Pre-Distribution: Bargaining over Property Rights

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October 7, 2019

*I would like to thank Jim Peck, Hulya Eraslan, Maria Montero, Arno Riedl, Ariel Rubinstein, Husseyin Yildirim, Thomas R. Palfrey, Vincent Anesi, Daniel Seidmann, and Todd Sandler for insightful discussions. I benefited greatly from participants at internal seminars of the departments of Economics at Maastricht University, The Ohio State University, and Universidad Pontificia Javeriana (Bogotá). This article has been presented at the 5^{th} World Congress of the Game Theory Society and the 2016 Association of Southern European Theorists (ASSET) in Thessaloniki. All errors remain mine.

Abstract

This article sets forth a model of multilateral negotiations in which members bargain over property rights prior to the creation of a joint surplus. We depart from the classical assumption embedded in the divide-the-dollar setting that the fund to distribute is either exogenous or the result of a cooperative interaction between players. The interplay between political constraints, technological features, and individual payoff maximizing incentives plays a central role in the characterization of equilibria. Novel results regarding the distribution of the surplus, the proposer's advantage, voting decisions, and efficiency in multilateral bargaining are derived and a comparison with the exogenous surplus case is provided. Importantly, we show that multiple stationary equilibibra may exist which departs from the established uniqueness result when the surplus is exogenous.

1. Introduction

Let there be one unit of wealth to distribute is the typical starting point in strategic models of bargaining, yet one may envision several situations in which negotiations involve the distribution of a surplus which has not yet been created. Partnership formations, joint ventures, military and political coalitions are all settings in which the parties involved are likely to negotiate the allocation of property rights prior to engaging in productive activities. In the canonical model of structured bilateral bargaining developed by Ariel Rubinstein (1982), the bargaining problem is described as if players negotiate over a set of contracts. Subsequently, the problem is modeled as the division of a unit of wealth representing the value of such contracts with no further elucidation upon the relationship between the total surplus achieved through the agreed contract. In the Baron and Ferejohn (1989) model of legislative bargaining the unit of wealth to divide represents the total number of cabinet posts which are to be assigned among parties conforming a coalitional government and in the model of offers and exit of Krishna and Serrano (1996) it represents a fixed budget.

With a few notable exceptions (Leblanc, Snyder, and Tripathi 2000, Battaglini and Coate 2007, 2008), the origin of the common fund and its division have been analyzed as separate problems. We depart from this tradition and set forth a game of *pre-distributive* multilateral bargaining in which economic agents first determine property rights and then engage in productive activities: profit-sharing agreements concurrently determine incentives to generate a common surplus. We seek to analyze how relevant bargaining institutional variables such as the committee size, voting rule, and discount factor affect the trade-off between appropriation and rent-generation incentives.

Our model depicts a setting in which it is not possible to specify complete contracts ex ante, all players have equal bargaining rights, and importantly, once property rights are determined, they are respected (no renegotiation). Naturally, any strategic model of bargaining will impose a structure to the negotiation process, thus, instead of proposing a new protocol, we shall focus on the widely-studied model by Baron and Ferejohn (1989; henceforth BF) which is a generalization of Rubinstein's game to three or more players.[1] The BF model has been central to the Political Economy literature and theorists have expanded it in several directions including a stochastically varying surplus (Merlo and Wilson 1995), asymmetric settings (Eraslan 2002), endogenous agenda-setting power (Yildirim 2007), policy preferences (Banks and Duggan 2000, Jackson and Moselle 2002), and a dynamic setting with an endogenous status quo (Baron 1996, Kalandrakis 2004, Bowen and Zahran 2012, Anesi and Seidmann 2015). Moreover, it has been experimentally studied in the laboratory for which qualitative support of the theoretical predictions has been obtained from multiple studies (Fréchette, Kagel, and Morelli 2005). Importantly, the origin of the fund has remained until now as an exogenous parameter. A review of the literature is provided in the next section.

In our model, players instead bargain over property rights (operationalized as equity shares) which determine simultaneously productive incentives and a profit-sharing rule. Once an agreement is reached, partners proceed to an investment stage in order to determine the total surplus. We consider a very simple production technology, linear and symmetric, because our aim is to describe the strategic interaction between the bargaining environment and the provision of productive incentives. At the onset it is clear that negotiating property rights is, indeed, a different problem because bargainers face a trade-off between rent-sharing and rent-generation absent in the divide-the-dollar setting. If a player attempts to offer a small share of equity to coalition partners, the size of the pie might shrink, rendering her large ownership percentage unprofitable. Solving the model is not a simple replication of the standard exogenous fund case because there is a meaningful interaction between political constraints (i.e. the fact that the voting quota must be met for approval of the equity agreement), the production or technological conditions (i.e. the fact that players should be sufficiently compensated to induce investments or effort), and self-interest (i.e. bargainers' desires to maximize own earnings). Since bargaining occurs prior to the total fund being

[1] In this game, one randomly chosen player proposes a distribution of a common fund which must be accepted by a majority through voting, or otherwise, the fund is discounted and the process repeats itself until approval.

created, we must also impose consistency conditions on players' expectations about the size of the fund in order for them to evaluate what an acceptable equity share is. As it turns out, there could be multiple stationary subgame perfect equilibria with different associated payoffs which is a novel result per se (see Eraslan 2002 for uniqueness of equilibrium payoffs in the exogenous fund case), yet we will focus on an equilibrium path which yields the highest payoff to the proposer. This will allow us to evaluate the maximum proposer power attainable when the fund is endogenous and compare it to the exogenous fund case.

Several aspects of our equilibrium are worth highlighting especially because they depart from the predictions of the exogenous fund case. First, the proposer forms a coalition characterized as a two-tier scheme: Some members are incentivized to invest (denoted hereafter as *productive members*) and others are simply offered a share that *buys* their vote without inducing investment. Productive members receive a payoff that is greater than the continuation value of the game, while non-productive coalition partners receive a share that induces a payoff equal to the ex ante value of the game. Hence, we are able to show that the proposer's rent-extraction aspirations are mitigated compared to the exogenous fund bargaining game due to the provision of productive incentives, and the proposer premium may totally dissipate under certain parameter values. Redundant members (those whose vote is not required for approval) receive zero equity, which also occurs in the exogenous fund case. In an extension of the model, we consider the case in which the total fund has an additional exogenous component and show that it can crowd out productive incentives and enhance the proposer's rents. In general, the proposer finds it more profitable to increase own equity holdings (at the expense of losing investing partners) as the exogenous portion of the fund increases. Finally, individual voting decisions in our model are contingent on the full distribution of ownership shares, and not only on the individual player's share as typically derived in models with an exogenous fund. This is due to the fact that the implied total surplus is a function of the entire distribution of shares.

In order to investigate how the production technology affects bargaining outcomes, we analyze the model for a Leontief production function which depicts a setting of perfect complementarities in effort. Here, a fully efficient equilibrium exists for all parameter values in which all players receive equity shares, but it does not entail unanimous approval because members' whose votes are not needed for passing the proposal receive a share that makes exerting effort beneficial but does not compensate them enough as to garner their support. Given that the complementarities in production require the proposer to offer everyone productive incentives, the proposer's premium is also lower compared to the exogenous fund case.

The article proceeds as follows. Section 2 presents a literature review mainly focused on multilateral bargaining games *a la* BF. Section 3 introduces the model. Section 4 solves the equilibrium and provides the comparative statics. Section 5 analyzes the mixed model in which part of the fund to distribute is exogenously given. In Section 6 we vary the production technology to allow for complementarities in production using a Leontief function. Section 7 concludes the paper. Appendices A and B contain the main proofs. Appendix C develops a model with complete contracts.

