VOLARIS
CAPITAL MANAGEMENT

- *We provide investors access to yield-enhancement and hedging opportunities in the options market.*
- *Our insights based on experience and active research and our customized technology applications are applied to pursue client's goals.*

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Machine Learning Approach to Options

Vivek Kapoor

Chief Investment Officer

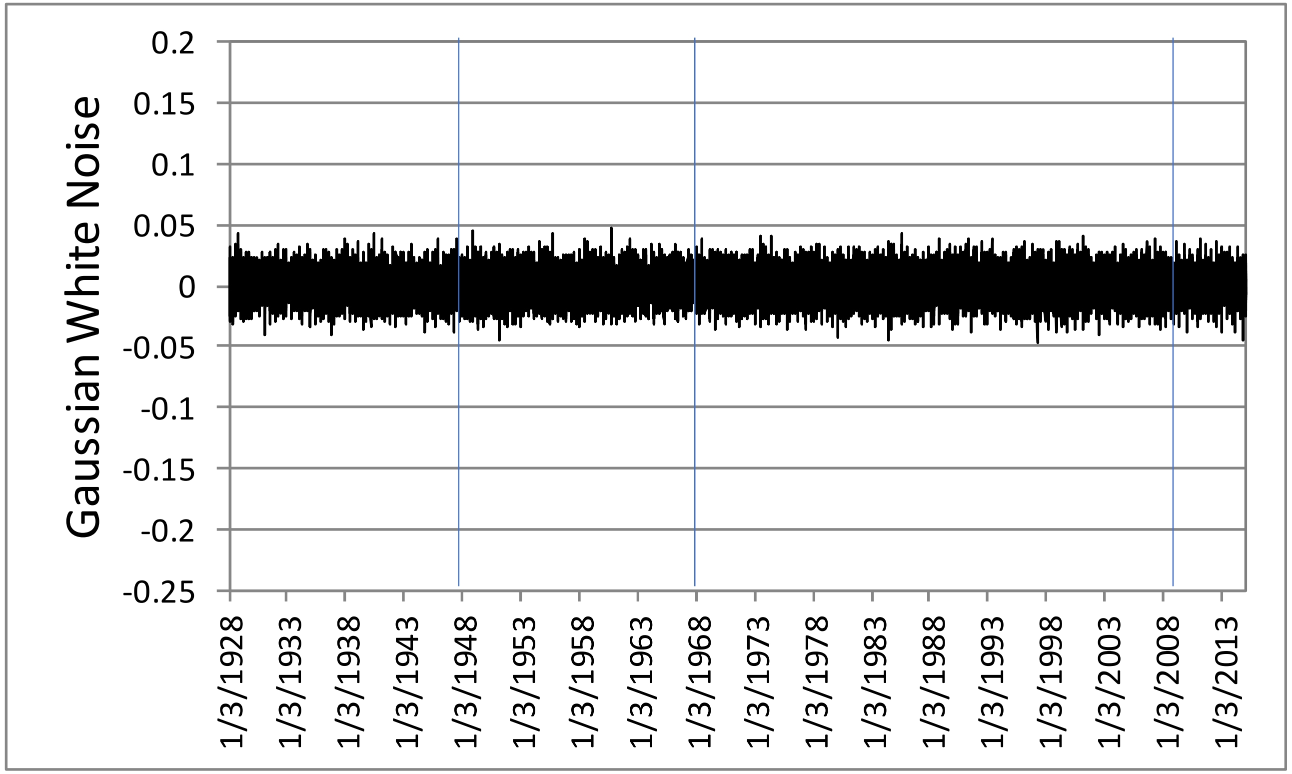
- How does my machine answer the question "Where am I?"
- Data driven description of Market Micro-Structure & its Time-Aggregation
- Non-Parametric Optimal Hedging Strategy
- Option Strike-Term Dependent Expected P&L and Residual-Risk Asymmetry
- Pursuit of Risk-Controlled-Yield and Carry-Controlled-Serendipity

Where am I?

Does it matter?

Are there temporal rhythms or rhymes embedded in the market returns?

Is White-Noise a plausible stochastic description of the market?



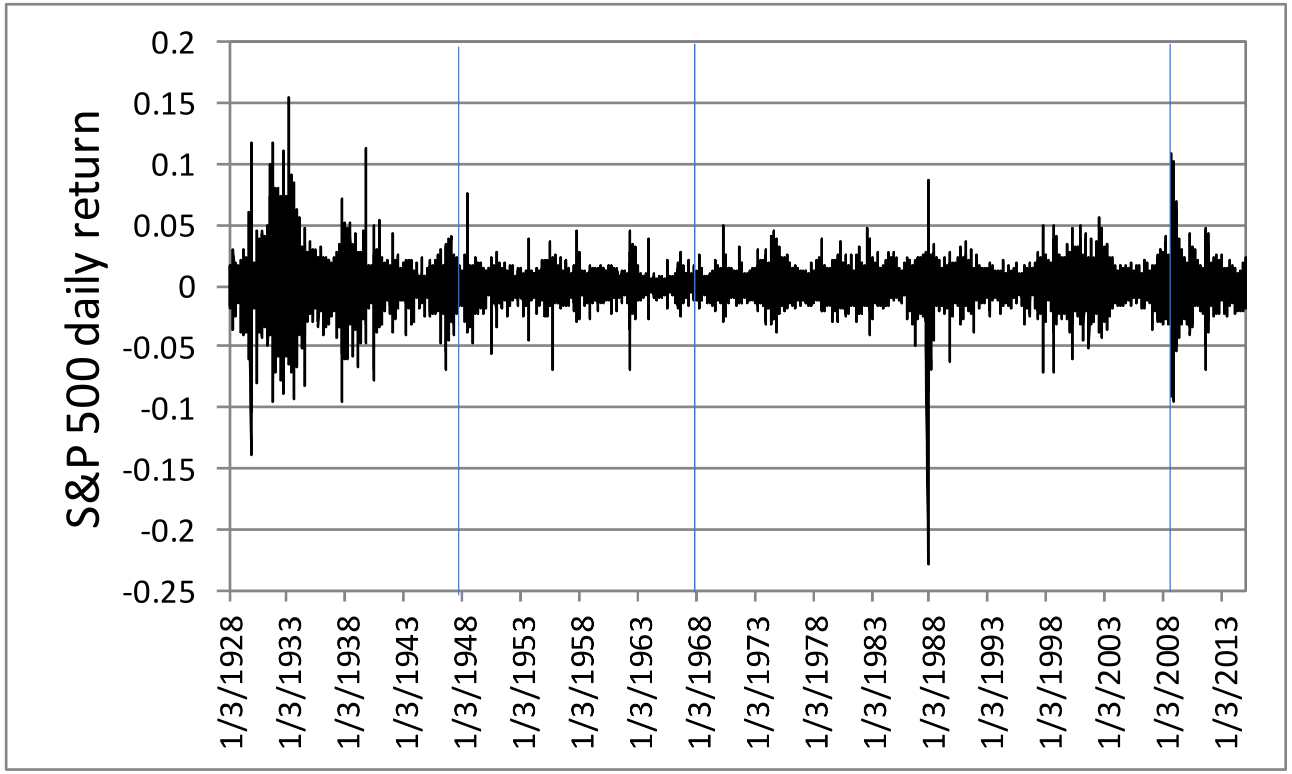
Gaussian-White-Noise Fit to Mean and Standard Deviation of S&P 500 Index Return

Where am I? Not Always in Kansas!

