Math 3331 - ODEs

Check List

1. <u>Separable</u> $\dfrac{dy}{dx} = f(x)g(y)$ separate and integrate $\displaystyle\int \dfrac{dy}{g(y)} = \int f(x)\,dx$

2(a). <u>Linear in x</u> $\dfrac{dy}{dx} + p(x)y = q(x)$ integrating factor $\mu = e^{\int p(x)\,dx}$

2(b). <u>Linear in y</u> $\dfrac{dx}{dy} + p(y)x = q(y)$ integrating factor $\mu = e^{\int p(y)\,dy}$

3(a). <u>Bernoulli in x</u> $\dfrac{dy}{dx} + p(x)y = q(x)y^n$ divide by y^n and let $u = \dfrac{y}{y^n} = \dfrac{1}{y^{n-1}}$

3(b). <u>Bernoulli in y</u> $\dfrac{dx}{dy} + p(y)x = q(y)x^n$ divide by x^n and let $u = \dfrac{x}{x^n} = \dfrac{1}{x^{n-1}}$

4. <u>Ricatti</u> $\dfrac{dy}{dx} = A(x)y^2 + B(x)y + C(x)$ we need one solution $y = y_1$

 The substitution $y = y_1 + \dfrac{1}{u}$ gives a linear ODE

5. <u>Homogeneous</u> $\dfrac{dy}{dx} = F\left(\dfrac{y}{x}\right)$ Let $y = xu$ substitute and separate

6. <u>Linear Fractional</u> $\dfrac{dy}{dx} = \dfrac{ax + by + f}{cx + dy + g}$

 Two Cases: (i) $ad - bc \neq 0$ Let $x = \bar{x} + \alpha, y = \bar{y} + \beta$

  (ii) $ad - bc = 0$ Let $u = ax + by$, or $u = cx + dy$

7. <u>Exact</u> $M(x,y)dx + N(x,y)dy = 0$ Test for Exactness $M_y = N_x$

 If so, let $f_x = M, f_y = N$, integrate partially giving the solution $f = c$.