2. Literature Review

The standard BF model under the closed amendment rule[2] has been generalized in many directions and a comprehensive survey is provided in Eraslan and McLennan (2013) and Eraslan and Evdokimov (2019). Eraslan (2002) considers the game in which players differ in their probability of being recognized as the proposer and hold different discount factors. It is shown that for a given parameter configuration, stationary subgame perfect equilibrium strategies (SSPE) need not be unique but the ex ante values of the game are. This implies that all equilibria yield the same vector of payoffs. Importantly, a player's ex ante value of the game is increasing in the probability of recognition and the discount

[2]The closed amendment rule refers to the situation in which a proposal on the floor is voted as is. In the open amendment rule protocol, another player can either challenge the proposal by submitting a new one or second the current proposal which then leads to a voting stage.

factor. Eraslan and McLennan (2013) show uniqueness of payoffs in a setting where the set of winning coalitions may vary for each proposer.

The BF game has also been studied in an environment in which the surplus to distribute fluctuates stochastically over time. Merlo and Wilson (1995) restrict attention to the unanimity voting rule and show that efficient delay might arise in equilibrium because players are better off by waiting for a larger fund. Eraslan and Merlo (2002) consider any voting rule and show that when not every member's vote is required for approval, delay can also arise but it will be inefficient because agreements can be reached too soon relative to the optimal delay. Eraslan and Merlo (2014) consider a case where each proposer *brings* a total fund of different magnitude to be distributed (i.e. the proposer and fund stochastic processes are perfectly correlated). In our model the fund may vary because anyone can contribute to expand the total fund, but the size of the fund is uniquely determined by the incentives embedded in the property rights agreement.

In a closely related article to the game of pre-distribution, Baranski (2016) presents a game with the reverse timing: Players first engage in a production stage followed by a profit-sharing game of bargaining.[3] Investments can be considered a sunk cost at the beginning of the bargaining stage (if strategies are restricted to be history-independent) and the resulting equilibrium prediction is that no one should contribute to the common fund. The reason is that the ex ante value of the bargaining game under the SSPE is equal to the average fund, which induces the same payoff structure as in a linear public goods game. The well-known free-rider problem obtains.

An application of the BF bargaining protocol to the firm context can be found in Britz, Herings, and Predtetchisnki (2013). In their setting, partners must agree unanimously over a future production plan for the firm in the midst of uncertainty. Differences in the risk attitudes of the committee members result in conflicts about which production plan to choose, but transfers that are payable prior to the realization of the state of nature are used to *grease*

[3]See Frankel (1998) for a model of production prior to bargaining in the Rubinstein (1982) bilateral bargaining framework.

the wheels of the bargaining process. Their main result is that the payoffs resulting from the equilibrium production plan and transfer scheme determined in the bargaining game are equivalent to the payoffs specified by a generalized Nash bargaining solution where the relative bargaining power weights are given by the probability of being the proposer for each player.[4] Their model can be interpreted as a bargaining game of risk sharing, an aspect that is not present in the game of equity bargaining which we pose here.

Battaglini and Coate (2007, 2008) study a dynamic model in which taxing and spending decisions are determined via legislative bargaining in a finite horizon version of the BF game.[5] The tax level uniquely determines optimal behavior in the labor market (i.e. how much leisure to consume and hours to work) which in turn determines the total budget available for public spending. In this sense, Battaglini and Coate endogenize the surplus to distribute in the bargaining game. Their setting is dynamic because policymaking is linked across periods via the accumulation of a durable public good. Besides differences in the focus of the research question and the dynamic nature of the Battaglini and Coate settings, our models are quite distinct in other dimensions as well. For example, workers receive a wage in Battaglini and Coate (a payment per unit of labor), while in our main model they receive ownership shares. Both models consider a linear production function, however Battaglini and Coate incorporate a convex disutility of labor which guarantees an interior optimum of labor choice. Also, bargaining can continue indefinitely until approval in our game while Battaglini and Coate impose a final period with a default distribution of resources, thus allowing them to solve by backward induction.

[4] Once they characterize the equilibrium in the payoff space, they show that there is a unique production plan and vector of transfers satisfying the Nash solution. This result holds in the limiting case where there is no discounting of future payoffs.

[5] Taxing is distortionary and the budget can be used for lump-sum transfers across districts (pork-barrel spending) or to fund a durable public good (or both). In their model, each player represents one district. This model generalizes Leblanc, Snyder, and Tripathi (2000) in several directions.

3. The Model

Let there be a committee with n players (odd) that are each endowed with 1 unit of wealth. In stage 1, members bargain by making offers (in a randomly alternating order) and voting on how to split the rents of a potential common fund that they will produce in stage 2. In each bargaining round (denoted by superscript $t \in \{1, 2, ...\}$) within stage 1, a player is randomly called upon (with equal probability) to propose a scheme on how to divide a common fund. Denote by $\mathbf{s}^t = (s_1^t, ...s_n^t)$ a division of the fund such that the sum of percentage shares satisfies $\sum s_j^t = 1$ (which we call the equity constraint) and each share $s_j^t \in [0, 1]$ for every player j. For each proposal on the floor, players vote to accept or reject and q votes are required for approval (including the proposer's vote). In case of rejection, the proposal and voting rounds are repeated. Once an allocation is approved in round τ, players proceed to stage 2 in which they simultaneously choose a contribution $c_i \in \{0, 1\}$. If a unit is contributed it is multiplied times $\alpha \in (1, q]$ and becomes part of the common fund.

Let h^t denote the history up to round $t-1$, which includes the list of previous proposers and their proposals, as well as the voting record. We denote by H^t the set of all possible histories of legth t ending right before another proposal. A strategy for a proposer in round t is given by $s : H^t \to [0, 1]^n$ and for voters it is given by $v : \{[0, 1]^n, H^t\} \to \{Yes, No\}$.

A contribution strategy is a function $c_j : [0, 1]^n \times H^t \to \{0, 1\}$ which is defined conditional on reaching an approval.[6] The total fund is given by $F = \alpha \sum_{i=1}^{n} c_i$.

Utility is linear in money. Given a profile of strategies $(\mathbf{s}, \mathbf{v}, \mathbf{c})$ in which an agreement was reached in period t, player j's utility is given by

$$u(\mathbf{s}, \mathbf{v}, \mathbf{c}) = \delta^{t-1} \left(s_j^t F - c_j + 1 \right) \ , \tag{1}$$

where $\delta \in (0, 1]$ is the discount factor.

[6] If the proposal is never approved, then by definition $\mathbf{c} = \mathbf{0}$ and payoffs are 0 to everyone.

4. Equilibrium

We restrict attention to stationary subgame perfect equilibria, meaning that bargaining strategies are history-independent. As usual in the literature, we assume that a member votes in favor if and only if she is offered a share that yields a payoff equal or greater than the continuation value of the game.

DEFINITION 1 A stationary subgame perfect equilibrium of the equity bargaining game, denoted by the stationary strategy profile $\boldsymbol{\sigma}^* := (\mathbf{s}^*, \mathbf{v}^*, \mathbf{c}^*)$ induces a vector of ex ante values of the game given by $\mathbf{V}^* = (V_1^*, ...V_n^*)$ such that $\boldsymbol{\sigma}^*$ and $\mathbf{V}^*$ satisfy:

1. For every player i we have that $u(\boldsymbol{\sigma}^*) \geq u(\tilde{\sigma}_i, \boldsymbol{\sigma}^*_{-i})$ for any other strategy $\tilde{\sigma}_i$;

2. s is only a function of the state of non-agreement, $v : \mathbf{s} \to \{Yes, No\}$, and $c : \mathbf{s} \to \{0, 1\}$;

3. $v = yes$ if and only if $u(\boldsymbol{\sigma}^*) \geq \delta V_i^*$;

4. The total fund generated by $\mathbf{c}^*$ given $\mathbf{s}^*$ is consistent with $\mathbf{V}^*$;

5. If there are multiple strategy profiles $\boldsymbol{\sigma}^*$ that satisfy (1)-(4), we select the one that yields the highest proposer payoff.

Points (1)-(3) in the definition above are standard in this setting, with the caveat that voting is typically defined as a function of own share only. In our setting it is a function of the vector of shares because the total surplus available depends of what others receive as well. The consistency requirement established in (4) is unique to our setting. Given that the equilibrium continuation value of the game will depend on the equilibrium size of the fund, it must be that the distribution of shares induces contribution levels consistent with $\mathbf{V}^*$.

