CONFORMAL METHOD TO ELIMINATE THE ADI-FDTD STAIR-CASING ERRORS

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Abstract

In this work, a novel ADI- CFDTD technique is presented. The proposed technique is a combination of ADI concept with CFDTD analysis method. The proposed method restores the second order accuracy with high stability than ADI-CFTD. ADI-CFDTD suits well to overcome the limitation of existed ADI-FDTD for analysis of curvilinear objects. By controlling wave propagation confining to the object boundary and avoids wave leakage performance will enhanced. In this study, ADI-CFDTD applied on circular microstrip patch antenna to evaluate the performance. Obtained results are compared with CST Microwave CAD model results. Both the results are good in agreement. The proposed ADI-CFDTD method helps to reduce the computational complexity.

I Introduction

In FDTD technique the time step '*Δt*' plays an important role; however it is limited by the CFL stability condition. The FDTD method becomes unstable when the step size exceeds the CFL limits. In order to overcome this drawback, the Alternative Direction Implicit (ADI) FDTD method is introduced. The ADI FDTD method [1-2] is unconditionally stable for any time step size. The spatial discretization in FDTD method provides inaccurate outcomes for curved objects which is known as staircase error. This staircase error becomes larger in ADI FDTD for curved objects. This limitation of ADI FDTD has been addressed in many research articles. In this chapter Conformal based ADI FDTD (ADI CFDTD) method is introduced. A circular patch antenna array is used to evaluate proposed conformal ADI FDTD. The proposed method provides good results as compared with antenna CAD tools.

In this work a novel ADI Conformal FDTD (ADI-CFDTD) technique [3] is proposed for restoring the second order accuracy with high stability in the ADI-FDTD technique. Mainly, the CFDTD is applied as a local correction to the ADI-FDTD technique. This correction enhances the local finite difference approximation accuracy from the zero order to the first order, and the global accuracy from the first order to the second order because of the super-convergence. The proposed method can be used for many CEM applications where the curved objects are integrated in a system. The proposed technique is capable in reducing the spatial sample density and computational complexity of CEM applications.

The ADI methods have been proposed n by the authors Peaceman and Rachford in the year 1955 [4]. These methods were mainly developed for getting the effective numerical outcomes of elliptic and parabolic partial differential equations.

The heat equation can be represented as follows

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \quad (1)$$

The best example of parabolic differential equation, over the mesh superimposed on the rectangular region $0 \leq x \leq a, 0 \leq y \leq b$.

By utilising the space time notation

$$u_{l-j}^n = u(i\Delta x, j\Delta y, n\Delta t) \quad (2)$$

The simple explicit finite-difference strategy for the solution is expressed as

$$\frac{u_{l.j}^{n+1} - u_{i.j}^n}{\Delta t} = \frac{u_{i+lj}^n - 2u_{i,j}^n + u_{i-l,j}^n}{\Delta x^2} + \frac{u_{i.j}^n - 2u_{i,j}^n + u_{i,j-l}^n}{\Delta y^2} \quad (3)$$

Even though the above explicit representation appears simple and straight forward for solutions, it is limited by the following CFL condition for stability i.e.

$$\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} \leq \frac{1}{2\Delta t} \quad (4)$$

The perfect time matching is achieved by selecting a very small Δt value which results in accurate outcome. In the implicit technique, the iterations are necessary to be long for a solution to settle. This is major drawbacks in implicit technique.

In the ADI approach [5], each time step is partitioned into two sub-portions i.e., the n^{th} and the *[n+1/2]*th steps. In the first half step, the second derivative, $\frac{\partial^2 u}{\partial y^2}$, is approximated at the n^{th} iteration by the finite difference replacement. The first, second order derivative, $\frac{\partial^2 u}{\partial x^2}$, is replaced by the *[n+1/2]*th iteration. A pair of simulation equations are implicit in the *x*-direction. In the process of converting from the intermediate iteration, *n+1/2* to *n+1* iteration, the difference equation gets implicit in the y-direction and explicit in the x-direction. Especially, the two sub-computations are:

$$\frac{u_{i,j}^{n+\frac{1}{2}} - u_{i,j}^{n}}{\Delta t/_2} = \frac{u_{i+1,j}^{n+\frac{1}{2}} - 2u_{i,j}^{n+\frac{1}{2}} + u_{i-1,j}^{n+\frac{1}{2}}}{\Delta x^2} + \frac{u_{i,j+1}^{n} - 2u_{i,j}^{n} + u_{i,j-1}^{n}}{\Delta y^2} \qquad (5)$$

$$\frac{u_{i,j}^{n+1} - u_{i,j}^{n+\frac{1}{2}}}{\Delta t/_2} = \frac{u_{i+1,j}^{n+\frac{1}{2}} - 2u_{i,j}^{n+\frac{1}{2}} + u_{i-1,j}^{n+\frac{1}{2}}}{\Delta x^2} + \frac{u_{i,j+1}^{n+1} - 2u_{i,j}^{n+1} + u_{i,j-1}^{n+1}}{\Delta y^2} \qquad (6)$$

The above equation is chosen as unconditionally stable and realization can be similar to that which has been utilized by the O'Brien, Hyman, and Kaplan [6].

