micro-rotation profile with the variation of Prandtl number

Fig. 13 Micro-inertia density profile with the variation of Magnetic parameter

Fig. 15 Concentration profile with the variation of $\Delta$
Fig. 16: Concentration profile with the variation of Schmidt number.

Fig. 17: Concentration profile with the variation of $\theta_c$.

Fig. 18: Temperature profile with the variation of $M$. 
Fig. 19 Temperature profile with the variation of Pr

Fig. 20 Temperature profile with the variation of $\theta_r$

Fig. 21 Temperature profile with the variation of $\Delta$
Numerical values of $f'(0), h'(0), g'(0), \theta'(0), c_f, N_u, S_h$:

### Table 1:

| Pr=0.7, Re=0.5, Gr=0.1, Sc=1, $\Delta = -10$, $\theta_y = -10$, K1=0.1 |
|---|---|---|---|---|---|---|---|
| M & $\Delta$ & f'(0) & g'(0) & h'(0) & $\theta'(0)$ & $C_f$ & $N_u$ & $S_h$ |
| 0.1 & 0.1 & 0.685411 & -1.33059 & 0.010906 & -0.52344 & 3.478774 & 0.189218 & 0.480992 |
| & 1 & 0.944901 & -1.4431 & 0.012057 & -0.55626 & 4.795805 & 0.200849 & 0.521662 |
| & 2.2 & 1.257048 & -1.55865 & 0.01338 & -0.58924 & 6.380097 & 0.213003 & 0.563432 |
| & 4 & 1.681497 & -1.69269 & 0.01518 & -0.62895 & 8.534367 & 0.227358 & 0.611884 |
| 0.3 & 0.1 & 0.661438 & -1.30667 & 0.010158 & -0.51112 & 3.357100 & 0.184762 & 0.472345 |
| & 1 & 0.920485 & -1.42179 & 0.011299 & -0.54397 & 4.671881 & 0.196639 & 0.513957 |
| & 2.2 & 1.231844 & -1.53936 & 0.012614 & -0.57815 & 6.252175 & 0.208994 & 0.556456 |
| & 4 & 1.655113 & -1.67521 & 0.014326 & -0.61841 & 8.400456 & 0.223547 & 0.605565 |
| 0.6 & 0.1 & 0.629299 & -1.27327 & 0.009155 & -0.49398 & 3.193982 & 0.178566 & 0.460269 |
| & 1 & 0.887343 & -1.3919 & 0.010276 & -0.5277 & 4.503671 & 0.190757 & 0.503154 |
| & 2.2 & 1.19725 & -1.51218 & 0.011572 & -0.56259 & 6.076591 & 0.203367 & 0.546632 |
| & 4 & 1.6185 & -1.65046 & 0.013266 & -0.60353 & 8.214631 & 0.218168 & 0.596619 |

### Table 2:

| Pr=0.7, Re=0.5, Gr=0.1, Sc=1, $\Delta = 0.10$, $\theta_y = -10$, K1=0.1 |
|---|---|---|---|---|---|---|---|
| M & $\theta_y$ & f'(0) & g'(0) & h'(0) & $\theta'(0)$ & $C_f$ & $N_u$ & $S_h$ |
| 0.1 & -10 & 0.685411 & -1.33059 & 0.010906 & -0.52344 & 3.478774 & 0.189218 & 0.480992 |
| & -6 & 0.70925 & -1.4472 & 0.010999 & -0.52652 & 3.414452 & 0.190329 & 0.493248 |
| & -2 & 0.817597 & -1.95385 & 0.011397 & -0.53963 & 3.152761 & 0.19507 & 0.517948 |
| 0.3 & -10 & 0.661438 & -1.30667 & 0.010158 & -0.51112 & 3.357100 & 0.184762 & 0.472345 |
| & -6 & 0.684244 & -1.42071 & 0.010243 & -0.51405 & 3.29407 & 0.185821 & 0.484219 |
| & -2 & 0.787854 & -1.91583 & 0.010606 & -0.52656 & 3.038066 & 0.190345 & 0.507869 |
| 0.6 & -10 & 0.629299 & -1.27327 & 0.009155 & -0.49398 & 3.193982 & 0.178566 & 0.460269 |
| & -6 & 0.650724 & -1.38372 & 0.009229 & -0.49671 & 3.132697 & 0.179552 & 0.471613 |
| & -2 & 0.747986 & -1.86276 & 0.009546 & -0.50836 & 2.88433 & 0.183765 & 0.493799 |