Since we are interested in comparing proposer power between the standard BF game with an exogenous fund, we will focus on proposer-optimal equilibria as indicated in point 5 of the definition above. This will inform us of the maximum surplus a proposer may obtain in our game with endogenous production. To clarify, we are not arguing that the first proposer chooses which equilibrium to play because this would violate the history-independence property of the stationarity refinement. Instead, we argue that players can coordinate on such equilibrium exogenously.

We show by example in Section that there can exist other equilibria which are more efficient in terms of production but yield a lower payoff to the proposer. In Appendix B, Proposition 4, we provide a general characterization of the conditions that must be met but we are unable to provide a closed-form solution. Thus, the characterization we pursue in the body is a constructive proof of existence.

In order to simplify our analysis we make one additional assumption about the parameter space. We require that the discount factor is not *too small*.

Assumption 1 The discount factor satisfies that $\frac{n}{n+\alpha-1} < \delta \leq 1$.[7]

The importance of this assumption will become clear in Lemma 2. It is a sufficient condition that ensures that all members in the proposer's coalition are offered a positive share, and rules out equilibria in which members receiving zero equity would also vote in favor. Note that when δ is very small, the continuation value given by δV_i^* could be lower than the unit endowment, thus players would accept any offer in order to avoid delays.

4.1 Stage 2: Investment Subgame

We start in stage two by characterizing the possible subgames after approval. For any approved distribution of shares in round τ of stage 1 ($\mathbf{s}^\tau$), a player's equilibrium strategy in stage 2 is given by

$$c_i^*(\mathbf{s}^\tau) = \begin{cases} 0 & \text{if} \quad \alpha s_i^\tau < 1 \\ 1 & \text{if} \quad \alpha s_i^\tau \geq 1 \end{cases} \tag{2}$$

which simply states that a player finds it optimal to invest if and only if she has a positive return. It is straightforward to show that at most $\lfloor \alpha \rfloor \leq q$ players can be induced to

[7]Note that this depends on the committee size and productivity parameter. In particular, as the committee size increases, the lower bound on δ tends to 1.

contribute since the sum of shares must be equal to 1. From now on, we say a member is *incentivized* or *productive* if she is given a share such that $c_i^*(\mathbf{s}^\tau) = 1$.

4.2 Stage 1: Bargaining

When players are bargaining over shares, they are implicitly bargaining over the associated payoffs. Hence, we study the *implicit bargaining game* in the payoff space for which there are well-established results in the literature that will be invoked throughout the process.[8]

Before proceeding, it should be noted that any allocation in which there is no production cannot be an equilibrium (see Lemma 4 in Appendix A). We will assume that the proposer is always a producer while solving the model.[9]

We denote the proposer's share by s_{Prop} and the share offered to incentivized members by s_{Cont}. We allow for the possibility that certain members are offered a positive share that does not induce contribution and denote such share by s_{Vote}.[10] In equilibrium it will be evident that all voting members receive the same s_{Vote} and all contributing members receive s_{Cont}.

Let k denote the number of productive members excluding the proposer (those to whom s_{Cont} is offered) and let m denote the number of voters to whom s_{Vote} is offered. It follows that the fund is given by

$$F(k) = \alpha(k+1) \quad . \tag{3}$$

[8]See Britz, Herings, and Predtetchinski (2013) for a similar approach.

[9]This is technically not an assumption and is only invoked to ease the exposition. As we show, productive members are weakly better off than non-investing members in equilibrium. Hence, the proposer will always be a productive member.

[10]Although it seems we are imposing a solution structure in solving the game, all we are doing is assuming that if two members are incentivized, then the offered shares are equal. In order to meet the voting quota, the proposer might require the votes of other non-incentivized partners whom are offered s_{Vote}. Again, we do not impose how many members receive this share.

We can rewrite the equity constraint as

$$s_{\text{Prop}} + ks_{\text{Cont}} + ms_{\text{Vote}} = 1 \quad . \tag{4}$$

The continuation value of the game (undiscounted) can be defined as

$$V(m,k;q,n,\alpha) := \frac{1}{n}s_{\text{Prop}}F(k) + \frac{k}{n}s_{\text{Cont}}F(k) + \frac{m}{n}(s_{\text{Vote}}F(k)+1) + \left(1 - \frac{1+k+m}{n}\right) \tag{5}$$

and it is a weighted average of the payoffs that a player receives in each possible role that she might find herself in.[11] The last term, $1 - \frac{1+k+m}{n}$, denotes the expected payoff from not being a‘ssigned equity. Using the equity constraint we obtain

$$V(m,k;q,n,\alpha) = \frac{F(k)}{n} + 1 - \frac{1+k}{n} = \frac{(\alpha-1)(k+1)}{n} + 1 \quad . \tag{6}$$

Equation (6) has the intuitive interpretation that the ex ante value of the game, for a given k, is equal to the average fund net of contributions plus the endowment. Hereafter we denote it by $V(k)$.

In order for the fund to be attainable (i.e. effectively produced), three production consistency conditions must be met:

$$1/\alpha \leq s_{\text{Prop}} \leq 1 \quad , \tag{7}$$

$$1/\alpha \leq s_{\text{Cont}} \leq 1 \quad , \tag{8}$$

$$0 \leq s_{\text{Vote}} < 1/\alpha \; . \tag{9}$$

Conditions (7) - (9) guarantee that there are exactly $k+1$ productive members (including the proposer). Furthermore, we require that members to whom a positive share is offered

[11] To calculate this, we are imposing symmetry (what player j offers i is what i offers j). We also assume that proposers randomize over whom to offer s_{Vote} and s_{Cont}. Hence, in k/n times a member is offered s_{Cont} and in m/n she is offerd s_{Vote}. Due to the stationarity assumption, this value remains the same for every bargaining period. We are imposing no delay in agreement.

find it optimal to vote in favor. Hence, the share offered must induce a payoff that is greater than or equal to the continuation value of the game. The voting consistency conditions are then:

$$s_{\text{Prop}} F(k) \geq \delta V(k) \quad , \tag{10}$$

$$s_{\text{Cont}} F(k) \geq \delta V(k) \quad , \tag{11}$$

$$s_{\text{Vote}} F(k) + 1 \geq \delta V(k) \quad , \tag{12}$$

$$m + k + 1 \geq q \quad , \tag{13}$$

where the last condition specifies that the proposal receives the necessary amount of votes. We are now ready to present the maximization problem that a proposer faces:

$$\max_{\{k, m, s_{\text{Prop}}, s_{\text{Cont}}, s_{\text{Vote}}\}} s_{\text{Prop}} \cdot F(k) \tag{14}$$
$$\text{s.t. conditions (4) and (7)-(13)} \quad .$$

A few observations will help us rewrite the problem more concisely in terms of k only. Note that s_{Prop} is decreasing in m and that $F(k)$ is not dependent on m, so that the proposer will restrict the amount of s_{Vote} offers made to exactly meet the voting quota. Also, recall that $k \leq \lfloor \alpha \rfloor \leq q$.[12] Hence, we have that $m = q - 1 - k$.

LEMMA 1 In equilibrium, $s_{\text{Cont}}(k) = \begin{cases} 1/\alpha & \text{if} \quad k \geq \tilde{k} \\ \delta V(k)/F(k) & \text{otherwise} \end{cases}$ where $\tilde{k} := \frac{\delta n}{n - \delta(\alpha - 1)} - 1$.