II Conventional ADI- FDTD Algorithm:

The CFL condition for FDTD results in upper bound value of the time step. To eliminating the CFL condition, the above mentioned ADI principle is integrated with FDTD method which leads to an unconditionally stable FDTD strategy, called ADI-FDTD technique.

2.1 Derivation of ADI-FDTD Scheme

As specified in the previous sections, in an isotropic medium with medium permittivity ε and medium permeabilityμ, the curl vector equation of Maxwell's equations can be expressed in six scalar partial differential equations in the Cartesian coordinates as specified in the previous equation. The initial equations are as follows:

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) \qquad (7a)$$

$$\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right) \qquad (7b)$$

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) \qquad (7c)$$

$$\frac{\partial H_x}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y}\right) \qquad (7d)$$

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right) \qquad (7e)$$

$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\right) \qquad (7f)$$

After applying ADI method to the equation 5.7a the time step of FDTD n to n+1 is segregated into two sub- step calculations which is nth to n+1/2 and n+1/2 to n+1 time step.

The two sub-step calculations are elaborated as follows

For the first sub-step $(i.e. at the (n+1/2) th time step), the$first patial derivation on the right-hand side of 5.7a, i.e $\frac{\partial H_z}{\partial y}$, is replaced with an implicit difference approximation of its unknown pivotal values at the [n+1/2]th time step, while the second derivatives on the right-hand side, i.e $\frac{\partial H_y}{\partial z}$, is replaced with an explicit finite difference approximation in its known values at the previous n-th time step. In the other words, equation 5.7a becomes

$$\frac{{E_x}_{i+\frac{1}{2},j,k}^{n-\frac{1}{2}} - {E_x}_{i+\frac{1}{2},j,k}^{n}}{\Delta t/_2} = \frac{1}{\varepsilon}\left[\frac{{Hz}_{i+\frac{1}{2},j+\frac{1}{2},k}^{n-\frac{1}{2}} - {Hz}_{i+\frac{1}{2},j-\frac{1}{2},k}^{n-\frac{1}{2}}}{\Delta y} - \frac{{Hy}_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n-\frac{1}{2}} - {Hz}_{i+\frac{1}{2},j,k-\frac{1}{2}}^{n-\frac{1}{2}}}{\Delta z}\right] \qquad (8)$$

For the second sub-step (i.e at (n+1)-the time step), the second term on the right-hand side, ${dHy}/_{dz}$, is replaced by an implicit finite-difference approximation of its unknown pivotal values at (n+1)-th time step; while the first term, ${dHz}/_{dy}$, is replaced with an explicit finite-difference approximation in its known values at the previous (n+1/2)-th time step.

Similarly, equation (5.7a) evolves to

$$\frac{{E_x}_{i+\frac{1}{2},j,k}^{n+\frac{1}{2}} - {E_x}_{i+\frac{1}{2},j,k}^{n+1/2}}{\Delta t/_2} = \frac{1}{\varepsilon}\left[\frac{{Hz}_{i+\frac{1}{2},j+\frac{1}{2},k}^{n+\frac{1}{2}} - {Hz}_{i+\frac{1}{2},j-\frac{1}{2},k}^{n+\frac{1}{2}}}{\Delta y} - \frac{{Hy}_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n+1} - {Hy}_{i+\frac{1}{2},j,k-\frac{1}{2}}^{n+1}}{\Delta z}\right] \qquad (9)$$

Note that the above two sub-steps represents the alternations in the FDTD recursive computation directions in the sequence of the terms, the first and the second terms. They result in the implicit formulations as the right-hand sides of the equations contain the field values which are unknown and need to be updated. The method is then termed “the Alternating Direction Implicit (ADI)” method.

By applying the same process to all the remaining scalar differential equations as mentioned in 5.13 and 5.14, one can acquire the entire set of the implicit unconditionally stable FDTD equations.