Does it matter? Yes

Are there temporal rhythms or rhymes in the market returns? Yes

Is White-Noise a plausible stochastic description of the market? No

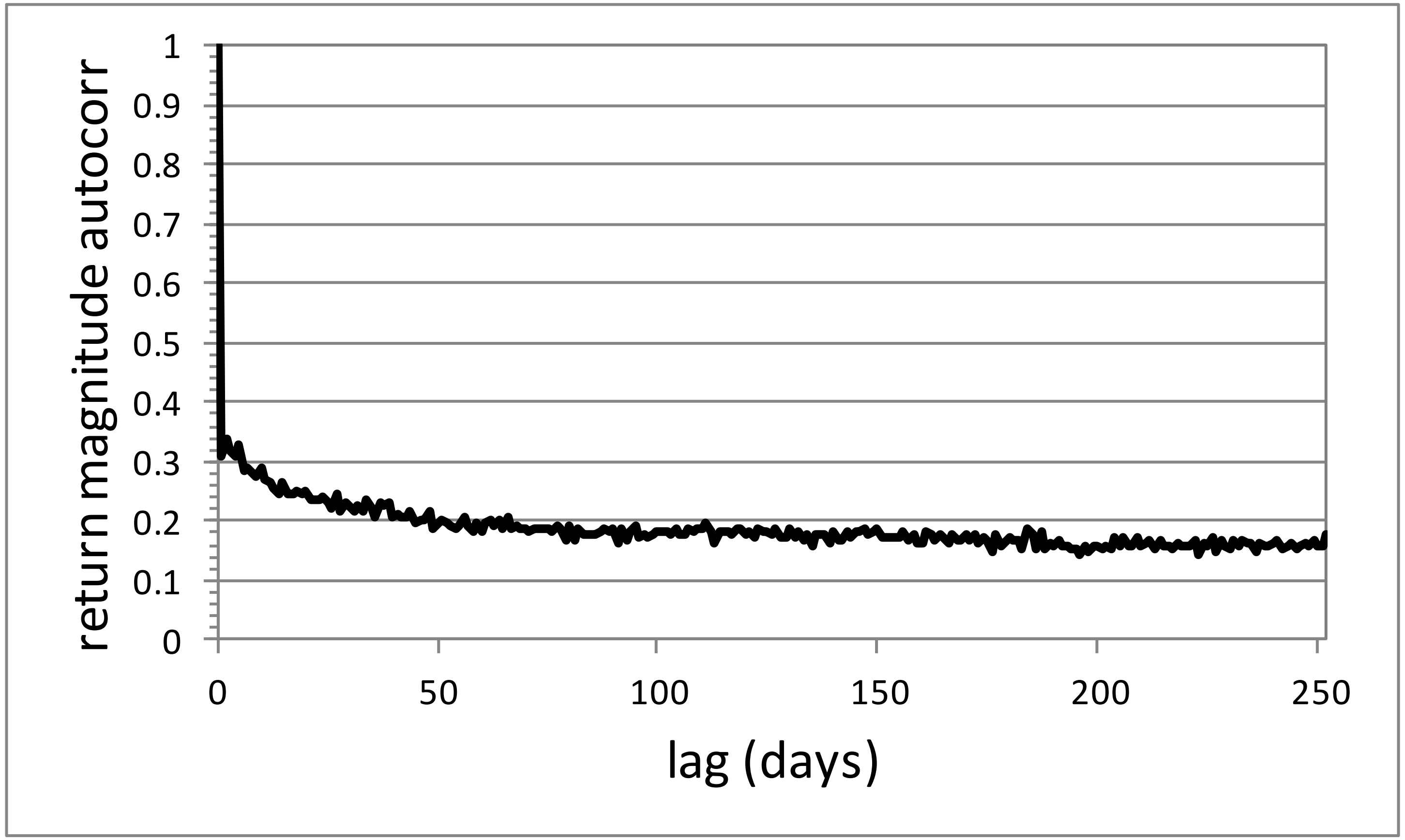


S&P 500 Index Daily Return

Market Rythms & Rhymes

S&P 500 Index return magnitude has significant temporal memory

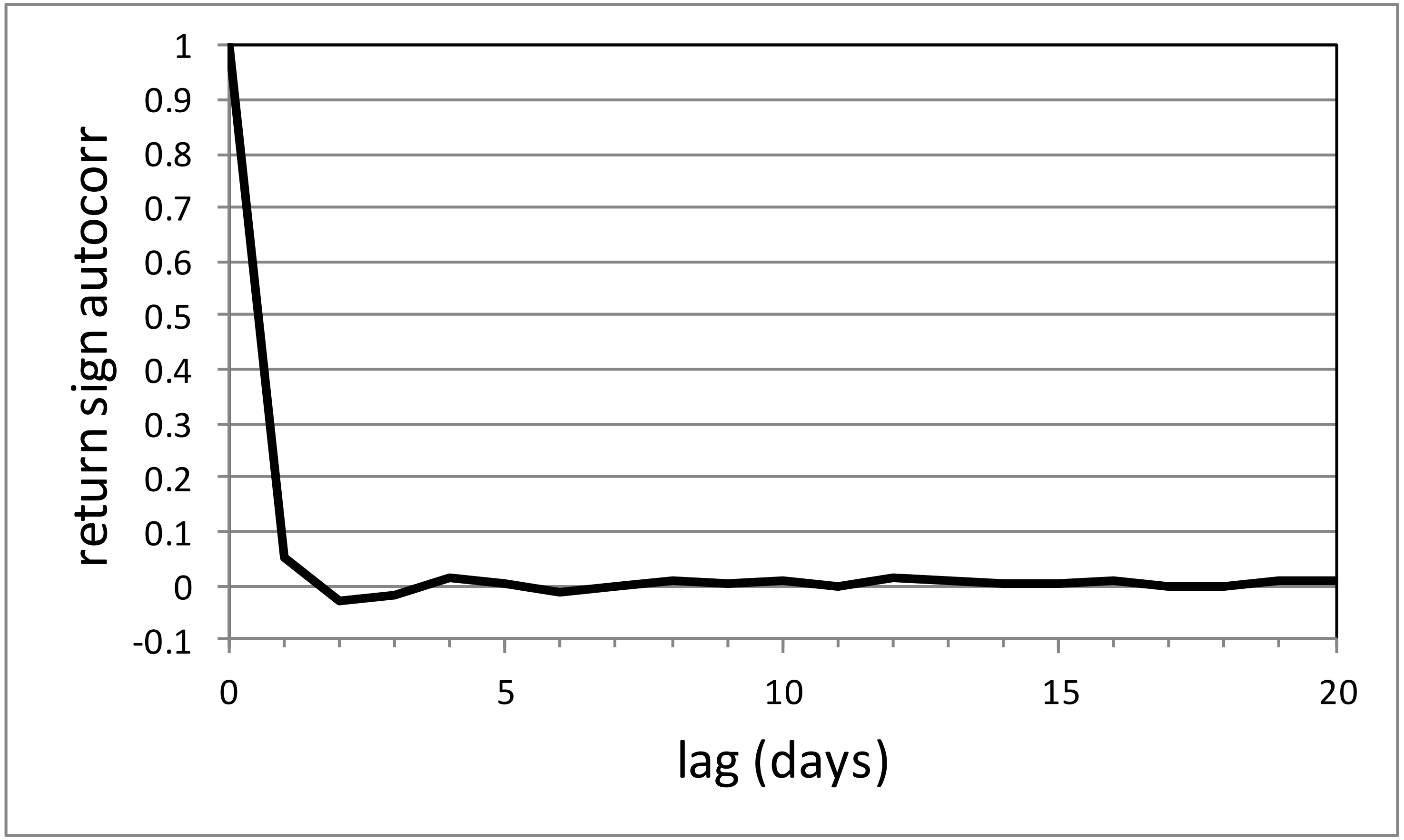
$$\rho_{|r|}(\tau)$$



Market Rythms & Rhymes

S&P 500 Index return sign exhibit much less temporal memory than its return magnitude