PROOF. Conditions (8) and (11) imply that $s_{\text{Cont}} \geq \max\{1/\alpha, \delta V(k)/F(k)\}$ and we can

[12] That fact that $k \leq \lfloor \alpha \rfloor$ follows from equations (7), (8), and (4). If $k > \lfloor \alpha \rfloor$, then $k \cdot \frac{1}{\alpha} + s_{\text{prop}} > 1$ violating (4).

show that

$$\begin{aligned} 1/\alpha &\geq \delta V(k)/F(k) \iff \\ k+1 &\geq \delta\left[\frac{(k+1)(\alpha-1)}{n}+1\right] \iff \\ k &\geq \frac{\delta n}{n-\delta(\alpha-1)}-1=\tilde{k} \ . \end{aligned}$$

Since the proposer's payoff decreases in s_{Cont}, in equilibrium it must be that constraints (8) or (11) bind (or both at $k=\tilde{k}$). ■

From condition (12) we solve for s_{Vote} to be

$$s_{\text{Vote}}(k)=\max\left\{\frac{\delta V(k)-1}{F(k)},0\right\}$$

and again it is straightforward to verify that this constraint binds. In the following lemma we make use of Assumption 1 to determine $s_{\text{Vote}}(k)$.

Lemma 2 If $\delta > \frac{n}{n+\alpha-1}$ (Assumption 1) then $\frac{\delta V(k)-1}{F(k)} > 0$ for all k.

Proof. We have that $\delta > \frac{n}{n+\alpha-1} = \frac{1}{1+(\alpha-1)/n} \geq \frac{1}{1+(k+1)(\alpha-1)/n} \implies \delta\left[1+(k+1)(\alpha-1)/n\right] = \delta V(k) > 1$ and the result follows. ■

We are now able to write problem 14 as a function of the number of incentivized members (k):

$$\max_{k\in\{0,\ldots,\lfloor\alpha\rfloor\}} \Pi(k) := s_{\text{prop}}\cdot F(k) = \left[1-k\cdot s_{\text{Cont}}(k)-(q-1-k)\left(\frac{\delta V(k)-1}{F(k)}\right)\right]\cdot F(k) \quad . \quad (15)$$

In calculating $s_{\text{Vote}}(k)$ and $s_{\text{Cont}}(k)$ we have substituted in the ex ante value of the game for an arbitrary k (as given by equation (6)). By doing so, we are imposing the consistency requirement that the ex ante value of the game for which members are willing to vote in favor, is indeed the one that will be induced by the proposer. Recall we are computing proposer-optimal equilibria, hence, players expect the continuation value of the game to be the one which maximizes any proposer's payoffs.

Temporarily, we drop the integrality requirement on k and solve the associated program (where $k \in \mathbb{R}$). By standard arguments, the solution to this problem exists. In Appendix A (Lemma 5) we show that the optimal solution is given by

$$\bar{k} := \frac{1}{2}\frac{n(\alpha+\delta) - \delta q(\alpha-1)}{n-\delta(\alpha-1)} - 1 \quad . \tag{16}$$

We now wish to characterize the optimal integer solution which is denoted by k^*. Given the concavity of the objective function, the proposer's payoff attains its highest value at one of the closest integers to $\bar{k}$ which are $\lfloor \bar{k} \rfloor$ and $\lfloor \bar{k} \rfloor + 1$. However, one needs to verify whether such integers are feasible or not. We denote by Ω the set of k that satisfy conditions (4) and (7)-(13). It is easy to see that $\lfloor \bar{k} \rfloor \in \Omega$ always and, by example, one can show that $\lfloor \bar{k} \rfloor + 1$ need not be feasible. Hence, we have that

$$k^* := \begin{cases} \lfloor \bar{k} \rfloor + 1 & \text{if} \quad \lfloor \bar{k} \rfloor + 1 \in \Omega \text{ and } \Pi(\lfloor \bar{k} \rfloor + 1) \geq \Pi(\lfloor \bar{k} \rfloor) \\ \lfloor \bar{k} \rfloor & \text{otherwise} \end{cases} \tag{17}$$

defines the optimal solution to problem (15).[13]

Regarding the timing of approval, it should be clear that players have no incentive to delay agreement because it would reduce payoffs whenever there is discounting. When there is no discounting, there is no rationale for delaying approval since players vote in favor whenever their share makes them greater than or equal to the continuation value of the game.

We summarize the equilibrium in the following proposition.

Proposition 1 The proposer optimal equilibrium of the equity bargaining game is as follows:

[13] Note that the above definition allows for the possibility that there are two integers (those closest to $\bar{k}$) that yield the same payoff to the proposer. In this case our definition chooses the most efficient one. We were not able to produce an example where this situation arises.

1. In every period, the proposer assigns $s^*_{\text{Cont}} := s_{\text{Cont}}(k^*)$ to k^* members, $q - k^* - 1$ other members receive $s^*_{\text{Vote}} := s_{\text{Vote}}(k^*)$, and the proposer assigns herself $s^*_{\text{Prop}} := 1 - k^* s^*_{\text{Cont}} - (q - k^* - 1)s^*_{\text{Vote}}$. The remaining $n - q$ members do not receive equity shares.

2. Player i votes in favor of any proposal $\mathbf{s}$ that implies k such that (i) $s_i F(k) \geq \delta V(k^*)$ and $1/\alpha \leq s_i \leq 1$ or (ii) $s_i F(k) \geq \delta V(k) - 1$ and $0 \leq s_i < 1/\alpha$.

3. $c_i(s_i) = 1$ if and only if $s_i \geq 1/\alpha$. In the equilibrium outcome, only the proposer and those who obtain a share s^*_{Cont} contribute all their endowment.

4. There is no delay in approval.

This equilibrium characterization presents three novel results in the multilateral bargaining literature. The first is that not every member of the coalition obtains the same payoffs. Note that non-productive coalition partners receive a share that yields exactly the continuation value of the game while productive members may receive $\frac{1}{\alpha}$ (depending on the parameter configuration) which yields a higher payoff. This takes us to the second feature: the need to provide productive incentives mitigates the proposer's rent-extraction capacities.

A third difference is that a member's voting decision does not depend only on the share she receives, but on the entire proposal. Recall that a member votes in favor if and only if her share yields a payoff greater than or equal to the discounted continuation value of the game. The total fund and the continuation value are a function of the number of productive members which is determined by the proposal (vector of shares), and not only the individual share. Thus, a member that receives s^*_{Vote} will vote against if the proposer did not offer s^*_{Cont} to k^* other partners.

In general, one can show that k^* is typically below the socially optimal level given by $k = \lfloor \alpha \rfloor - 1$. By socially optimal production we mean the level that can be implemented by a social planner who distributes shares without forcing investments. Note that when $\alpha < 2$ the socially optimal production coincides with $k^* = 0$ because there is only enough equity to incentivize one member (the proposer). The socially optimal level can also be achieved in equilibrium for high productivity as α approaches q and q approaches n. A formal argument is presented in the Appendix (see Corollary 6).

Fully productive partnerships may arise in equilibrium, but this requires a high productivity level and the unanimous voting rule. We now show that there is a range of values of the discount factor for which the proposer-optimal equilibrium coincides with a fully productive equilibrium (i.e. when $k = n - 1$ which means everyone invests).

Corollary 1 Let $\alpha = q = n$. Then, there exists $\delta(n)$ such that $k^* = n - 1$ for all $\delta \in [\delta(n), 1]$.

Letting $\delta = 1$ and $\alpha = q = n$ one can easily verify that $\bar{k} = n - 1$ which implies that all members are investing. This is because the proposer offers $1/n = 1/\alpha$ to all of them and a reduction of anyone's share would reduce the fund and not lead to approval. As δ decreases, the discounted continuation value of the game falls, hence the proposer can potentially offer a share $s_{\text{vote}} < 1/\alpha$ to some members and attempt to obtain their vote while he increases his own equity holdings. However, if δ is very close to 1, the integer solution need not change because the proposer's increase in equity from downgrading a productive member's share is not enough to compensate the loss in total production. In the Appendix we provide a formal proof.

4.3 Comparative Statics

We are now ready to present various comparative statics results regarding the equilibrium size of the fund. We will focus on the equilibrium region where $s_{\text{Cont}} = 1/\alpha$ and in which we remain within the parameter space stated in assumption 1.