For converting the nth time step equation to the [n+1/2]th time step:

$$\frac{Ex^{n+\frac{1}{2}}_{i+\frac{1}{2},j,k}-Ex^{n}_{i+\frac{1}{2},j,k}}{\Delta t/2}=\frac{1}{\varepsilon}\left[\frac{Hz^{n+\frac{1}{2}}_{i-\frac{1}{2},j+\frac{1}{2},k}-Hz^{n+\frac{1}{2}}_{i+\frac{1}{2},j-\frac{1}{2},k}}{\Delta y}-\frac{Hy^{n-\frac{1}{2}}_{i+\frac{1}{2},j,k+\frac{1}{2}}-Hy^{n}_{i+\frac{1}{2},j,k-\frac{1}{2}}}{\Delta z}\right] \quad (10a)$$

$$\frac{Ey^{n+\frac{1}{2}}_{i,\frac{1}{2}+j,k+\frac{1}{2}}-Ey^{n}_{i,\frac{1}{2}+j,k}}{\Delta t/2}=\frac{1}{\varepsilon}\left[\frac{Hx^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k+\frac{1}{2}}-Hx^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k-\frac{1}{2}}}{\Delta z}-\frac{Hz^{n}_{i+\frac{1}{2},j+\frac{1}{2},k}-Hz^{n}_{i-\frac{1}{2},j+\frac{1}{2},k}}{\Delta x}\right] \quad (10b)$$

$$\frac{Ez^{n+\frac{1}{2}}_{i,j,k+\frac{1}{2}}-Ez^{n}_{i,j,k+\frac{1}{2}}}{\Delta t/2}=\frac{1}{\varepsilon}\left[\frac{Hy^{n+\frac{1}{2}}_{i+\frac{1}{2},j,k+\frac{1}{2}}-Hy^{n+\frac{1}{2}}_{i-\frac{1}{2},j,k+\frac{1}{2}}}{\Delta x}-\frac{Hx^{n}_{i,j+\frac{1}{2},k+\frac{1}{2}}-Hx^{n}_{i,j-\frac{1}{2},k+\frac{1}{2}}}{\Delta y}\right] \quad (10c)$$

$$\frac{Hx^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k+\frac{1}{2}}-Hx^{n}_{i,j+\frac{1}{2},k+\frac{1}{2}}}{\Delta t/2}=\frac{1}{\mu}\left[\frac{Ey^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k+1}-Ey^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k}}{\Delta z}-\frac{Ez^{n}_{i,j+1,k+\frac{1}{2}}-Ez^{n}_{i,j,k+\frac{1}{2}}}{\Delta y}\right] \quad (10d)$$

$$\frac{Hy^{n+\frac{1}{2}}_{i+\frac{1}{2},j,k+\frac{1}{2}}-Hy^{n}_{i+\frac{1}{2},j,k+\frac{1}{2}}}{\Delta t/2}=\frac{1}{\mu}\left[\frac{Ez^{n+\frac{1}{2}}_{i+1,j,k+\frac{1}{2}}-Ez^{n+\frac{1}{2}}_{i,j,k+\frac{1}{2}}}{\Delta x}-\frac{Ex^{n}_{i+\frac{1}{2},j,k+1}-Ex^{n}_{i+\frac{1}{2},j,k}}{\Delta z}\right] \quad (10e)$$

$$\frac{Hz^{n+\frac{1}{2}}_{i+\frac{1}{2},j+\frac{1}{2},k}-Hz^{n}_{i+\frac{1}{2},j+\frac{1}{2},k}}{\Delta t/2}=\frac{1}{\mu}\left[\frac{Ex^{n+\frac{1}{2}}_{i+\frac{1}{2},j+1,k}-Ex^{n+\frac{1}{2}}_{i+\frac{1}{2},j,k}}{\Delta y}-\frac{Ey^{n}_{i+1,j+\frac{1}{2},k}-Ey^{n}_{i,j+\frac{1}{2},k}}{\Delta x}\right] \quad (10f)$$

Conversion of [n+1/2]th to the [n+1]th time step

$$\frac{Ex^{n+1}_{i+\frac{1}{2},j,k}-Ex^{n+\frac{1}{2}}_{i+\frac{1}{2},j,k}}{\Delta t/2}=\frac{1}{\varepsilon}\left[\frac{Hz^{n+\frac{1}{2}}_{i+\frac{1}{2},j+\frac{1}{2},k}-Hz^{n+\frac{1}{2}}_{i+\frac{1}{2},j-\frac{1}{2},k}}{\Delta y}-\frac{Hy^{n+1}_{i+\frac{1}{2},j,k+\frac{1}{2}}-Hy^{n+1}_{i+\frac{1}{2},j,k-\frac{1}{2}}}{\Delta z}\right] \quad (11a)$$

