Math 4315/5315 - PDEs Home Work 5

Due: Wed. Oct. 27, 2021

1. Transform the following PDEs to standard form. In the case of hyperbolic, transform to only modified standard form.

$$
\begin{aligned}
(i) \quad & u_{xx} + 2u_{xy} + u_{yy} = 0, \\
(ii) \quad & y^2 u_{xx} + 2xyu_{xy} + x^2 u_{yy} - 2xu_x = 0, \\
(iii) \quad & 2u_{xx} - 3u_{xy} + u_{yy} = u_x + u_y, \\
(iv) \quad & x^2 u_{xx} - 3xyu_{xy} + 2y^2 u_{yy} = 0, \\
(v) \quad & 4u_{xx} + 4u_{xy} + 5u_{yy} = 1, \\
(iv) \quad & u_{xx} + (1+y^2)^2 u_{yy} = 0.
\end{aligned}
$$

Grad Students and Bonus for Undergrads
The PDE

$$x^2 u_{xx} - 4xyu_{xy} + 4y^2 u_{yy} + xu_x = 0$$

is parabolic. Introducing new coordinates

$$r = x^2 y, \quad s = y,$$

reduces the PDE to

$$u_{ss} - \frac{r}{s^2} u_r = 0.$$

In fact, any choice of

$$r = f\left(x^2 y\right), \quad s = g(x,y),$$

will transform the original PDE to one that is in parabolic standard form. Can the choice of f and g be made such that we can transform to

$$u_{ss} = u_r$$

or

$$u_{ss} = 0?$$