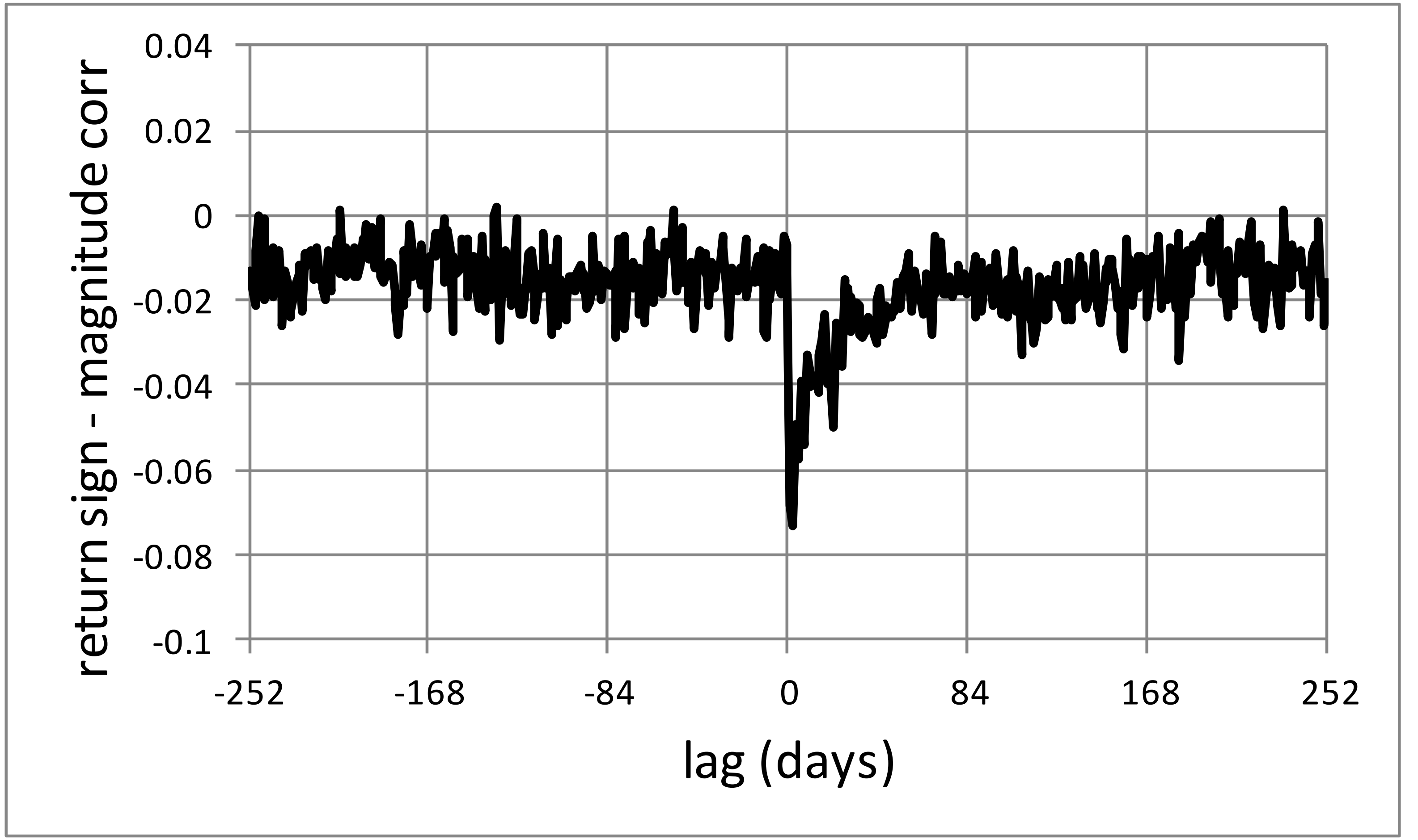
$$\rho_I\left(\tau\right)$$



Market Rythms & Rhymes

S&P 500 Index return sign leads the return magnitude

$$\rho_{I|r|}(\tau)$$



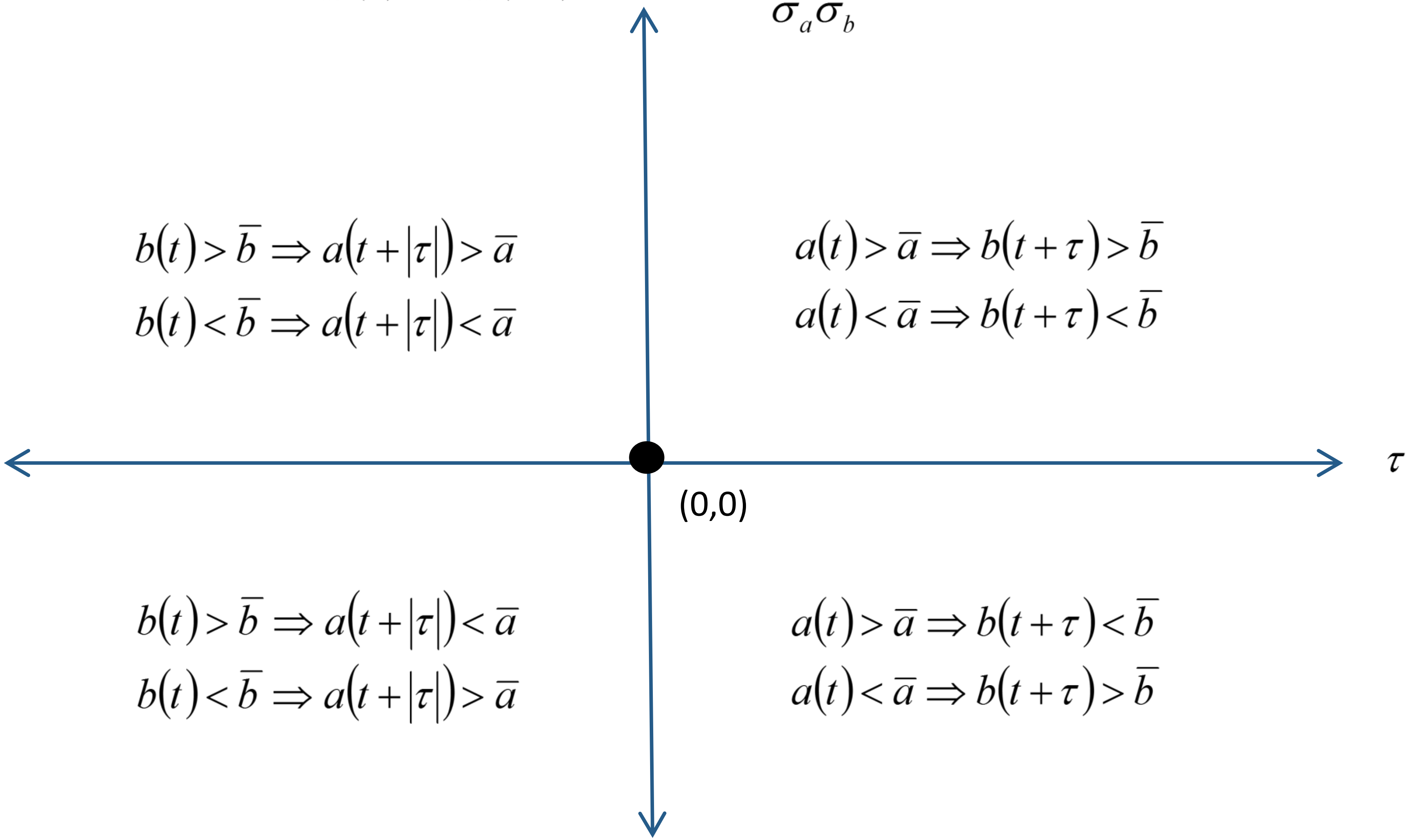
S&P 500 Index Downturns Presage High Volatility

Lead-Lag and Cross-Correlation Function at Different Lags

$$\rho_{ab}(\tau) = \rho_{ba}(-\tau) \equiv \frac{E\left[\left(a(t)-\bar{a}\right)\left(b(t+\tau)-\bar{b}\right)\right]}{\sigma_a \sigma_b}$$

$$b(t) > \bar{b} \Rightarrow a(t+|\tau|) > \bar{a}$$
$$b(t) < \bar{b} \Rightarrow a(t+|\tau|) < \bar{a}$$

$$a(t) > \bar{a} \Rightarrow b(t+\tau) > \bar{b}$$
$$a(t) < \bar{a} \Rightarrow b(t+\tau) < \bar{b}$$



$$b(t) > \bar{b} \Rightarrow a(t+|\tau|) < \bar{a}$$
$$b(t) < \bar{b} \Rightarrow a(t+|\tau|) > \bar{a}$$

$$a(t) > \bar{a} \Rightarrow b(t+\tau) < \bar{b}$$
$$a(t) < \bar{a} \Rightarrow b(t+\tau) > \bar{b}$$

Stochastic Market Model

Everything should be made as simple as possible, but not simpler.
Albert Einstein

Takes Two to Tango
Al Hoffman & Dick Manning

Sometimes it takes three to tango
Anonymous

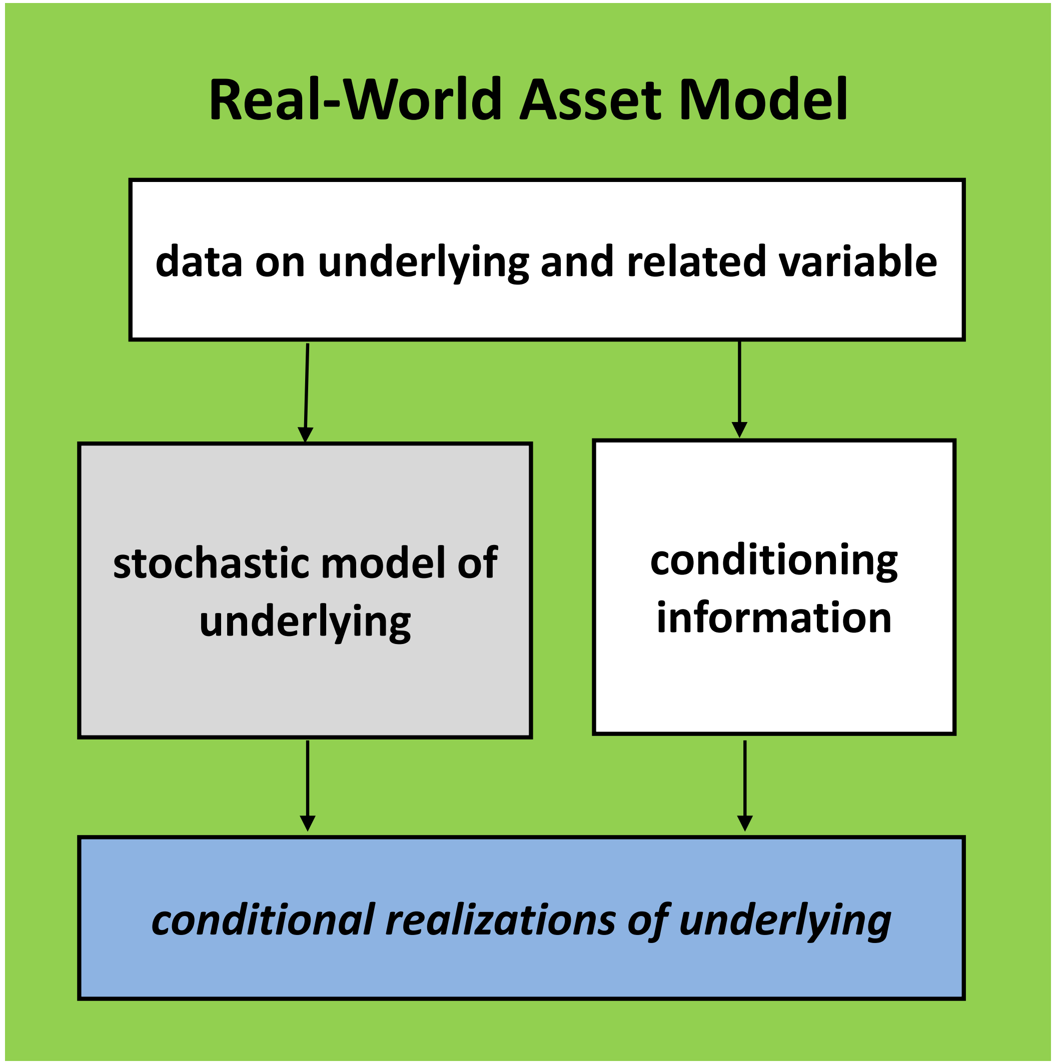
Economics ended up with the theory of rational expectations, which maintains that there is a single optimum view of the future, that which corresponds to it, and eventually all the market participants will converge around that view. This postulate is absurd, but it is needed in order to allow economic theory to model itself on Newtonian Physics.
George Soros