$$\frac{Ey^{n+1}_{i,\frac{1}{2}+j,k}-Ey^{n+\frac{1}{2}}_{i,\frac{1}{2}+j,k}}{\Delta t/2}=\frac{1}{\varepsilon}\left[\frac{Hx^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k+\frac{1}{2}}-Hx^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k-\frac{1}{2}}}{\Delta z}-\frac{Hz^{n+1}_{i+\frac{1}{2},j+\frac{1}{2},k}-Hz^{n+1}_{i-\frac{1}{2},j+\frac{1}{2},k}}{\Delta x}\right] \quad (11b)$$

$$\frac{Ez^{n+1}_{i,j,k+\frac{1}{2}}-Ez^{n+\frac{1}{2}}_{i,j,k+\frac{1}{2}}}{\Delta t/2}=\frac{1}{\varepsilon}\left[\frac{Hy^{n+\frac{1}{2}}_{i+\frac{1}{2},j,k+\frac{1}{2}}-Hy^{n+\frac{1}{2}}_{i-\frac{1}{2},j,k+\frac{1}{2}}}{\Delta x}-\frac{Hx^{n+1}_{i,j+\frac{1}{2},k+\frac{1}{2}}-Hx^{n+1}_{i,j-\frac{1}{2},k+\frac{1}{2}}}{\Delta y}\right] \quad (11c)$$

$$\frac{Hx^{n+1}_{i,j+\frac{1}{2},k+\frac{1}{2}}-Hx^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k+\frac{1}{2}}}{\Delta t/2}=\frac{1}{\mu}\left[\frac{Ey^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k+1}-Ey^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k}}{\Delta z}-\frac{Ez^{n+1}_{i,j+1,k+\frac{1}{2}}-Ez^{n+1}_{i,j,k+\frac{1}{2}}}{\Delta y}\right] \quad (11d)$$

$$\frac{Hy_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n+1} - Hy_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n+\frac{1}{2}}}{\Delta t/2} = \frac{1}{\mu}\left[\frac{Ez_{i+1,j,k+\frac{1}{2}}^{n+\frac{1}{2}} - Ey_{i,j,k+\frac{1}{2}}^{n+\frac{1}{2}}}{\Delta x} - \frac{Ex_{i+\frac{1}{2},j,k+1}^{n+1} - Ex_{i+\frac{1}{2},j,k}^{n+1}}{\Delta z}\right] \quad (11e)$$

$$\frac{Hz_{i+\frac{1}{2},j+\frac{1}{2},k}^{n+1} - Hz_{i+\frac{1}{2},j+\frac{1}{2},k}^{n+\frac{1}{2}}}{\Delta t/2} = \frac{1}{\mu}\left[\frac{Ex_{i+\frac{1}{2},j+1,k}^{n+\frac{1}{2}} - Ex_{i+\frac{1}{2},j,k}^{n+\frac{1}{2}}}{\Delta y} - \frac{Ey_{i+1,j+\frac{1}{2},k}^{n+1} - Ey_{i,j+\frac{1}{2},k}^{n+1}}{\Delta x}\right] \quad (11f)$$

Here the field components are represented with the notations $E_{\propto}{}_{i,j,k}^{n}$ and $H_{\propto}{}_{i,j,k}^{n}$ $with \propto = x, y, z$. The grid positions of the field components are similar to those of the conventional FDTD of the Yee's method. When a large step size is considered which results a large number of numerical dispersion errors in ADI-FDTD technique. The spatial discretization in ADI-FDTD technique provides accurate results for first order discontinuous media. The increased error in the spatial discretization is coupled to the large error temporal discretization which increase the error rate in ADI-FDTD technique for discontinues media. The ADI-FDTD technique fails when conformal structures are considered as an object. In order to overcome these drawbacks the ADI technique is integrating with CFDTD technique for the reason of enhancing the accuracy in the conventional ADI-FDTD technique.

III Proposed ADI-CFDTD technique

Integrating ADI FDTD with CFDTD technique can helpful to enhance accuracy for the analysis of conformal structures. With this scope ADI CFDTD is proposed and same is discussed here. In the ADI-CFDTD technique the updating equations at regular grid points are similar to the ADI method. The electric field equations which are Ex, Ey and Ez updating equations are also identical to the ADI method at conformal boundaries.

The update equation for E_x from the time step n to n+1/2 is expressed in equation

$$E_x^{n+1/2}(i,j,k) = A(i,j,k).E_x^n(i,j,k) + B(i,j,k).\left[\frac{H_Z^{n+1/2}(i,j,k) - H_Z^{n+1/2}(i,j-1,k)}{\Delta y} - \frac{H_y^n(i,j,k) - H_y^n(i,j,k-1)}{\Delta z}\right] \quad (12)$$

The update equation for E_x from the time step n+1/2 to n+1 is expressed in equation

$$E_x^{n+1}(i,j,k) = A(i,j,k).E_x^{n+1/2}(i,j,k) + B(i,j,k).\left[\frac{H_Z^{n+1/2}(i,j,k) - H_Z^{n+1/2}(i,j-1,k)}{\Delta y} - \frac{H_y^{n+1}(i,j,k) - H_y^{n+1}(i,j,k-1)}{\Delta z}\right] \quad (13)$$

Similarly update equations for $E_y and E_z$are similar to the ADI- FDTD technique.