Corollary 2 In equilibrium, the following results hold about $F(k^*)$:

1. it is weakly decreasing in q;

2. it is weakly increasing in n;

3. it is weakly decreasing in δ if $n < (q-\alpha)(\alpha-1)$ and weakly increasing if $n \geq (q-\alpha)(\alpha-1)$;

4. it is increasing in α.

Results (1), (2) and (4) of Corollary 2 are straightforward to verify by computing $\partial\bar{k}/\partial q$, $\partial\bar{k}/\partial n$, and $\partial\bar{k}/\partial\alpha$.[14] When more votes are required for approval, the proposer must weigh the benefits of adding an extra productive or voting member and satisfy the feasibility constraint. It turns out that as q increases, adding an extra productive member is too expensive in terms of equity and the overall effect is a reduction of the fund.

The positive relationship between the equilibrium fund and the committee size is not expected, because typically as the size of the group increases, problems of collective action tend to worsen. To see why this is the case, note that when q is fixed, adding more members to the committee does not alter the voting constraint, and that as n gets larger, the ex ante value of the game becomes smaller. This implies that the share offered to a non-productive voter decreases and it can reach a point where the proposer has enough available equity to upgrade a non-productive coalition partner's share so that she becomes productive.

The relationship between the optimal fund and the discount factor is guided by more subtle dynamics. In equilibrium, as δ increases, the share offered to a non-productive coalition member (s_{Vote}) increases (for a fixed k), while s_{Cont} is constant. This means that the proposer must give up own equity if he wants to sustain the same level of production. Alternatively, he can take one of two paths: (1) sacrifice a productive member and replace her by a voting member or (2) replace a voting member by a productive member. Summarizing, the proposer must weigh the payoffs from maintaining the level of output by sacrificing own equity, increasing output and sacrificing equity, or reducing the fund and augmenting his

[14]The comparative statics in (1)-(3) specify weakly monotonic relations is due to the fact that a change in the variable in question might not be enough to induce a discrete change in k^*. We omit the proof because these are simple arithmetic calculations.

own equity.[15] The optimal choice will depend on the parameter configuration, which we proceed to explain.

Fixing q and n, the inequality $n > (q-\alpha)(\alpha-1)$ is more likely to hold when either α is large (close to q) or small (close to 1), which set us in the region where the fund increases with δ. A large α makes s_{Cont} relatively cheap in terms of equity, thus making it more attractive to enhance a non-productive partner's share to productive levels. When α is small, replacing a voting member by a productive member induces a big loss of equity to the proposer but this loss is less than proportional to the percentage increase in production. Note that for a small α, the total fund is small as well. Thus, adding a productive member generates a large proportional increase in the fund.

For intermediate values of α, adding a productive member is no longer as cheap as it is for large α, nor does it induce a large proportional change in the size of the fund as it happens for small values of α. Hence, as δ increases, the proposer finds it optimal to replace a productive member by a voting member in this region, a decision that entails a reduction in the total fund.

Example 1 Let $n = 15$. For $q = 11$ and $\alpha = 11$ we have that $k^* = 6$ if $\delta = 1$ and $k^* = 5$ if $\delta = 0.8$. Hence, the fund decreases as δ decreases. The opposite effect happens when $n = 21$, $q = 18$ and $\alpha = 15$ because we have that $k^* = 5$ if $\delta = 1$ and $k^* = 6$ when $\delta = 0.8$. These parameters satisfy assumption 1.

We now examine the effect of committee size on the total fund for the simple majority and unanimity voting rules.

Corollary 3 Larger committees yield (weakly) lower efficiency under the unanimity and simple majority voting rules.

A proof can be found in Appendix A. Recall that in Corollary (2) we had fixed the voting quota and considered a change in the size of the committee. Here, the voting quota

[15] When sacrificing a productive member, the proposer is able to increase his equity because $s_{\text{Vote}} < s_{\text{Cont}}$.

is pegged to the size of the committee. Therefore, increasing the committee size entails that more votes need to be *bought* in order to obtain approval. Although non-productive coalition partners become cheaper, the decrease in equity that must be disbursed to them is surpassed by the increase in equity disbursed to non-productive voters as the committee size increases.

4.4 Other Equilibria

In the equilibrium analysis we focused on proposer optimal equilibria, but now we turn to an example in which we show that there may exist another stationary equilibrium, in particular a more efficient one.

Consider the following example with a five person committee. Let $\alpha = q = 3$ and $\delta = 1$. Plugging these values into equation (17) we obtain that $k^* = 1$. This implies that $F(k^*) = 6$, $s_{\text{Prop}} = 8/15$, $s_{\text{Vote}} = 2/15$, and $s_{\text{cont}} = 1/3$, and the ex ante value of the game given by $V(1) = 9/5$. The proposer's payoff is $6 \times 8/15 = 48/15$.

Now I show that $k = 2$, in which three members produce and receive equal shares (s_{cont} $=1/3$, and $s_{\text{Prop}} = 1/3$) is a stationary equilibrium as well, but it yields a lower payoff to the proposer. When $k = 2$, the fund is equal to 9 and the stationary value of the game is $V(2) = 11/5$. The payoff to each member of the coalition, including the proposer, is 3.

Can the proposer increase her earnings and at the same time offer equity shares to a miminum winning coalition such that those in it will vote in favor? We show this is not possible and focus on the most profitable deviation which occurs when the proposer increases own equity and takes a non-productive coalition member against the continuation value. The proposer must offer a share s_{vote} such that $s_{\text{vote}} \cdot 6 + 1 = V(2) = 11/5 \Rightarrow s_{\text{vote}} = 1/5$.

Recall that a single deviation does not change the continuation value of the game due to stationarity. The proposer keeps 7/15 shares and obtains a payoff of $42/15 < 3$. This implies that the deviation was not profitable.

In this example it is clear that the multiplicity of equilibria is not due to the fact when optimizing the 15 one may obtain a $\bar{k}$ that is not an integer because the proposer optimal equilibria is exactly $k^* = \bar{k} = 2$ and in the more efficient equilibria is $k = 3$.

The conditions for finding the most efficient equilibrium are stated in Appendix B in Proposition 4. Unfortunately, we are unable to obtain a closed-form solution as we do for the proposer-optimal equilibrium.

5. Discussion on Bargaining over Endogenous and Exogenous Funds

In this section we analyze two issues. First we compare the distribution of final payoffs between the BF game with an exogenous fund and the game of predistribution in order to analyze when they coincide and when they differ. Second, we introduce an exogenous fund into the equity bargaining game in order to analyze how bargaining outcomes and efficiency may be affected. In particular we are able to show that, in the presence of an exogenous fund, rent-extraction incentives can significantly crowd out productive incentives.

5.1 Relationship between Exogenous and Endogenous Fund Bargaining Payoffs

In order to make the bargaining games with an endogenous and exogenous fund comparable, we must transform the standard BF game to allow for players to hold an initial endowment which cannot be consumed until an agreement has been reached. Also, the surplus to distribute will be defined as the surplus (total fund net of investments) that would be available in equilibrium under the predistributive equity bargaining game.

Proposition 2 Let there be n players, with a q voting rule and $\delta \in (0, 1]$ each endowed with one unit of wealth. Let $(k + 1)(\alpha - 1)$ with $\alpha > 1$ and k a non-negative integer be the total surplus to distribute. The equilibrium payoffs of the bargaining game including the initial wealth endowment are as follow:

1. The proposer offers $q-1$ members $\frac{\delta(k+1)(\alpha-1)}{n}$ and thus they earn a final payoff of $1+\frac{\delta(k+1)(\alpha-1)}{n}$;

2. $n-q$ members are offered 0 and thus they earn a final payoff of 1;

3. The proposer keeps the rest and earns $1+(k+1)(\alpha-1)-(q-1)\frac{\delta(k+1)(\alpha-1)}{n}$.

Corollary 4 The distribution of payoffs resulting from the Baron and Ferejohn game defined in Proposition 2 with $k=k^*$are equivalent to the distribution of payoffs in equity bargaining game if and only if $\delta=1$ and $k^*\leq\tilde{k}$.