Goals

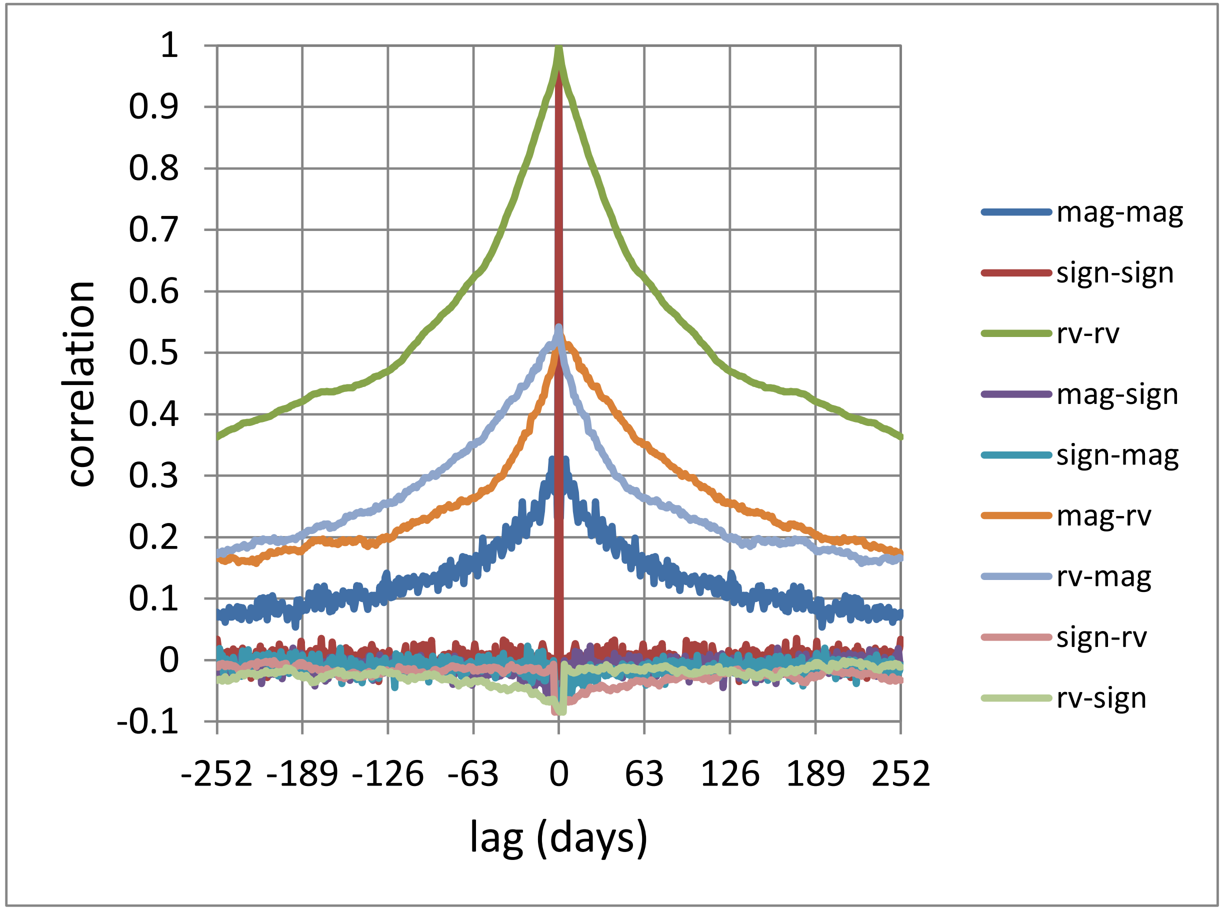
- *incorporate data-driven lead-lag relation among market direction, return magnitude, & a related variable*
- *impart realistic term structure of return skewness and kurtosis*
- *make available description of ensemble of outcomes consistent with long term and recent market behavior*

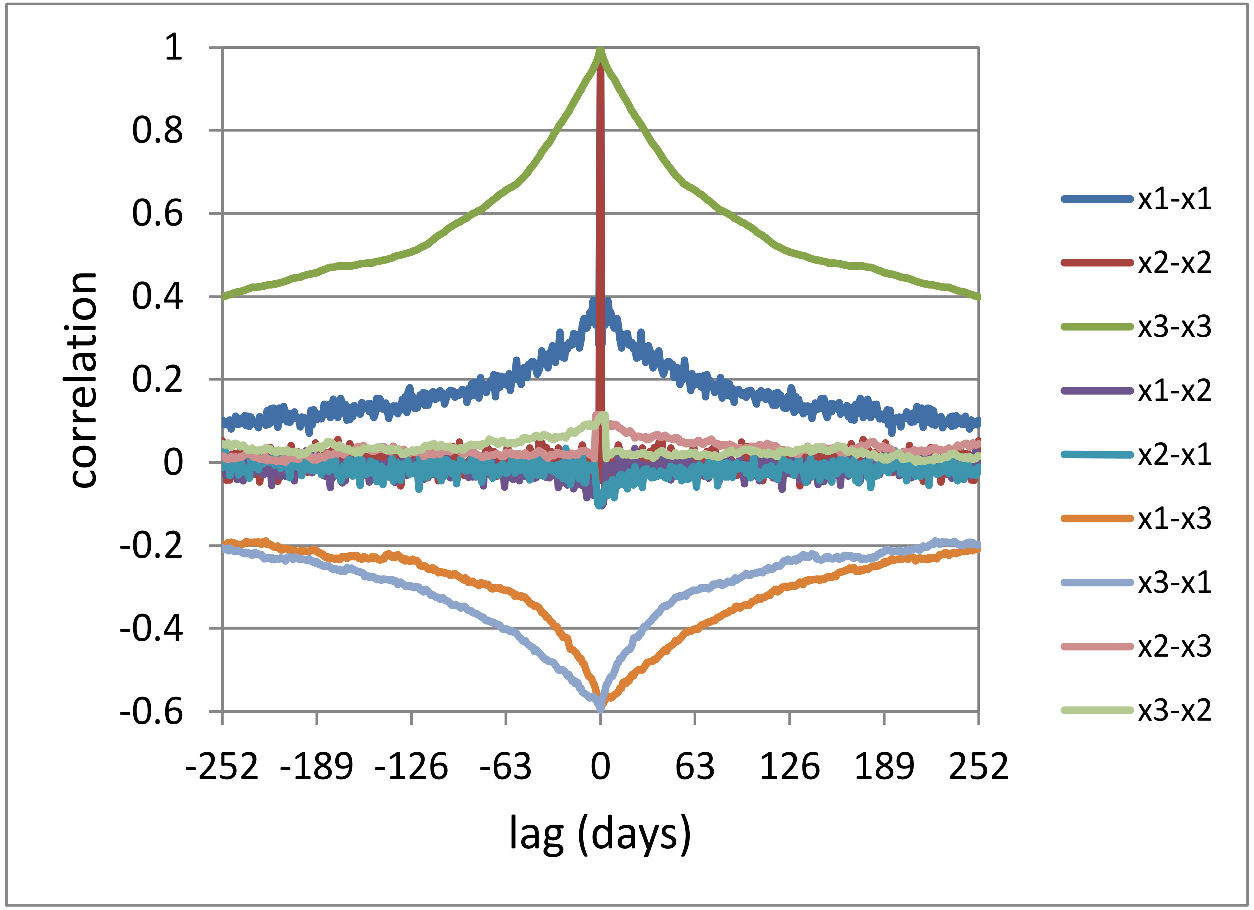
Stochastic Market Model

- Model return magnitude and *related variable* as *functions* of autoregressive processes – classical jointly Gaussian with explicitly specifiable covariance structure
  - Base period Non-Gaussian nature captured by empirically defined *functions*
- *Autocorrelation of return magnitude process controls the modeled term structure of return kurtosis*
- The return sign is modeled employing a threshold on an autoregressive stochastic process
- *The cross-correlation of return magnitude and return sign process controls the term structure of return skewness*