In the proposed ADI- CFDTD technique the Magnetic field components are modified at conformal gird points and corresponding H-Field updated equations are mentioned below:

The magnetic field equations which are Hx, Hy and Hz from the time step n to n+1/2 can be written as :

$$H_x^{n+1/2}(i,j,k) = H_x^n(i,j,k) + \frac{C(i,j,k)}{S_{yz}(i,j,k)}.\left[E_y^{n+1/2}(i,j,k+1)l_y(i,j,k) - E_y^{n+1/2}(i,j,k)l_y(i,j,k) - E_z^n(i,j+1,k)l_z(i,j,k) + E_z^n(i,j+1,k)l_z(i,j,k)\right] \quad (14)$$

$$H_y^{n+1/2}(i,j,k) = H_y^n(i,j,k) + \frac{C(i,j,k)}{S_{zx}(i,j,k)}.\left[E_z^{n+1/2}(i+1,j,k)l_z(i+1,j,k) - E_z^{n+1/2}(i,j,k)l_z(i,j,k) - E_x^n(i,j,k+1)l_x(i,j,k+1) + E_x^n(i,j,k)l_x(i,j,k)\right] \quad (15)$$

$$H_z^{n+1/2}(i,j,k) = H_z^n(i,j,k) + \frac{C(i,j,k)}{S_{xy}(i,j,k)}.\left[E_x^{n+1/2}(i,j+1,k)l_x(i,j+1,k) - E_x^{n+1/2}(i,j,k)l_x(i,j,k) - E_y^n(i+1,j,k)l_y(i+1,j,k) + E_y^n(i,j,k)l_y(i,j,k)\right] \quad (16)$$

In the same way, the H-field updated equations from n+1/2 to n+1

$$H_x^{n+1}(i,j,k) = H_x^{n+1/2}(i,j,k) + \frac{C(i,j,k)}{S_{yz}(i,j,k)}.\left[E_y^{n+1/2}(i,j,k+1)l_y(i,j,k+1) - E_y^{n+1/2}(i,j,k)l_y(i,j,k) - E_z^{n+1}(i,j+1,k)l_z(i,j+1,k) + E_z^{n+1}(i,j,k)l_z(i,j,k)\right] \quad (17)$$

$$H_y^{n+1}(i,j,k) = H_y^{n+1/2}(i,j,k) + \frac{C(i,j,k)}{S_{zx}(i,j,k)}.\left[E_z^{n+1/2}(i+1,j,k)l_z(i+1,j,k) - E_z^{n+1/2}(i,j,k)l_z(i,j,k) - E_x^{n+1}(i,j,k+1)l_x(i,j,k+1) + E_x^{n+1}(i,j,k)l_x(i,j,k)\right] \quad (18)$$

$$H_z^{n+1}(i,j,k) = H_z^{n+1/2}(i,j,k) + \frac{C(i,j,k)}{S_{xy}(i,j,k)}.\left[E_x^{n+1/2}(i,j+1,k)l_x(i,j+1,k) - E_x^{n+1/2}(i,j,k)l_x(i,j,k) - E_y^{n+1}(i+1,j,k)l_y(i+1,j,k) + E_y^{n+1}(i,j,k)l_y(i,j,k)\right] \quad (19)$$

Where the coefficients are:

$$A = \frac{4\epsilon - \sigma\Delta t}{4\epsilon + \sigma\Delta t} B = \frac{2\Delta t}{4\epsilon + \sigma\Delta t} C = \frac{\Delta t}{2\mu}$$

$l_x, l_y and l_z$ are Cell lengths along x, y, and z directions

$S_{xy}, S_{yz} and S_{zx}$ are Surface areas which are present in the irregular cell parallel to xy, xz, and yz planes.

Substituting the values of magnetic field back to the electric filed will lead to an implicit tri-diagonal updating equation for the electric field and explicit updating equation for the magnetic field at every sub-time step.