The proof to the corollary relies on the fact that $s_{\text{Cont}}=\frac{\delta V}{F(k)}$ for $k\leq\tilde{k}$ and that non-productive coalition partners are offered a share $s_{\text{Vote}}=\frac{\delta V-1}{F(k)}$. These conditions imply that all coalition partners (except the proposer) earn a payoff that is equal to the continuation value of the game. It follows that $q-1$ coalition partners (except the proposer) earn $\delta V=\delta\left[\frac{(k+1)(\alpha-1)}{n}+1\right]$ which is equal to $\frac{\delta(k+1)(\alpha-1)}{n}+1$ (the payoff of a coalition member in the BF game with an exogenous fund) if and only if $\delta=1$.

5.2 Equity Bargaining with a Partially Exogenous Fund

In our equity bargaining model the total fund is determined by the sum of contributions multiplied times the productivity parameter α. Here we consider

$$F(k)=\alpha(k+1)+x \tag{18}$$

where $x\geq 0$ is an exogenous amount that is available to distribute regardless of players' investments and publicly known prior to the start of the bargaining rounds. In this case, the ex ante value of the game for any k is given by

$$V(m,k;q,n,\alpha)=\frac{F(k)}{n}+1-\frac{1+k}{n}=\frac{(\alpha-1)(k+1)}{n}+\frac{x}{n}+1 \tag{19}$$

which is equal to the one derived in equation (6) plus x/n (the average exogenous fund).[16] Solving the problem in the same manner as we did for $x = 0$ we obtain the optimal solution to problem (15) which is given by

$$\bar{k}_x := \bar{k} - \frac{x(n - \delta\alpha)}{2\alpha(n + \delta - \delta\alpha)} \quad . \tag{20}$$

In a similar fashion as before, one can easily specify the integer solution k_x^*.[17]

COROLLARY 5 In equilibrium we have that:

1. k_x^* is non-increasing in x;

2. $\lim_{x \to 0} k_x^* = k^*$.

The results are quite intuitive. If x is large enough, the proposer is better off by sacrificing productive members whose contributions to the total fund do not compensate for the extra equity that must be disbursed to them in order to incentivize investments. The proposer receives a larger payoff by offering them a lower share which is just enough to obtain their vote. Thus, the existence of an exogenous fund crowds out efficiency by exacerbating the proposer's rent-extraction incentives.

It should be noted that in the particular case in which $q = \alpha = n$ and $\delta = 1$ the total fund is constant in x and $k_x^* = k^*$. The reason is that in equilibrium, regardless of x, each member receives a share equal to $1/n = 1/\alpha$ which implies that everyone invests.

To explain part (2) of the previous corollary, note that $x = 0 \Rightarrow \bar{k}_x = \bar{k}$. However, the integer solution might coincide for small values of x. For this reason we have stated our equivalence result in terms of the limit as x approaches 0.

[16] Here we will be analyzing the case for which at least the proposer finds it optimal to invest. It should be noted that with a very large exogenous fund the proposer might not be able to retain a share large enough that incentivizes himself to invest. In order to derive a clear comparison with the previous findings we focus on the case in which the proposer produces. The more general case does not add any new insight nor does it change our results in Corollary 5.

[17] This is given by $\lfloor \bar{k}_x \rfloor$ or $\lfloor \bar{k}_x \rfloor + 1$ whichever is feasible and yields the largest payoff to the proposer. Whenever $\bar{k}_x < 0$ then $k_x^* = 0$.

6. Complementarities in Production

Thus far, we have assumed that there are no complementarities in production: investment choices by one player do not affect the marginal productivity of other players' investments. In this section we will reanalyze our model with a Leontief production function, an extreme case of synergies in which the minimum investment determines the total output. Specifically we consider the total fund given by

$$F = \alpha n \min\{c_1, ..., c_n\} \quad . \tag{21}$$

6.1 Equilibrium

Lemma 3 Let $\mathbf{s}^{\tau}$ be any vector of equity shares which has been approved. Then, the following hold:

1. If $\exists i$ such that $s_i < 1/\alpha n$ then $c_j^*(\mathbf{s}^{\tau}) = 0 \ \forall j$.

2. If $\forall i \ s_i \geq 1/\alpha n$ then any symmetric contribution choice is an equilibrium.

In what follows we will focus in subgames where a positive fund can be supported in equilibrium ($c_i = 1$ for all i). In such case, the total fund is given by

$$F = \alpha n \tag{22}$$

and each player's earnings (undiscounted) are given by $s_i \alpha n$. In a symmetric stationary equilibrium, the continuation value of the game is then given by

$$V(c; q, n, \alpha) = \alpha \tag{23}$$

which is the average fund.

Notice that in the additive setting we could potentially have players that vote in favor but do not invest, and these were denoted by s_{Vote}. This is no longer relevant here because everyone invests, thus we will now use s_{vote} to denote a player that votes in favor but also invests. For a proposal to be implemented it must be that at least q members including the proposer receive a share satisfying

$$s_{\text{vote}}F + 1 - c \geq \max\{\delta V, 1/\alpha n\} \quad . \tag{24}$$

This inequality states that the payoff resulting from the profit-sharing agreement must yield a payoff greater than or equal to the continuation value of the game. It will be binding in equilibrium because the proposer does not gain anything from offering more since no one will change her contribution.

Before we set up our maximization problem, we denote by m the number of members who receive s_{vote}. We then require that the equity feasibility constraint holds:

$$s_{\text{prop}} + m s_{\text{vote}} + (n - m - 1)\left(\frac{1}{\alpha n}\right) = 1 \quad . \tag{25}$$

The proposer's problem becomes

$$\max_{s_{\text{prop}}, s_{\text{vote}}, m} s_{\text{prop}}F - c + 1 \tag{26}$$

$$\text{s.t. } (25) \quad . \tag{27}$$

PROPOSITION 3 If the production function is characterized by a symmetric Leontief function, then the unique SSPE consistent with positive production satisfies the following properties

1. The proposer assigns $s^*_{\text{vote}} = \begin{cases} \frac{\delta}{n} & \text{if } \delta \geq 1/\alpha \\ \frac{1}{\alpha n} & \text{otherwise} \end{cases}$ to exactly $q - 1$ members. The remaining $n - q + 1$ members receive $\frac{1}{\alpha n}$ and the proposer keeps the rest.

2. All members receiving $s_i \geq s^*_{\text{vote}}$ vote in favor.

3. There is no delay in approval.

The equilibrium characterization is quite intuitive. If players are very impatient ($\delta < 1/\alpha$) then the proposer needs to worry about offering only the smallest productive incentive to all members, because a rejection would entail a high cost. In such case, all members receive $1/\alpha n$ and vote in favor. If players are patient enough then the proposer offers a share that guarantees a payoff equal to the continuation value of the game to a minimum winning coalition of members who vote in favor.

Notice that in the linear case, for each possible distribution of shares, there exists a unique vector of equilibrium contributions. In this case, if all members receive a share greater than or equal to $1/\alpha n$ players face a coordination dilemma at the production stage. Thus, even if the approved equity scheme is consistent with the SSPE predictions, no player investing remains an equilibrium of the subgame.

7. Discussion and Concluding Remarks

The trade-off between productive and appropriative activities has been the object of study in several contexts of political economy. For example, Hirschleifer (1991), Skaperdas (1992), and Grossman and Kim (1995) develop models in which players can invest in *swords* as a means to appropriate others' production or invest in *plowshares* that yield a given output. In these settings, too many swords are useless when there is little production, so that in equilibrium the marginal investment in appropriation activities yields the same return as the marginal investment in production. Autocrats face a similar trade-off when determining property rights over their subjects in order to maximize their rents. Olson (1993) explains how a stable monarchy will have stronger long run incentives to create and respect property rights compared to a "roving bandit". North and Weingast (1989) argue that "it is not always in the ruler's interest to use power arbitrarily or indiscriminately; by striking a bargain with constituents that provides them some security, the state can often increase its revenue (pg. 806)".[18] Our model assumes that property rights will be respected, hence we have focused on how rights will be assigned and not the extent to which they will be upheld. Moreover, players may only appropriate what has been contributed to the common project through a preestablished agreement of ownership but may not appropriate other's endowments.