Stochastic Market Model

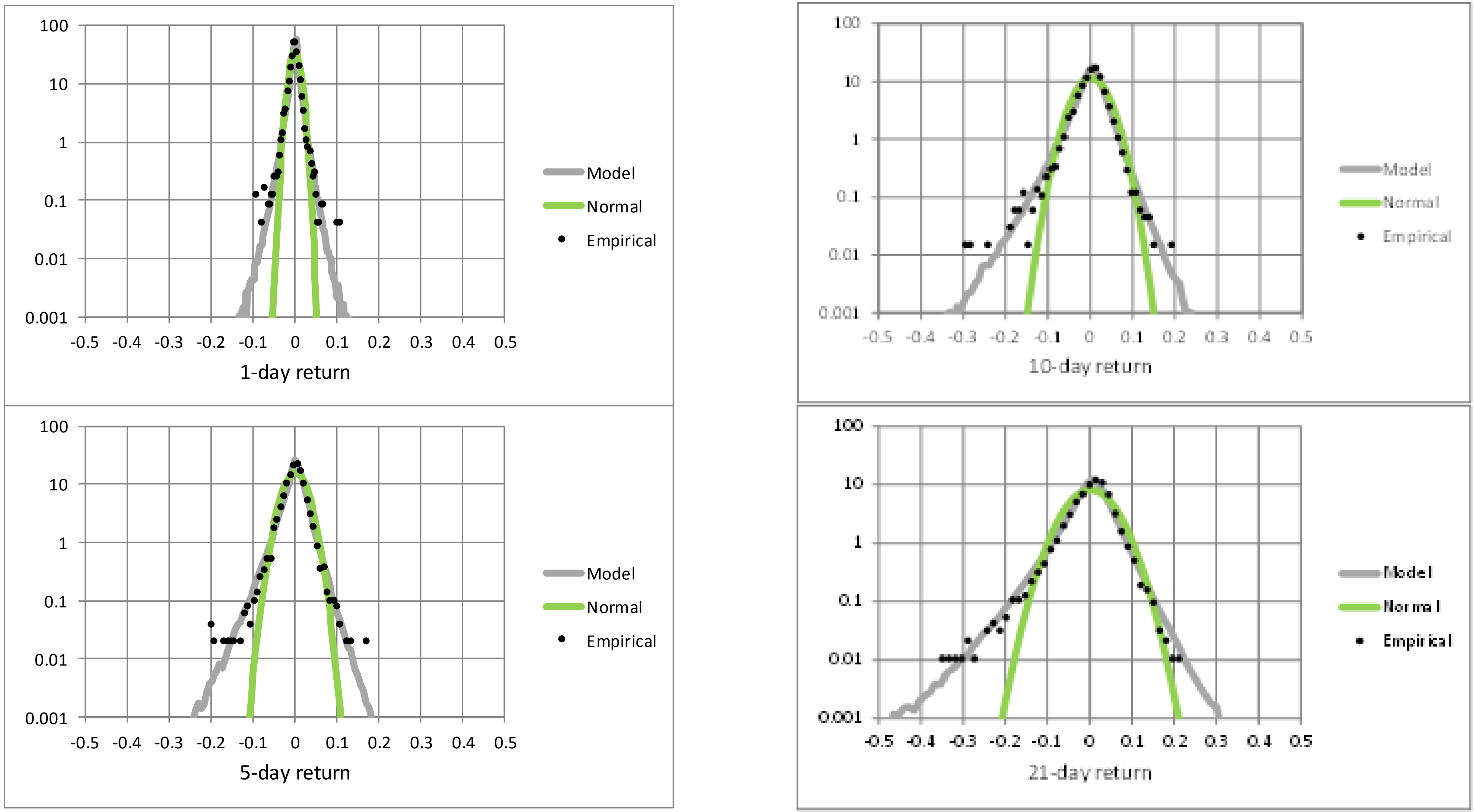




Direct Observables Lead-Lag Structure

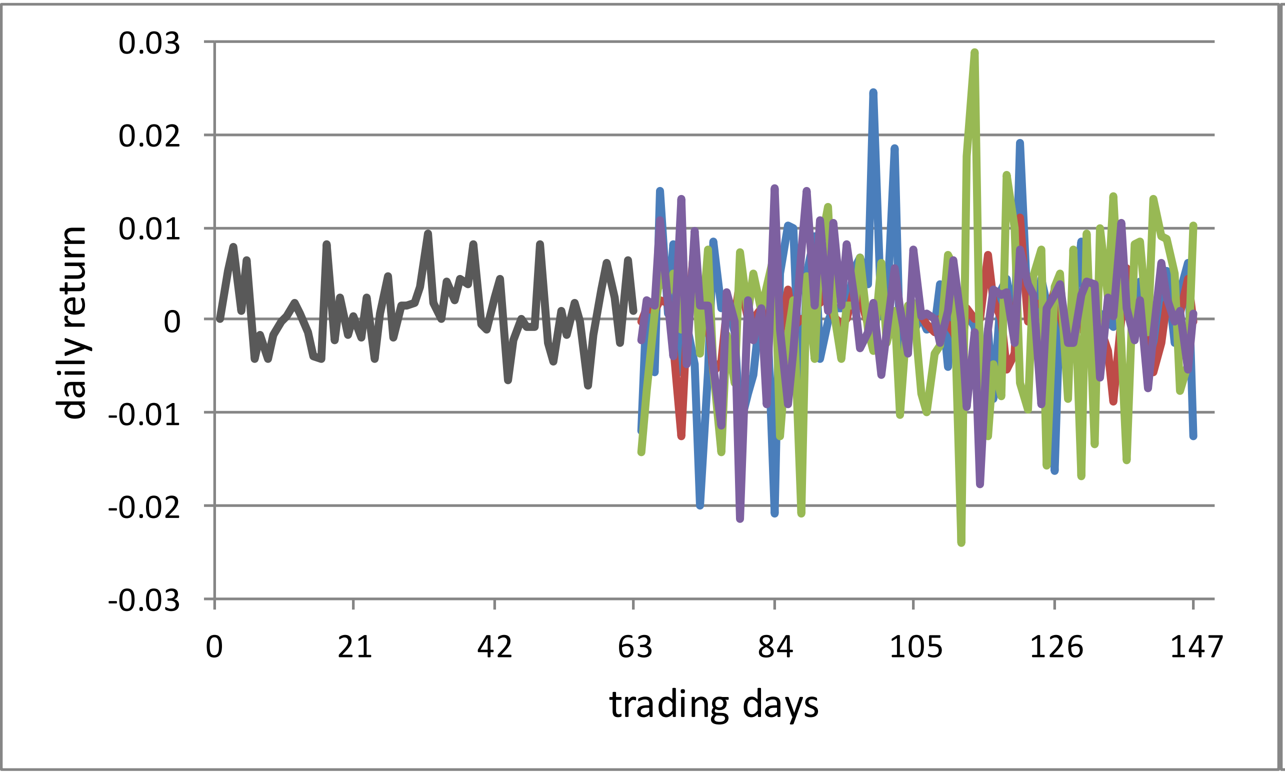
Transformed Process Lead-Lag Structure

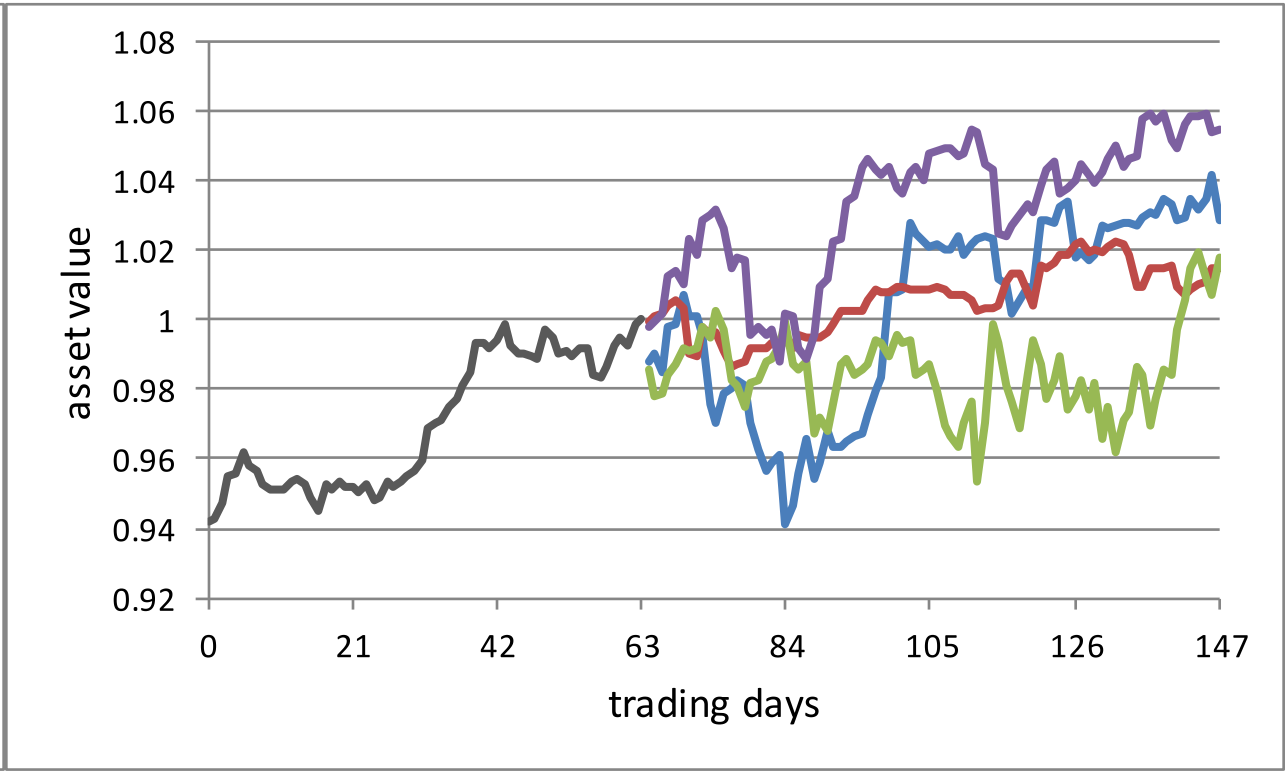
Stochastic Market Model: Unconditional Results



S&P 500 Return Distribution Term Structure

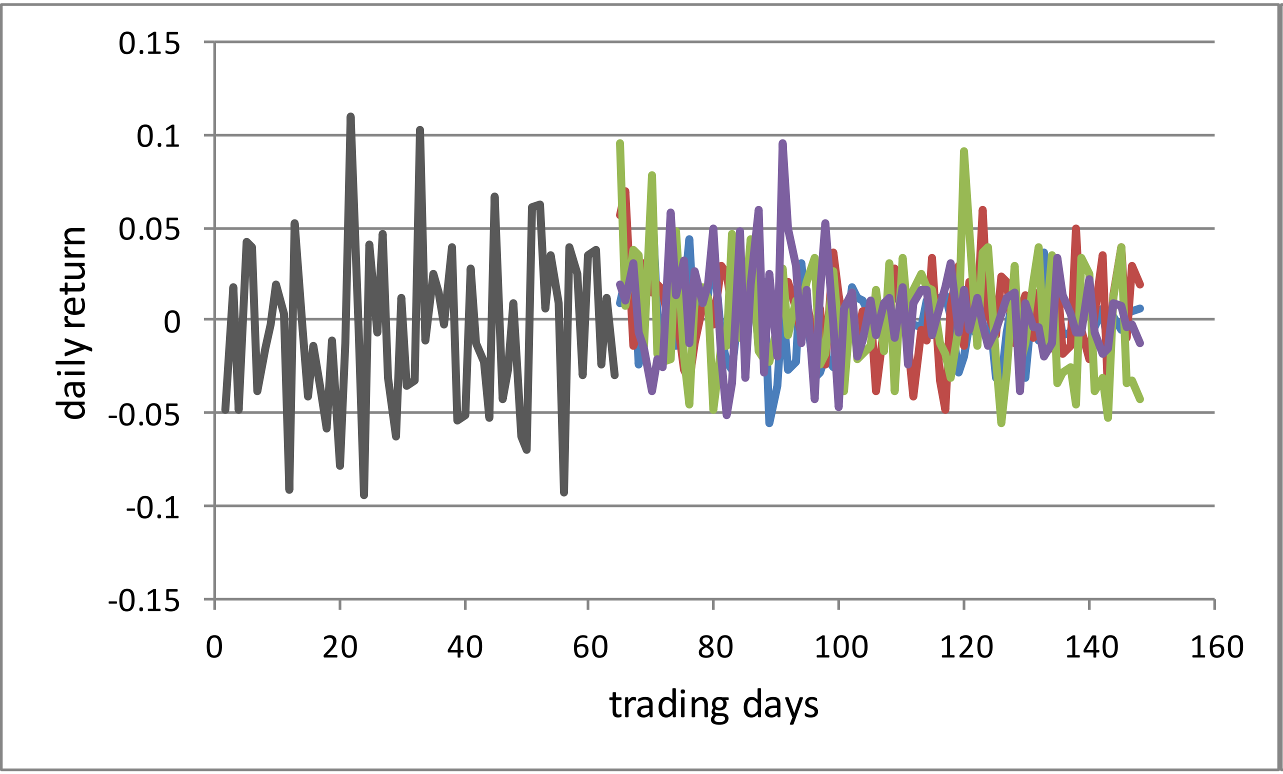
Stochastic Market Model: Conditional Results

