Math 2496 - Calc I

Consider

$$\frac{d}{dx}\sqrt{x^2+1} \qquad \text{(a composite fct)}$$

so if $u = x^2+1$ then

$$\frac{d}{dx} u^{1/2} = \frac{d}{du} u^{1/2} \cdot \frac{du}{dx} \qquad \text{Chain Rule}$$

Does there exist a type of "chain Rule" for integrals.

Consider

$$\int 2x(x^2+1)dx \qquad \text{let's expand}$$

$$\int (2x^3+2x)dx = \frac{2x^4}{4} + \frac{2x^2}{2} + c$$

$$= \frac{x^4}{2} + x^2 + c$$

However if $\int 2x(x^2+1)^5 dx$ expanding & $\int$ would be labor. intensive

Note $= \frac{x^4}{2} + x^2 + c$ c is arb so absorb $\frac{1}{2}$

$= \frac{1}{2}(x^4+2x^2+1) + c - \frac{1}{2}$

$= \frac{1}{2}(x^2+1)^2 + c$

if $\int x\,dx = \frac{x^2}{2} + c$

$\int u\,du = \frac{u^2}{2} + c$ compare

maybe if we let $u = x^2+1$ we could get the answer easier. This method is called the "u-substitution" method

again $\int 2x(x^2+1)\,dx$

$u = x^2+1$ to change this in terms of u only we will need a du

This is were differentials come in

if $u = x^2+1$

$du = 2x\,dx$

$$\int 2x(x^2+1)\,dx = \int \underbrace{(x^2+1)}_{u}\underbrace{2x\,dx}_{du}$$

$$= \int u\,du = \frac{u^2}{2} + c = \frac{1}{2}(x^2+1)^2 + c$$

The book says if

$$\int f(g(x))\cdot g'(x)\,dx = \int f(u)\,du + c$$

if $u = g(x)$

we pretty much will do by example.

Ex $\displaystyle\int e^{-x}dx = \int -e^{u}du = -e^{u} + c$ ↲ back sub

$= -e^{-x} + c$

let $u = -x$

$du = -dx$

Ex $\int \sin^2 x \cos x \, dx$

↑ hard part

let $u = \sin x$ $\quad du = \cos x \, dx$

then $\int u^2 du = \frac{1}{3} u^3 + C = \frac{1}{3} \sin^3 x + C$

Ex $\int \frac{x^2}{\sqrt{x^3+1}} dx$

let $u = x^3+1$ $\quad du = 3x^2 dx$ $\quad x^2 dx = \frac{1}{3} du$

$$\int \frac{\frac{1}{3} du}{\sqrt{u}} = \frac{1}{3} \int u^{-1/2} du = \frac{1}{3} \frac{u^{1/2}}{1/2} + C$$

$$= \frac{2}{3} \sqrt{x^3+1} + C$$

Ex $\int \sqrt{2x+1} \, dx = \int u^{1/2} \frac{du}{2} = \frac{u^{3/2}}{2 \cdot 3/2} + C$

let $u = 2x+1$

$du = 2dx$ $\quad = \frac{1}{3} (2x+1)^{3/2} + C$

Ex $\int x(x+1)^4\,dx$ (don't expand)

let $u = x+1$, so $du = dx$

$\int x\,u^4\,du$

what about this — back to sub.

$x = u-1$

so $\int (u-1)\,u^4\,du$ ← is this any easier - Yes!

$\int u^5 - u^4\,du = \frac{u^6}{6} - \frac{u^5}{5} + c$

$= \frac{1}{6}(x+1)^6 - \frac{1}{5}(x+1)^5 + c$

The question of how to deal with definite integrals is considered next lecture.