The tri-diagonal updating equation for E_x can be written as:

$$\propto_{ijk} E_x^{n+1/2}(i,j-1,k) + \beta_{ijk}E_x^{n+1/2}(i,j,k) + \gamma_{ijk}E_x^{n+1/2}(i,j+1,k) = b_{ijk} \quad (20)$$

Where

$$\propto_{ijk} = -\frac{B(i,j,k)}{\Delta y}\frac{C(i,j-1,k)}{S_{xy}(i,j-1,k)}l_x(i,j-1,k)$$

$$\beta_{ijk} = 1 + \frac{B(i,j,k)}{\Delta y}l_x(i,j,k).\left[\frac{C(i,j,k)}{S_{xy}(i,j,k)} + \frac{C(i,j-1,k)}{S_{xy}(i,j-1,k)}\right]$$

$$\gamma_{ijk} = -\frac{B(i,j,k)}{\Delta y}.\frac{C(i,j,k)}{S_{xy}(i,j,k)}l_x(i,j+1,k)$$

$$b_{ijk} = A(i,j,k)E_x^{n+1/2}(i,j,k) + \frac{B(i,j,k)}{\Delta y}.\left[-E_y^n(i+1,j,k)\frac{C(i,j,k)}{S_{xy}(i,j,k)}l_y(i+1,j,k) + E_y^n(i,j,k)\frac{C(i,j,k)}{S_{xy}(i,j,k)}l_y(i,j,k) + E_y^n(i+1,j-1,k)\frac{C(i,j-1,k)}{S_{zy}(i,j-1,k)}l_y(i+1,j-1,k) - E_y^n(i,j-1,k)\frac{C(i,j-1,k)}{S_{zy}(i,j-1,k)}l_y(i,j-1,k)\right] + \frac{B(i,j,k)}{\Delta y}.\left[H_z^n(i,j,k) - H_z^n(i,j-1,k)\right] - \frac{B(i,j,k)}{\Delta z}.\left[H_y^n(i,j,k) - H_y^n(i,j,k-1)\right] \quad (21)$$

The tri-diagonal updating equation can be written as a matrix form as show in the equation

$$T_x\begin{bmatrix} E_x^{n+1/2}(i,j,k) \\ \vdots \\ E_z^{n+1/2}(i,j-1,k) \\ E_x^{n+1/2}(i,j,k) \\ E_z^{n+1/2}(i,j+1,k) \\ \vdots \\ E_z^{n+1/2}(i,j-2,k) \\ E_z^{n+1/2}(i,j-1,k) \end{bmatrix} = b_x$$

Where

$$Tz = \begin{bmatrix} \beta i1k\gamma i1k00 \ldots 0 \\ \alpha i2k\beta i2k\gamma i2k0 \ldots 0 \\ \vdots \\ 0 \ldots 0\alpha i(J-2)k\beta i(J-2)k\gamma i(J-2)k \\ 0 \ldots 00\alpha i(J-1)k\beta i(J-1)k \end{bmatrix}$$

The tri-diagonal matrix having dimension $(J-1) \times (J-1)$, the PML boundary condition is applied at the outer boundary of the computation domain.