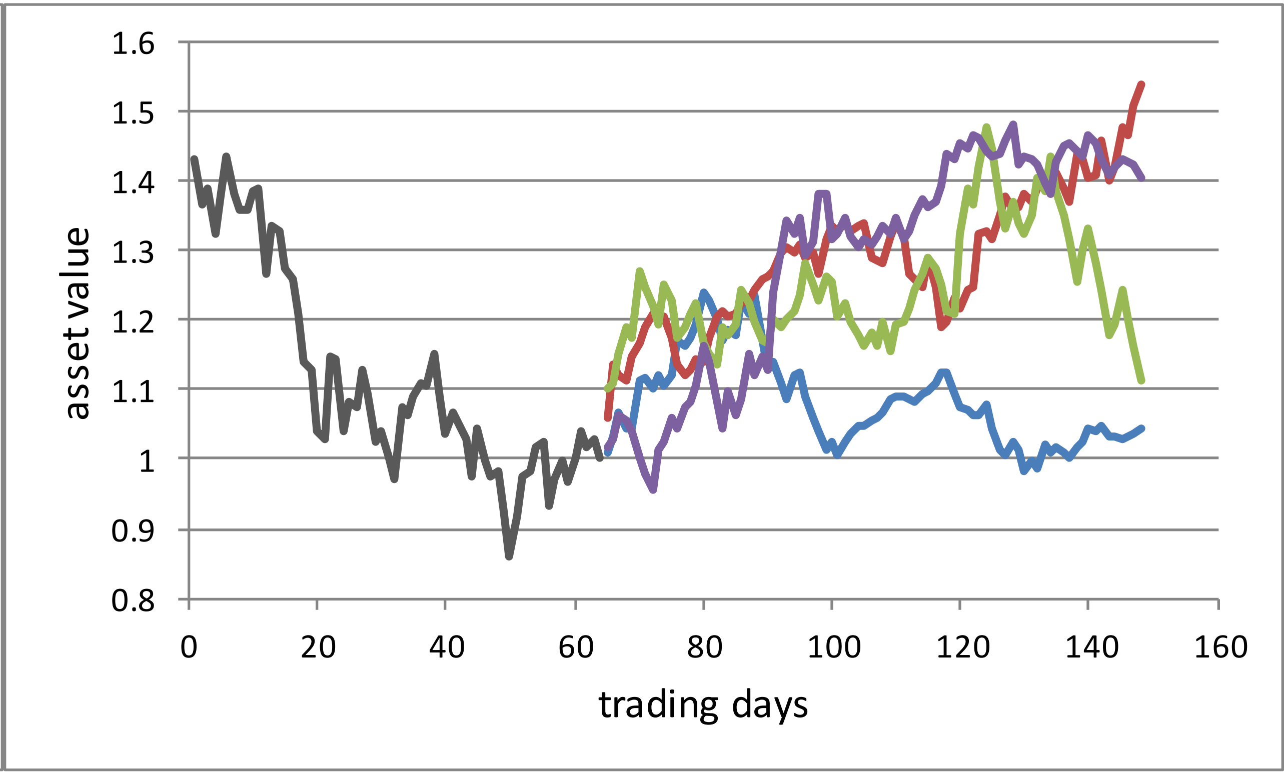




S&P 500 conditional realizations with low volatility conditioning information

Stochastic Market Model: Conditional Results





S&P 500 conditional realizations with high volatility conditioning information

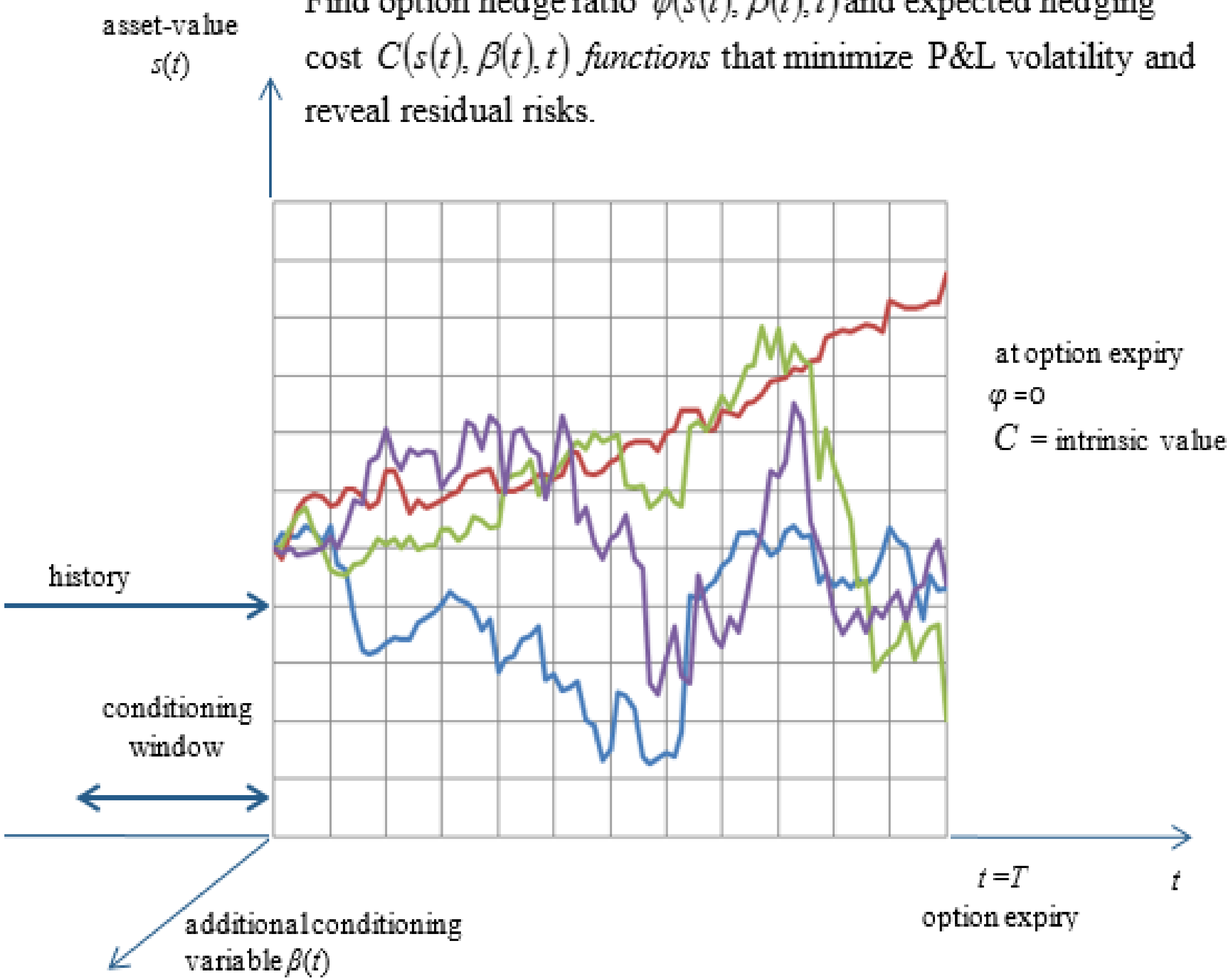
Non-Parametric Optimal Hedging Strategy

And for every problem that is muddled by over-complexity, a dozen are muddled by over-simplifying.
Sydney J. Harris

The eternal mystery of the world is its comprehensibility…The fact that it is comprehensible is a miracle.
Albert Einstein

Recognizing reflexivity has been sacrificed to the vain pursuit of certainty in human affairs, most notably in economics, and yet uncertainty is the key feature of human affairs.
George Soros

Find option hedge ratio $\phi(s(t), \beta(t), t)$ and expected hedging cost $C(s(t), \beta(t), t)$ *functions* that minimize P&L volatility and reveal residual risks.



Non-Parametric Optimal Hedging Strategy

$$\Delta W_{t_k}^{option}(t_k, t_{k+1}) = C(s(t_k), \beta(t_k), t_k) - G(t_k) \qquad G(t_k) = C(s(t_{k+1}), \beta(t_{k+1}), t_{k+1}) df(t_k, t_{k+1}) + P(t_{k,i}) df(t_k, t_{k,i})$$

$$\Delta W_{t_k}^{hedge}(t_k, t_{k+1}) = \phi(s(t_k), \beta(t_k), t_k) H(t_k) \qquad H(t_k) = \left[s(t_{k+1}) - \frac{s(t_k)}{DF(t_k, t_{k+1})} \right] df(t_k, t_{k+1}) + \pi_i df(t_k, t_i)$$

$$\Delta W_{t_k}^{tc}(t_k, t_{k+1}) = -\left[\delta \left| \phi(s(t_{k+1}), \beta(t_{k+1}), t_{k+1}) - \phi(s(t_k), \beta(t_k), t_k) \right| \right] df(t_k, t_{k+1})$$

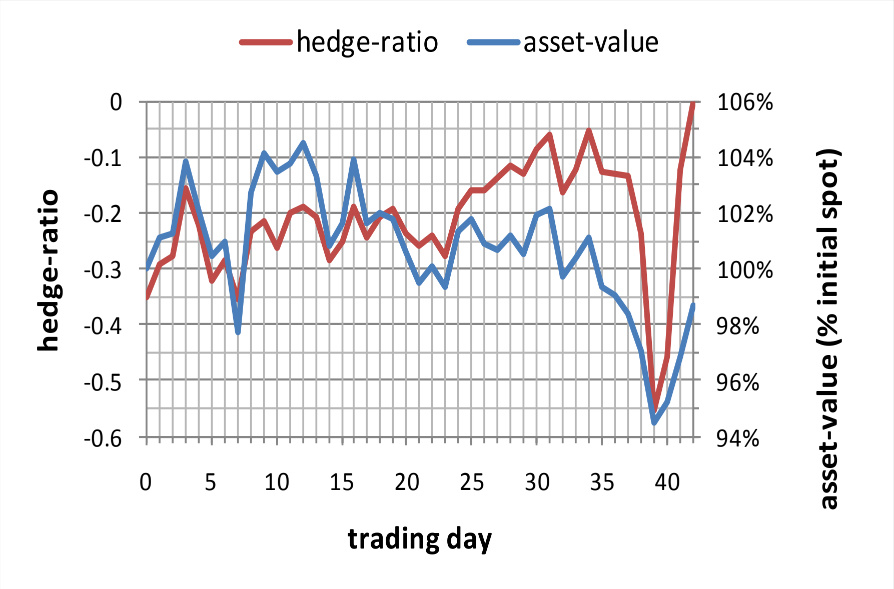
$$\Delta W_{t_k}(t_k, t_{k+1}) = \Delta W_{t_k}^{option}(t_k, t_{k+1}) + \Delta W_{t_k}^{hedge}(t_k, t_{k+1}) + \Delta W_{t_k}^{tc}(t_k, t_{k+1})$$

