Math 1496 - Calc 1

Applied Min / Max Probs - 2

When the admission price for a baseball game is \$6 per ticket 36,000 tickets were sold. When the price was raised to \$7 33,000 were sold. At what price should you sell the tickets for max. profit

Sol: $R = PN$ Price & N-number

Now $P = 6 + x$ x is in dollar

$N = 36{,}000 - 3000x$ a reduction of 3000 per dollar

$R = (6+x)(36{,}000 - 3{,}000x)$

$= 3000(x+6)(12-x) = 3000(72 + 6x - x^2)$

$R' = 3000(6 - 2x)$ $R' = 0$ $x = 3$

$R'' = -6000 < 0$ max

Sell at $P = \$9$

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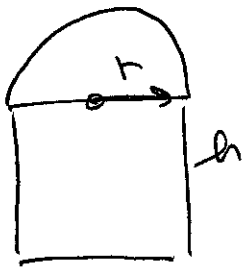
4.7 Applied Min/Max Problems 2

Ex 1 Suppose we have a norman window (½ circle on top of a rectangle) with a perimeter of 18 m. Find the dimensions with max area

Soln



$$P = \frac{1}{2}2\pi r + 2h + 2r = 18$$

$$A = 2rh + \frac{1}{2}\pi r^2$$

Now $2h = 18 - 2r - \pi r$

$$\Rightarrow A = r(18 - 2r - \pi r) + \frac{1}{2}\pi r^2$$

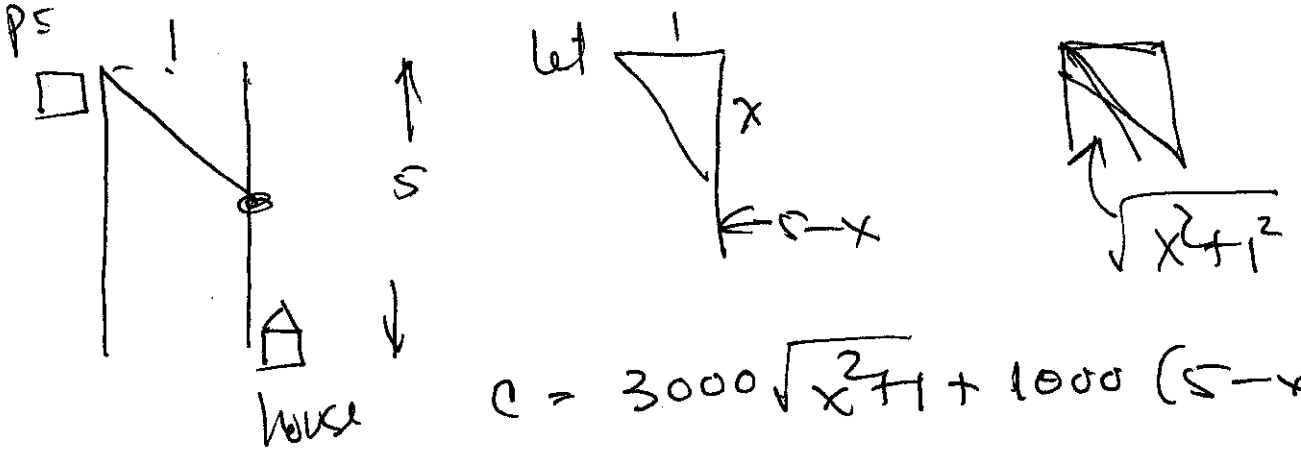
$$= 18r - \left(2 + \frac{\pi}{2}\right) r^2$$

$$A' = 18 - 2\left(\frac{4+\pi}{2}\right) r \qquad A' = 0 \qquad r = \frac{18}{\pi + 4}$$

$$A'' = -(4+\pi) < 0$$

So a max so $r = \frac{18}{\pi+4}$, $h = \frac{18}{\pi+4} = 2.52$ m.

Ex 3 Suppose you want to run an underground cable from a power station on 1 side of a river 1 mile wide to a house on the other side 5 miles down stream. Suppose it costs $1000/mile to lay cable underground & $3000/mile to lay cable underwater. How should you lay the cable to min cost



$$C = 3000\sqrt{x^2+1} + 1000(5-x)$$

$$C' = \frac{3000(x\cdot 2x)}{2\sqrt{x^2+1}} - 1000$$

$C' = 0$ when $\frac{x\,3000}{\sqrt{x^2+1}} = 1000$

$$3x = \sqrt{x^2+1} \qquad 9x^2 = x^2+1$$

$$\Rightarrow \quad 8x^2 = 1 \Rightarrow x = \pm\frac{1}{2\sqrt{2}}$$ the + is the only one makes sense

So $x = \frac{1}{2\sqrt{2}}$

Now $c'' = 3000 \left\{ \frac{\sqrt{x^2+1} - x \cdot \frac{2x}{2\sqrt{x^2+1}}}{(x^2+1)} \right\}$

$= 3000 \cdot \frac{x^2+1-x^2}{(x^2+1)^{3/2}}$

$= 3000 \cdot \frac{1}{(x^2+1)^{3/2}}$

$c'' > 0$ when $x = \frac{1}{2\sqrt{2}}$ so a min

$c\left(\frac{1}{2\sqrt{2}}\right) = 3000\sqrt{\frac{1}{8}+1} + 1000\left(5 - \frac{1}{2\sqrt{2}}\right)$

$= \$7828.43$