In our main analysis we have employed a linear and symmetric production technology. The symmetry assumption has been useful in solving the equilibrium in the outcome space without having to explicitly characterize bargaining strategies. As such, we have benefitted from well-established results about the stationary value of the game in a symmetric setting. If one were to consider asymmetric partners, the equilibrium strategies must be explicitly solved for (i.e., the probability of each player of being included or not in a winning coalition and the resulting stationary value of the game for each player as a function of her characteristics). For example, a partner with very high productivity is more valuable when properly incentivized to produce compared to a low-productivity partner. The highly productive partner would then be enticed to ask for a higher portion of the pie, but such an increase in the demand for rents might decrease his probability of inclusion in the winning coalition. Recall that in our setting, a partner can be productive or non-productive (and payoffs resulting from each role are generally different) which makes the problem more complex. We conjecture that partners with higher productivities or endowments are more likely to be invited to the coalition as productive than as non-productive partners.

Our results confirm a common intuition that committees with higher voting requirements are less efficient. In our model, this trade-off takes place because much of the available equity must be used for buying votes instead of fostering investments, which evidences how the bargaining process can take a toll on efficiency. Importantly, this result relies on the fact that partner productivity is not affected by the number of agreeing partners which again

[18] The authors provide a historical account of how the possibility to limit the English monarch's confiscatory power after the Glorious Revolution lead to an increase in capital flows in the eighteenth century.

underscores the relevance of productive synergies.

In many examples of teamwork or joint production that one can conceive of, complementarities are certainly present. We solve the model using a Leontief function where the smallest investment determines the total output which is clearly an extreme case of synergies. In this case, we identify a set of equity agreements that can sustain positive production and solve for the SSPE equity scheme. Here, all members receive a positive share because if at least one member is not offered a share that incentivizes production the total output would be zero. However, only the minimum required number of voters receives a share large enough to obtain approval, again creating a two-tier scheme of equity owners.

While contract theory has mostly been focused on how to provide the right incentives to create a surplus, bargaining theory has been mostly concerned with how to redistribute an existing surplus. The model of pre-distributive bargaining is a first step in filling this gap. Future research could focus on varying the production technology, information structure, or bargaining protocol to better understand the trade-off between rent-generation and rent-extraction in the multilateral bargaining context, a setting that resembles many economic and political activities.

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A. Proofs

A.1 Positive production in equilibrium

Lemma 4 In equilibrium, at least one member will produce.

Proof. Suppose that $\mathbf{s}$ is an equilibrium allocation of shares such that $\alpha_i s_i < 1$ for all i. Then, the equilibrium payoffs of the game are 1 for each player. A proposer can unilaterally deviate by giving ϵ to $q-1$ members where ϵ is small and keeping $1-q\epsilon > 1/\alpha$. The proposal will be approved and it yields a higher payoff to some members because the proposer is incentivized to produce. ■

A.2 Derivation of the Optimal k.

Lemma 5 For $k \in [0, \lfloor\alpha\rfloor - 1] \subset \mathbb{R}$ the function

$$\Pi(k) := \begin{cases} \left[1 - k \cdot \frac{1}{\alpha} - (q-1-k)\left(\frac{\delta V - 1}{F(k)}\right)\right] \cdot F(k) & \text{if} \quad k \in [\tilde{k}, \lfloor\alpha\rfloor] \\ \left[1 - k \cdot \frac{\delta V}{F(k)} - (q-1-k)\left(\frac{\delta V - 1}{F(k)}\right)\right] \cdot F(k) & \text{if} \quad k \in [0, \tilde{k}] \end{cases} \tag{28}$$

has the following properties:

Lemma 6

1. It is linear and increasing in $k \in [0, \tilde{k}]$.
2. It is quadratic in $k \in [\tilde{k}, \lfloor\alpha\rfloor]$with a maximum at $\bar{k}$.
3. It is continuous at $k = \tilde{k}$.

4. $\tilde{k} \leq \bar{k}$ with equality if and only if $\delta = 1$ and $q = n$.

Proof. The proofs for (1)-(3) are simple arithmetic computations. For (4) we have that

$$\begin{aligned}
\tilde{k} &\leq \bar{k} \iff \\
\frac{\delta n}{n-\delta(\alpha-1)} - 1 &\leq \frac{1}{2}\frac{n(\alpha+\delta)-\delta q(\alpha-1)}{n-\delta(\alpha-1)} - 1 \iff \\
\delta q(\alpha-1) &\leq n(\alpha-\delta)
\end{aligned}$$

and the result follows because $\delta q \leq n$ and $\alpha - 1 \leq \alpha - \delta$. Equalities only hold when $\delta = 1$ and $q = n$. ∎

A.3 Proof of Corollary 3

Let $\bar{k}^M$ and $\bar{k}^U$ denote $\bar{k}$ given by (16) under the simple majority and unanimity rules. For the majority rule we are evaluating at $q = (n+1)/2$ and for the unanimity at $q = n$. We have that

$$\frac{\partial \bar{k}^M}{\partial n} = -\frac{(\alpha-1)[\alpha(2-\delta)+3\delta-1)]}{4(n+\delta-\alpha\delta)^2}$$

and this is always negative because $\alpha(2-\delta)+3\delta-1 > 0$ given that $\alpha > 1$ and $2-\delta > 1$ thus $\alpha(2-\delta) > 1$.

Similarly for the unanimity rule we have that

$$\frac{\partial \bar{k}^U}{\partial n} = -\frac{(\alpha-1)[\alpha(1-\delta)+2\delta)]}{2(n+\delta-\alpha\delta)^2}$$

and this equation is always negative.

A.4 Upper bound for $\bar{k}$

Corollary 6 We have that $\alpha - 1 \geq \bar{k}$.

Proof. By Corollary 2 $\bar{k}$ can be increasing or decreasing in δ, thus we examine the lowest and highest possible values of $\bar{k}$. We define $\bar{k}_1 := \bar{k}\big|_{\delta=1}$ and $\bar{k}_2 := \bar{k}\big|_{\delta=\frac{n}{n+\alpha-1}}$. Now we compute

$$
\begin{aligned}
g_1 &: = \alpha - 1 - \bar{k}_1 = \frac{(\alpha-1)(n+q-2\alpha)}{2(n-\alpha+1)} \\
g_2 &: = \alpha - 1 - \bar{k}_2 = \frac{(\alpha-1)(q+n-\alpha)}{2n}
\end{aligned}
$$

and conclude that when $\alpha \to 1$ both g_1 and g_2 approach 0 meaning that efficient production will coincide with the proposer optimal solution. When $\delta = 1$ we have that the proposer optimal solution will coincide with the socially optimal production for cases where α is close to q and q is close to n. ■

A.5 Proof of Corollary 1

PROOF. Let $k = n-1$. It follows that the proposer's payoff is $\pi_{\text{Prop}}(k = n-1) = n$. Now let $k = n-2$, one member is offered $s_{\text{vote}} < 1/n$. The proposer's payoff is given by $\pi_{\text{Prop}}(k = n-2) = (2n^2 - n - d + dn - dn^2)/n$. We have that

$$
\pi_{\text{Prop}}(k = n-1) \geq \pi_{\text{Prop}}(k = n-2) \iff (-n^2 + n + d - dn + dn^2)/n > 0
$$

if and only if $\delta \geq (n-1)n/(1-n+n^2) = \delta(n)$. One can easily show that $\delta(n)$ increases in n and is bounded from above by 1 and from below by 6/7 (because $n \geq 3$). ■

B. Other Stationary Equilibria

We are not able to provide a closed-form solution to the problem of finding the most efficient stationary equilibrium but in the following proposition we provide a characterization of equilibrium strategies.