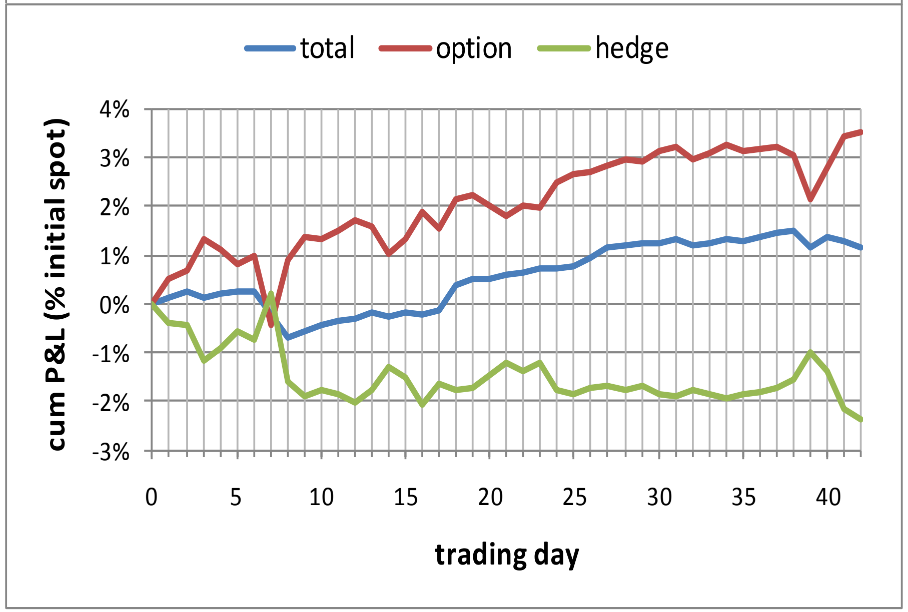
Solve for $C(s(t_k), \beta(t_k), t_k)$ and $\phi(s(t_k), \beta(t_k), t_k)$ so that

$$E[\Delta W_{t_k}(t_k, t_{k+1})] = \overline{\Delta W_{t_k}(t_k, t_{k+1})}$$

minimize $\sigma^2_{\Delta W_{t_k}(t_k, t_{k+1})} \equiv E[(\Delta W_{t_k}(t_k, t_{k+1}) - \overline{\Delta W_{t_k}(t_k, t_{k+1})})^2]$

Non-Parametric Optimal Hedging Strategy: Sample Results





Median total P&L sample path hedge performance from OHMC analysis of sell-hedge 42 day 95% strike SPX put in high volatility regime

The hedging strategy is cognizant of transaction costs and is conditioned on the trailing 10 day realized volatility. The hedging strategy also seeks to maintain an *expected* P&L over each hedging interval to achieve a target Sortino-Ratio of 1.

The attempted replication is full of slips even in this relatively benign outcome. The risk-premium charged by the seller-delta-hedger towards the goal of maintaining a Sortino-Ratio of 1 every day is fulfilled insofar as the total P&L at the end of 42 days slightly exceeds the initially expected P&L in pricing the put. The P&L outcome is of course uncertain, and can be far less favorable if the underlying moves sharply.

Non-Parametric Optimal Hedging Strategy: Sample Results

Hedge P&L Distribution of a 38 day 97.5% strike SPX Put in a Low Volatility Regime

P&L Variance Optimal Hedge Ratio: 26.5%

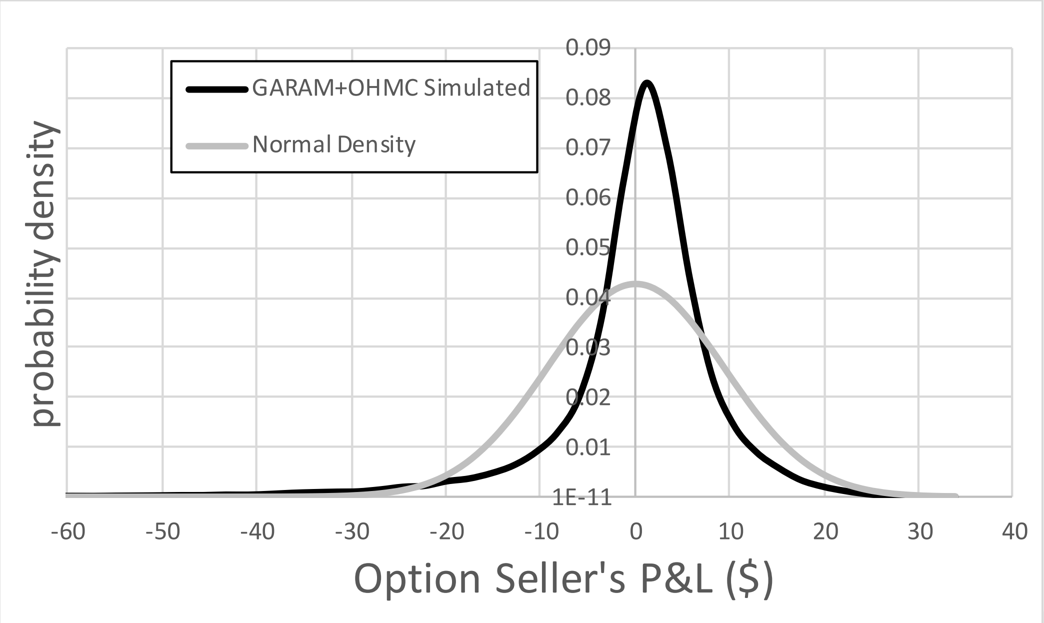
Expected cost of hedging: $10.53

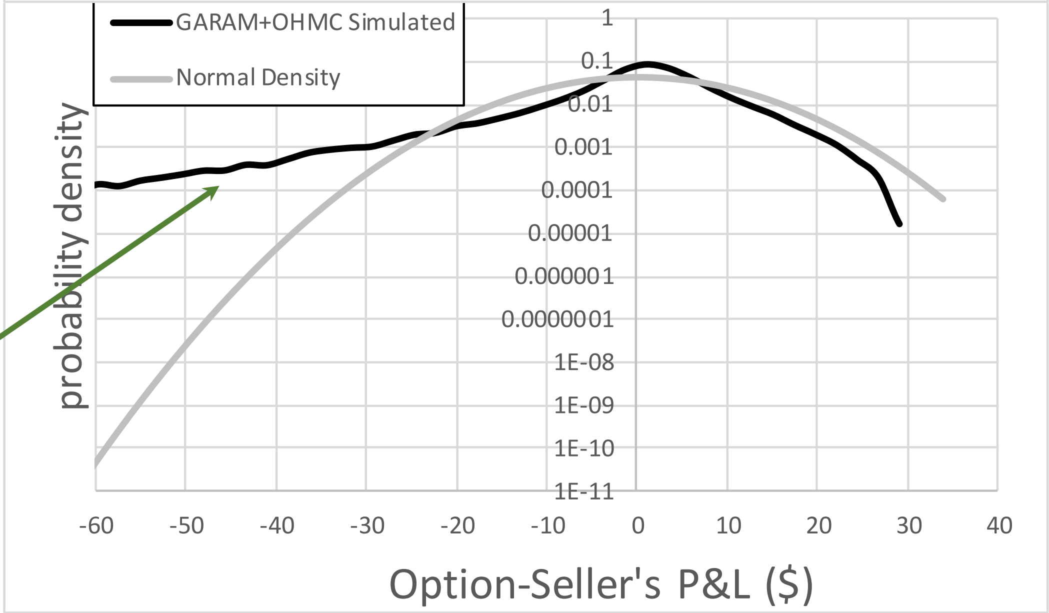
Standard deviation of cost of hedging: $ 9.36

Downside deviation of cost of hedging: $12.30

Bid – Ask: $21.2 - $21.6

Fat Loss Tail of The Option Seller-Delta Hedger: Genesis of the Option Risk Premium

