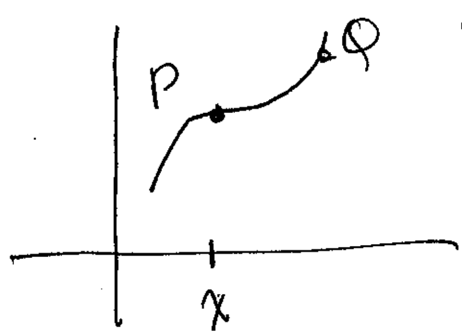
Math 1496 - Calc 1

3.1 The Derivative & the tangent Problem

we considered this back week 2



consider a pt $P(x, f(x))$ on the curve $y = f(x)$ then move a bit ~~to~~ $x+h$ so we have

a new pt $Q(x+h, f(x+h))$

The slope of this line is

$$\frac{\Delta y}{\Delta x} = \frac{f(x+h) - f(x)}{x+h-x} = \frac{f(x+h)-f(x)}{h}$$

Now we take the limit as $h \to 0$

so $$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = m$$

this gives the slope of the tangent line

we now give this a name -

Defⁿ - Derivative

the derivative of f(x) at x is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

other names $\frac{df}{dx}$, y', $\frac{dy}{dx}$ ($D_x[y]$ I never use this)

ex $f(x) = 3x+2$

Use the defⁿ to find f'

$$f'(x) = \lim_{h \to 0} \frac{3(x+h) + 2 - (3x+2)}{h}$$ expand

$$= \lim_{h \to 0} \frac{\cancel{3x} + 3h + \cancel{2} - \cancel{3x} - \cancel{2}}{h}$$

$$= \lim_{h \to 0} \frac{3\cancel{h}}{\cancel{h}} = 3$$

Ex $f(x) = -x^2+2x$

$$f'(x) = \lim_{h\to 0} \frac{-(x+h)^2+2(x+h)-(-x^2+2x)}{h}$$

$$= \lim_{h\to 0} \frac{-\cancel{x^2}-2xh-h^2+\cancel{2x}+2h+\cancel{x^2}-\cancel{2x}}{h}$$ cancel

$$= \lim_{h\to 0} \frac{\cancel{h}(-2x-h+2)}{\cancel{h}}$$ cancel

$$= \lim_{h\to 0} -2x-h+2 = -2x+2$$

Note: alway keep lim until you do the limit

Ex $f(x) = \frac{1}{x}$

$$f'(x) = \lim_{h\to 0} \frac{\frac{1}{x+h}-\frac{1}{x}}{h} = \lim_{h\to 0} \frac{\frac{x}{x(x+h)}-\frac{x+h}{x(x+h)}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x}-\cancel{x}-h}{hx(x+h)} = \lim_{h\to 0} \frac{-\cancel{h}}{\cancel{h}(x)(x+h)}$$

$$= \lim_{h \to 0} \frac{-1}{x(x+h)} = -\frac{1}{x^2}$$

Ex $f(x) = \sqrt{x}$

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$ rationalize

$$= \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \lim_{h \to 0} \frac{\cancel{x}+\cancel{h} - \cancel{x}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

Note

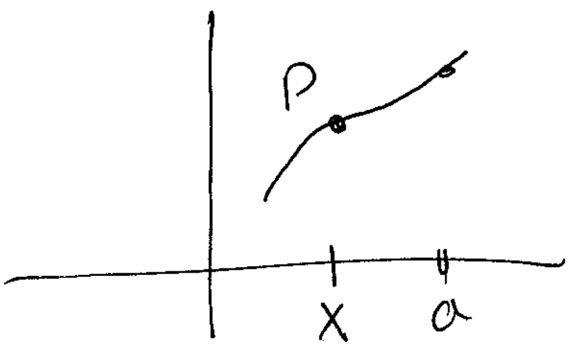
$f(x) = x$ $\quad f' = 1$

$f(x) = x^2$ $\quad f' = 2x$

$f(x) = x^{1/2}$ $\quad f' = \frac{1}{2}x^{-1/2}$

is there a pattern here?

Derivative at a point



consider 2 pts on f(x)

P (x, f(x))

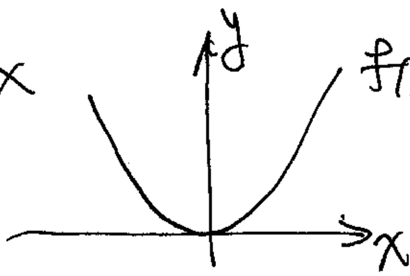
Q (a, f(a))

one at x and the other at a

slope $\lim_{x \to a} \frac{f(x) - f(a)}{x - a} = f'(a)$ ← called derivative at a pt $x = a$

$x = a$

Ex



$f(x) = x^2$ find $f'(2)$

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$$

$$= \lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}} = 4$$

Note if $f(x) = x^2$ $f'(x) = 2x$

$f'(2) = 2 \cdot 2 = 4$ same

A function is said to be differentiable at $x = a$ if

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a} \text{ exists}$$

and f is said to be differentiable ~~at~~ on (a, b) if the derivative exists at each pt in (a, b)

Ex $\quad f(x) = \sqrt{x} \qquad f'(x) = \dfrac{1}{2\sqrt{x}}$

So f is cont^s on $[0, \infty)$

but only diff^{ble} on $(0, \infty)$

since $f'(x)$ DNE at $x = 0$