PROPOSITION 4 Let $\hat{\boldsymbol{\sigma}} = (\hat{\boldsymbol{\sigma}}_i, \hat{\boldsymbol{\sigma}}_{-i})$ be a stationary strategy profile where $\hat{k}$ members are incentivized and the stationary values of the game are given by $\hat{V} := V(\hat{k})$. Then $\hat{\boldsymbol{\sigma}}$ is the most efficient equilibrium if the equilibrium strategies satisfy following conditions:

1. $s_{\text{Prop}}(\hat{k}, \hat{V}) = 1 - \hat{k} \cdot s_{\text{Cont}}(\hat{k}, \hat{V}) - m s_{\text{Vote}}(\hat{k}, \hat{V})$, $s_{\text{Cont}}(\hat{k}, \hat{V}) = \max \left\{ \frac{1}{\alpha}, \frac{\delta \hat{V}}{\hat{k}} \right\}$, $s_{\text{Vote}}(\hat{k}, \hat{V}) = \frac{\delta \hat{V} - 1}{F(\hat{k})}$.

2. A member votes in favor if and only if $s_{\text{Cont}} F(\hat{k}) \geq \delta \hat{V}$, $s_{\text{Vote}}(\hat{k}, \hat{V}) F(\hat{k}) + 1 \geq \delta \hat{V}$, or $s_{\text{Prop}}(\hat{k}, \hat{V}) F(\hat{k}) \geq \delta \hat{V}$ and the proposal receives q votes ($q = m + \hat{k} + 1$).

3. Given $\hat{\boldsymbol{\sigma}}_{-i}$, $\hat{k} = \arg\max_{k \in \{0, \ldots, \lfloor \alpha \rfloor - 1\}} s_{\text{Prop}}(k, \hat{V}) F(k)$ subject to conditions (1) and (2) above.

4. $\hat{k} = \arg\max_{k \in \{0, \ldots, \lfloor \alpha \rfloor - 1\}} F(k)$ subject to conditions (1), (2), and (3) above.

Proof. Conditions (1) and (2) state that no resources should be wasted, otherwise, this could result in lower efficiency or the proposer could improve his position (for a given $\hat{V}$) without failing to obtain the majority vote. Hence, productive members are offered the smallest share that induces contribution and non-productive coalition partners are offered a share that yields exactly the continuation value. Condition (3) states that the proposer cannot deviate to any other equity scheme and earn a higher profit while still obtaining the majority vote. Finally, condition (4) states that we choose the highest amount of productive members for which the previous conditions hold. Existence is guaranteed because the equilibrium characterized in Proposition 1 satisfies (1)-(3), thus we know that $\hat{k} \geq k^*$. ■

C. Bargaining over Complete Contracts

In the body of the article, we considered a setting in which member's compensations could not be conditioned on contribution levels. We now relax this assumption and model a situation in which the proposer can offer members a contract of the form $f_i(c_i) = a_i c_i$ where a_i specifies the compensation per unit contributed by player i. A proposal in period t is denoted by $\mathbf{a}^t = (a_2^t, ...a_n^t)$ where the i^{th} entry is player i's contract. Without loss of generality we identify the proposer with the player index $i = 1$. In order to avoid unnecessary theoretical complications, we simply define the proposer as the residual claimant, i.e. his payoff is defined as the amount remaining after paying out contracts based on contributions.

At a terminal bargaining node τ in which the approved proposal is $\mathbf{a}^\tau$, a player's optimal contribution strategy is given by

$$c_i^*(\mathbf{a}^\tau) = \begin{cases} 0 & \text{if} \quad a_i^\tau < 1 \\ 1 & \text{if} \quad a_i^\tau \geq 1 \end{cases} . \tag{29}$$

The total fund is given by $F(k_c) = \alpha(1 + k_c)$ where k_c is the number of productive members excluding the proposer (the subscript c is used to denote the fact that complete contracts are feasible).

Proposition 5 In equilibrium, $n - q$ members receive $a_i = 1$; $q - 1$ members receive $a_i = \max\{\delta\alpha, 1\}$ and $k_c^* = n$. A member votes in favor if and only if $a_i \geq \delta\alpha$. There is no delay in approval.

Intuitively, note that the smallest a that induces contributions is $a = 1$. For each possible number of productive members, it always pays to incentivize an additional partner because the proposer appropriates a portion of the generated rents and loses nothing. Hence, everyone will obtain a contract in which a is at least 1. In order to meet the voting quota the proposer must offer a contract such that $q - 1$ members will vote in favor. The minimum winning coalition receives a contract that yields a payoff equal to the continuation value of the game. The proof is below.

Proof. We will solve the model where the proposer's (assigned player index 1) contract is defined as a transfer irrespective of his contribution. Thus, we have that the proposer's payoff is given by $\sum_{i=1}^{n} \alpha_i c_i - \sum_{i=2}^{n} a_i c_i$. This simplifies the analysis because it guarantees that the partnership is always solvent to honor the contracts offered (we allow for negative earnings). Define $\Lambda := \{i > 1 \text{ s.t. } a_i \geq \delta V\}$, which represents the set of those who will vote in favor and $\Omega := \{i > 1 \text{ s.t. } a_i \geq 1\}$ represents the set of incentivized members besides the proposer. Clearly, $\Lambda \subset \Omega$ because $\delta V > 1$ (Assumption 1).

As before, we are characterizing equilibria in which the proposer produces. Let $k_c := |\Omega|$ and $F(k_c) = \alpha(k_c + 1)$ denote the size of the fund.The voting approval constraint is given by

$$|\Lambda| \geq q - 1 \tag{30}$$

and the proposer's maximization problem is given by

$$\begin{aligned} \max_{k_c, \{a_i\}_{i=2}^n} & F(k_c) - \sum_{i \in \Omega} a_i \\ \text{s.t. } & (30) \quad . \end{aligned} \tag{31}$$

In the problem above we have imposed equilibrium behavior in the subgame in the sense that $c_i = 1$ if $i \in \Omega$.

It is clear that condition (30) binds because the proposer's payoff decreases in a_i. Hence, we have that there will be $q-1$ members receiving $\bar{a}_i(k_c) = \max\left\{\delta\left[\frac{(\alpha-1)(k_c+1)}{n} + 1\right], 1\right\}$. It is useful to note that $F(k_c) - \sum_{i \in \Omega} a_i$ is increasing in k_c as long as $a_i \leq \alpha$ (i.e. the contract offers a compensation lower than the member's productivity). It follows that $k_c^* = n - 1$. Given that the proposer already has the necessary votes and that his payoff decreases in a_i, he chooses $a_i = 1$ for the remaining $n - q$ partners.

In equilibrium $\bar{a}_i^* := a_i(k_c^*) = \max\{\delta\alpha, 1\}$ and the proposer's payoff if given by $\alpha n - (q - 1)\max\{\delta\alpha, 1\} - (n - q - 1)$. It is straightforward to show that his payoff increases in n and decreases in q (constant in q when $\delta\alpha < 1$). ■

For comparison with the equity bargaining model, Corollary 7 contains the comparative statics.

Corollary 7 In equilibrium, the following results hold about $F(k_c^*)$:

1. it is constant in q
2. it is increasing in n;
3. it is constant in δ;
4. it is increasing in α.

The equivalence of payoff distribution between the contract bargaining game and the standard BF redistributive game holds for a broader range of parameters than for the equity bargaining game but it is still a necessary condition that $\delta = 1$.[19]

Corollary 8 The distribution of payoffs resulting from the Baron and Ferejohn game defined in Proposition 2 with $k = k_c^*$ are equivalent to the distribution of payoffs in contract bargaining game if and only if $\delta = 1$.

[19] If we are only concerned about the distribution of the surplus, namely $\frac{\delta(k+1)(\alpha-1)}{n}$ (i.e. we do not take into accound initial endowments), then the only required condition for equivalence between the endogenous and exogenous fund cases is that $k^* \leq \tilde{k}$.