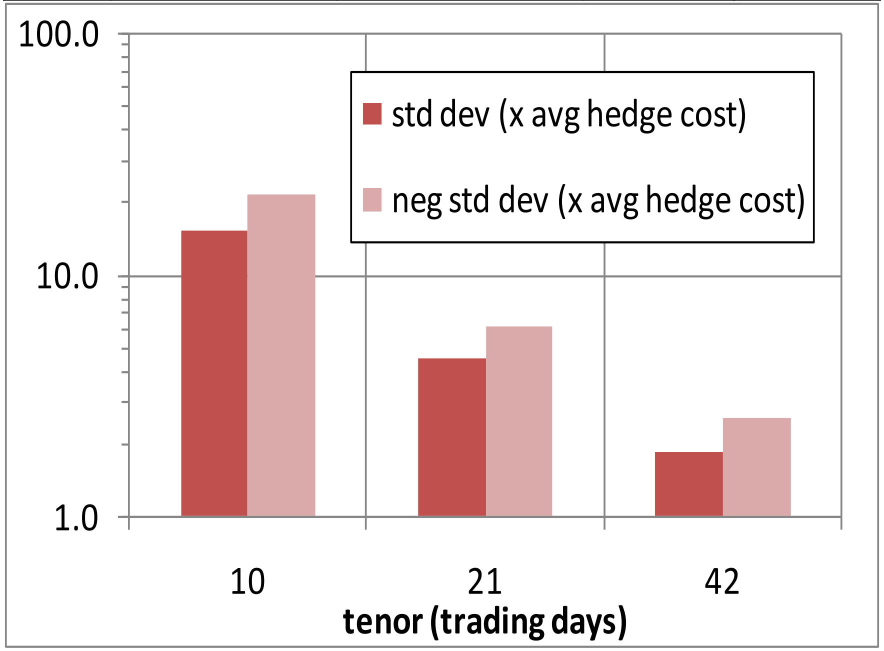




Non-Parametric Optimal Hedging Strategy: Term Dependence

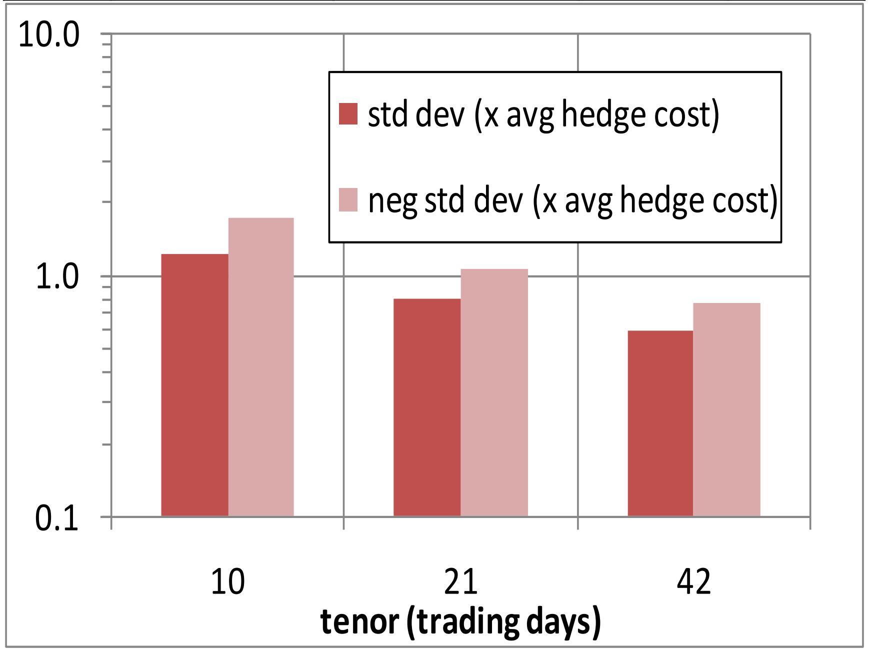
Low-Volatility Regime

tenor (days)	std dev (x avg hedge cost)	neg std dev (x avg hedge cost)	skewness	kurtosis
10	15.5	21.6	-23	1061
21	4.5	6.1	-6.1	113
42	1.85	2.56	-3.8	42



High-Volatility Regime

tenor (days)	std dev (x avg hedge cost)	neg std dev (x avg hedge cost)	skewness	kurtosis
10	1.23	1.72	-3.8	38
21	0.79	1.07	-2.9	30
42	0.59	0.77	-2.4	18

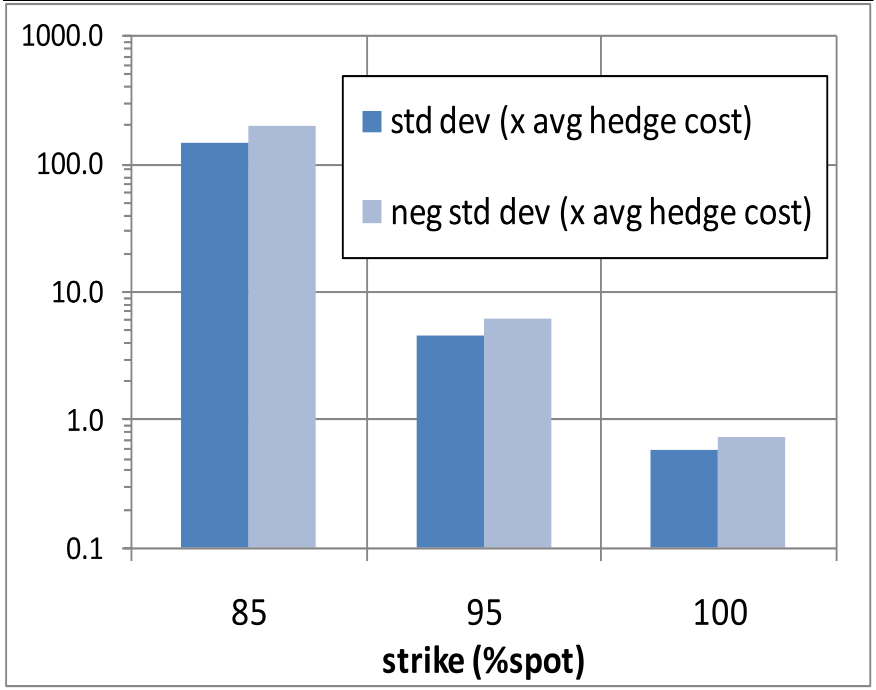


Term-dependence of residual risk for a seller-optimal-hedger of a 95% strike put

Non-Parametric Optimal Hedging Strategy: Strike Dependence

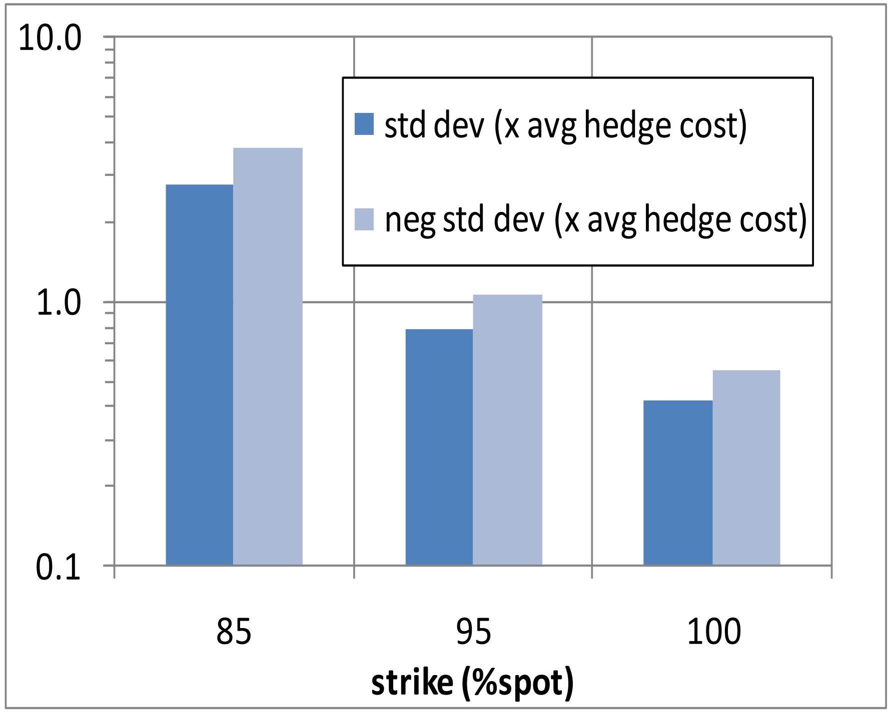
Low-Volatility Regime

strike (% spot)	std dev (x avg hedge cost)	neg std dev (x avg hedge cost)	skewness	kurtosis
85	147	200	-168	41086
95	4.5	6.1	-6.1	113
100	0.58	0.74	-2.3	19



High-Volatility Regime

strike	std dev (x avg hedge cost)	neg std dev (x avg hedge cost)	skewness	kurtosis
85	2.8	3.8	-5.5	91
95	0.79	1.1	-2.9	30
100	0.42	0.55	-2.4	21



Strike-dependence of residual risk for a seller-optimal-hedger of a 21-day put

Risk Neutral Tautology

Option Price =

Unique Hedging Cost

=

negative P&L incurred while hedging

- Sponsored by OTC derivatives sales to book day 1 P&L
- Aided and abetted by academics to show off martingale mathematics
- Inconsistent with the slowly decaying term structure of real return kurtosis – hedging error is the norm and not the exception and is at least leading order

Real World Interpretation

Option Price =

Expected Hedging Cost

=

expected negative P&L incurred while hedging

Risk Premium

=

compensation for irreducible hedge-slippage that drives margin/risk-capital

- Intended to help risk-takers understand risk-return
- Aided and abetted by hedgers and those bearing residual risks
- Attempt to discern absolute risk return consistent with realistic stochastic behavior of market

Machine Learning in Options: Opportunities

"For Machine Learning to gain credibility in investment management it must supplement its forecasts with an error bar that reflects the markets, the current environment, and the forecast horizon" ***Anonymous***

- **Stochastic Models Capable of Capturing Real-World Term-Structures**
  - exploit cross-sectional information
  - swarms of heterogeneous agents with behavioral plausibility
    - *analytical approaches for model specification*

- **Risk-Preference in the Face of Asymmetry**
  - how much of an expected gain is needed to compensate adverse asymmetry?
    - *What is the underpinning of a risk premium?*

- **High Performance Computing**
  - solving Variational-Calculus problems in unbounded domains
  - MC simulation with long memories

Machine Learning in Options: Dead Ends

- **Option Price = Expected Payoff Under Risk Neutral Measure**
  - **this is a vacuous tautology**
    - *hedge slippage is at least of the same order of magnitude as average hedge cost*
    - *highly asymmetric hedge slippage distribution*
  - **irresponsible from perspective of provider of risk capital (buyside client = investor)**
    - *inconsistent with obligations of fiduciary*
  - **used to recognize OTC derivative P&L by invoking immaculate replication**
    - *accounting naiveté and poor risk-capital regimes can fuel "creativity" in derivatives*

Machine Learning in Options: Dead Ends

- **Slow Scripting Language(s)**
  - **machine learning for options is not a trivial problem**
    - *dense-inner loops are everywhere*
    - *nonlinear constraints require iteration*
    - *slow-glue is a recipe for retardation*
  - **machine learning needs thinkers, authors, and doers and not software copycats**
    - *time-to-code is not the limiting ingredient !*

References

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