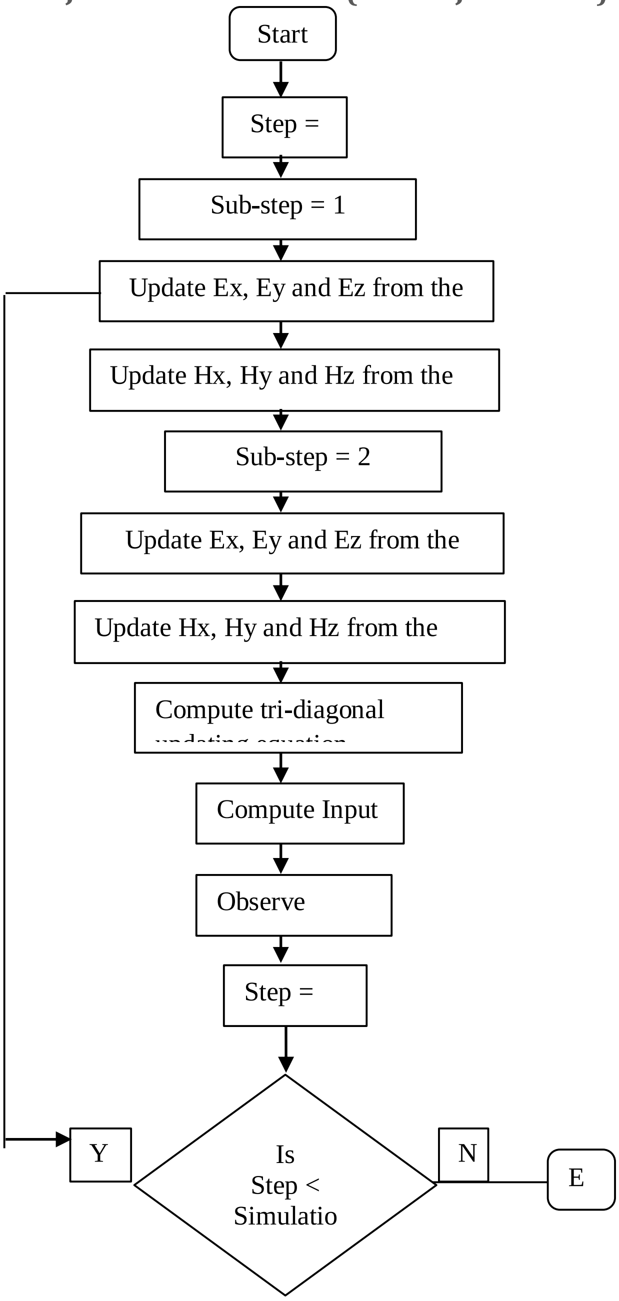

Figure 5.1. Flowchart of ADI-CFDTD Algorithm

IV Results

Developed ADI-CFDTD technique is validated by using circular micro-strip patch antenna as conformal object. The same object is designed and simulated in CST microwave studio as shown in Figure 5.2.to operate at 2.1GHz .

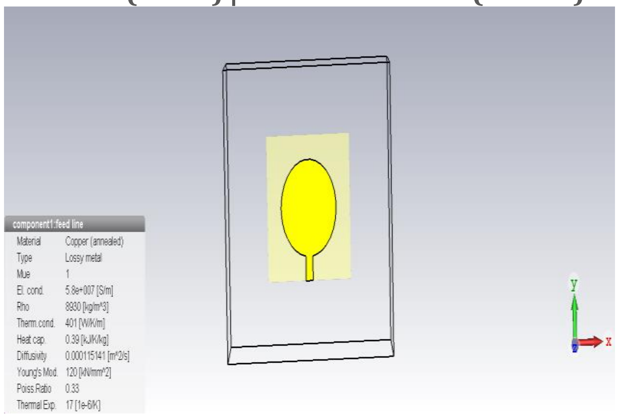

Figure: 5.2 Circular patch antenna modeled in CST Microwave Studio

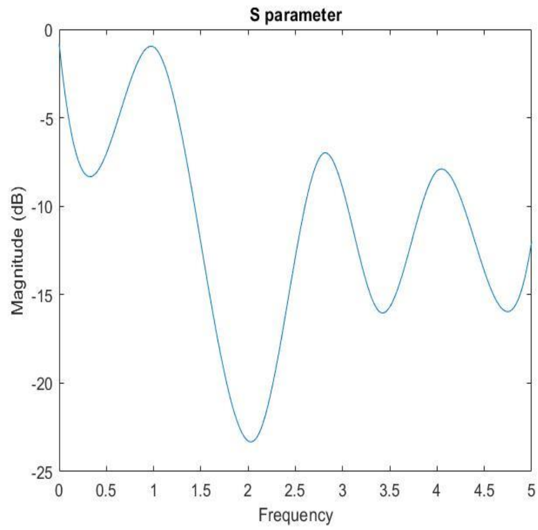

Figure5.3 S11 observed in Matlab at 2.1 GHz frequency

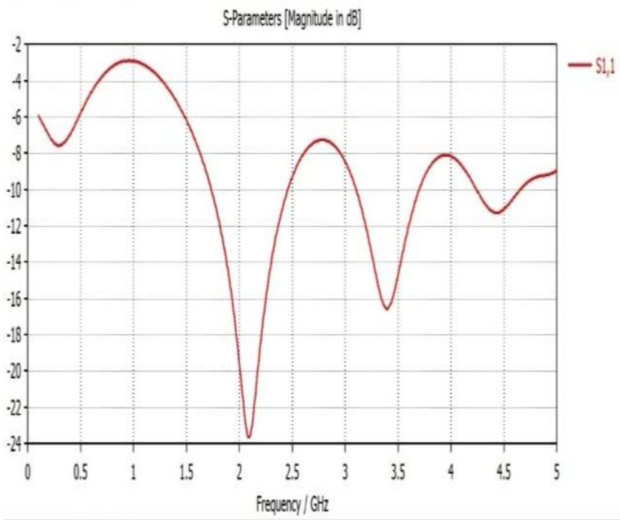

Figure 5.4 S11 observed in CST MW Studio at 2.1 GHz frequency.

Above two S11 figures depict the results obtained by using CST Microwave studio simulation and Matlab computation. The results achieved using CST Microwave studio considered as benchmark. Then same circular micro strip antenna has developed in Matlab and applied ADI-CFDTD with PML boundaries to obtain results. The obtained results are good in agreement with bench mark results. Hence, the developed ADI-CFDTD can be used to studies about the curvilinear structures.

During the computation in Matlab, The applied input sine wave signal and achieved output signal are shown in Figure 5.5 and 5.6.