### Table 3:

| Pr=0.7, Re=0.5, Gr=0.1, Sc=1, $\Delta = 0.10$, $\theta_y = -10$, K1=0.1 |
|---|---|---|---|---|---|---|---|
| M & $\theta_y$ & f'(0) & g'(0) & h'(0) & $\theta'(0)$ & $C_f$ & $N_u$ & $S_h$ |
| 0.1 & -10 & 0.685411 & -1.33059 & 0.010906 & -0.52344 & 3.478774 & 0.189218 & 0.480992 |
| & -6 & 0.680594 & -1.32919 & 0.010738 & -0.54119 & 3.45325 & 0.184455 & 0.480845 |
| & -2 & 0.65968 & -1.32287 & 0.010022 & -0.62208 & 3.348179 & 0.164908 & 0.4782 |
| 0.3 & -10 & 0.673148 & -1.31846 & 0.010523 & -0.51719 & 3.416535 & 0.186956 & 0.476606 |
| & -6 & 0.656846 & -1.30523 & 0.009991 & -0.52842 & 3.333797 & 0.180103 & 0.471822 |
| & -2 & 0.636904 & -1.2987 & 0.009281 & -0.60729 & 3.232582 & 0.160987 & 0.469461 |
| 0.6 & -10 & 0.629299 & -1.27327 & 0.009155 & -0.49398 & 3.193982 & 0.178566 & 0.460269 |
| & -6 & 0.625039 & -1.27179 & 0.008992 & -0.51069 & 3.172357 & 0.174058 & 0.459734 |
| & -2 & 0.606519 & -1.2651 & 0.008296 & -0.58685 & 3.07836 & 0.155567 & 0.457318 |
Table 4:

Re=0.5, Gr=0.1, Sc=1, Δ = 0.10, θr = −10, θr = −10, K1=0.1

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Table 5:

Re=0.5, Gr=0.1, Pr=0.7, Δ = 0.10, θr = −10, θr = −10, K1=0.1

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Table 6:

Re=0.5, Gr=0.1, Pr=0.7, Δ = 0.10, θr = −10, M = 0.1, K1=0.1

<table>
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<th>θr</th>
<th>S_r</th>
<th>f'(0)</th>
<th>g'(0)</th>
<th>h'(0)</th>
<th>θ'(0)</th>
<th>C_f</th>
<th>N_u</th>
<th>S_p</th>
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</table>
Nomenclatures:

\( \beta \) = Volumetric coefficient of thermal expansion

\( \beta^* \) = Volumetric co-efficient of expansion with concentration

\( \lambda \) = Gravitational acceleration

\( \lambda_c \) = Thermal conductivity

\( \mu \) = Dynamic viscosity

\( \mu_\infty \) = Dynamic viscosity of the ambient fluid

\( \nu \) = Kinematic viscosity

\( \nu_\infty \) = Kinematic Viscosity of the ambient fluid

\( \kappa \) = Vortex viscosity

\( \gamma \) = Spin-gradient or micro rotation viscosity

\( c_p \) = Specific heat

\( \eta \) = Dimensionless co-ordinates

\( u \) = Velocity in the \( x \) – direction

\( v \) = Velocity in the \( y \) – direction

\( f \) = Dimensionless velocity

\( h \) = Dimensionless micro-rotation

\( \theta \) = Dimensionless temperature

\( \theta_c \) = Dimensionless reference temperature corresponding to viscosity parameter

\( \theta_v \) = Dimensionless reference temperature corresponding to thermal conductivity parameter

\( C_f \) = Skin-friction coefficient

\( T \) = Temperature

\( C \) = Species concentration

\( T_\infty \) = Ambient temperature

\( T_w \) = Wall temperature

\( C_w \) = Species concentration at the wall

\( C_\infty \) = Species concentration far from the wall

\( J \) = Micro rotation density

\( \sigma \) = Electrical conductivity

\( \rho \) = Density

\( D \) = Chemical molecular diffusivity

\( G_r \) = Local Grashoff number for heat transfer

\( S_c \) = Schmidt number

\( P_r = \frac{\nu \rho_\infty \beta^*}{\lambda_\infty} \) = Prandtl number

\( M = \frac{\sigma B_e^2}{\rho \gamma_\infty c^2} \) = Hartmann Number

\( K_c = \frac{k}{\rho \gamma_\infty} \) = coupling constant parameter

\( \Delta \) = Diffusion parameter

\( Nu \) = Nusselt number

\( S_h \) = Sherwood number
**Subscripts:**

\( w \), the condition at the wall

\( \infty \), the condition far away from the surface

**Superscripts:**

\( \gamma \), Differentiation with respect to \( \eta \)

**REFERENCES:**


**Author’s Brief Biography:**

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