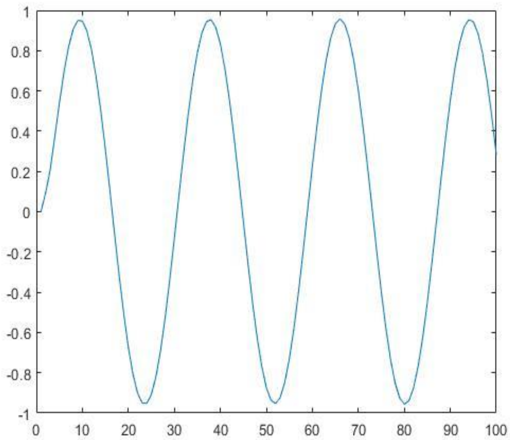

Figure 5.5 Input pulse applied at 150,100

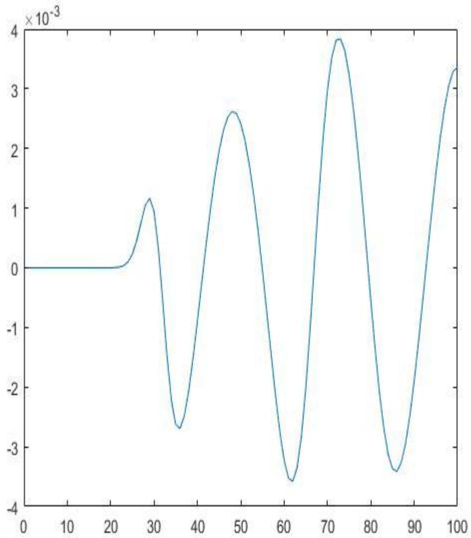

Figure 5.6 Output pulse at 100,100

Electromagnetic wave propagation has observed at distinct time steps. Among them, two are presented here. In case study 1,From Figure 5.7, at time step 73 the EM wave propagates outside of the computational domain since the conventional ADI-FDTD is failure to proper meshing of domain since it is in curvilinear nature.

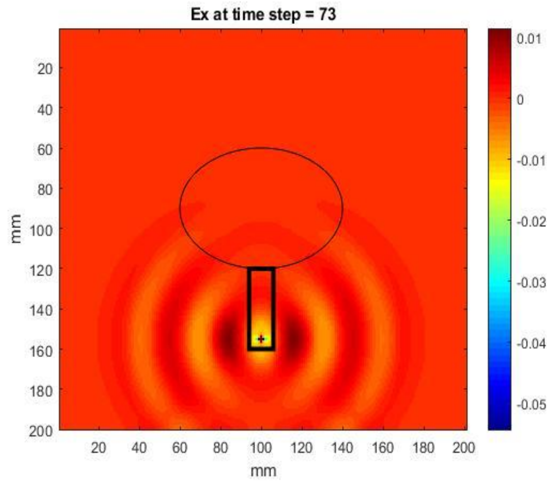

Figure 5.7 ADI-FDTD without PML

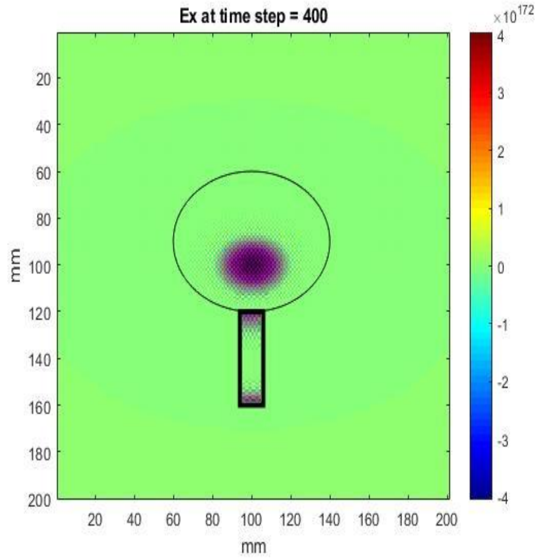

Figure 5.8 ADI-CFDTD method with PML

While in other case study, ADI-CFDTD is applied to the domain to compute the object. I.e. shown in figure 5.8, it is observed that the wave is totally confined within the object boundaries, thus helps to get accurate results.

Conclusion

This paper discusses about the Stability improvement of modified ADI algorithm for conformal structures. In general, CFDTD becomes unstable when large step size is considered. Namki proposed ADI FDTD technique which is unconditionally stable irrespective of the step size. In this research, the conventional ADI technique is modified to suit circular objects and further improved accuracy at the boundaries using modified PML. The Proposed ADI-CFDTD with PML is validated with circular patch antenna and compared with CST Mw studio.

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