# Chapter 6
# Exponential and Logarithmic Functions

## Section 6-6
## Solving Exponential and Logarithmic Equations

## Solving Exponential Equations

**Exponential equations** are equations in which variable expressions occur as exponents. The result below is useful for solving certain exponential equations.

### Core Concept

**Property of Equality for Exponential Equations**

**Algebra** If $b$ is a positive real number other than 1, then $b^x = b^y$ if and only if $x = y$.

**Example** If $3^x = 3^5$, then $x = 5$. If $x = 5$, then $3^x = 3^5$.

Tutorial

**EXAMPLE 1** Solving Exponential Equations

Solve each equation.

**a.** $100^x = \left(\frac{1}{10}\right)^{x-3}$ **b.** $2^x = 7$

**SOLUTION**

**a.**

| | |
|---|---|
| $100^x = \left(\frac{1}{10}\right)^{x-3}$ | Write original equation. |
| $(10^2)^x = (10^{-1})^{x-3}$ | Rewrite 100 and $\frac{1}{10}$ as powers with base 10. |
| $10^{2x} = 10^{-x+3}$ | Power of a Power Property |
| $2x = -x + 3$ | Property of Equality for Exponential Equations |
| $x = 1$ | Solve for $x$. |

**Check**

$$100^1 \stackrel{?}{=} \left(\frac{1}{10}\right)^{1-3}$$

$$100 \stackrel{?}{=} \left(\frac{1}{10}\right)^{-2}$$

$$100 = 100 \checkmark$$

**b.**

| | |
|---|---|
| $2^x = 7$ | Write original equation. |
| $\log_2 2^x = \log_2 7$ | Take $\log_2$ of each side. |
| $x = \log_2 7$ | $\log_b b^x = x$ |
| $x \approx 2.807$ | Use a calculator. |

**Solve the equation.**

**1.** $2^x = 5$

**2.** $7^{9x} = 15$

**3.** $4e^{-0.3x} - 7 = 13$

## Solving Logarithmic Equations

**Logarithmic equations** are equations that involve logarithms of variable expressions. You can use the next property to solve some types of logarithmic equations.

### Core Concept

**Property of Equality for Logarithmic Equations**

**Algebra** If $b$, $x$, and $y$ are positive real numbers with $b \neq 1$, then $\log_b x = \log_b y$ if and only if $x = y$.

**Example** If $\log_2 x = \log_2 7$, then $x = 7$. If $x = 7$, then $\log_2 x = \log_2 7$.

The preceding property implies that if you are given an equation $x = y$, then you can exponentiate each side to obtain an equation of the form $b^x = b^y$. This technique is useful for solving some logarithmic equations.

Tutorial

### EXAMPLE 3 Solving Logarithmic Equations

Solve (a) $\ln(4x - 7) = \ln(x + 5)$ and (b) $\log_2(5x - 17) = 3$.

**SOLUTION**

**a.**

| | |
|---|---|
| $\ln(4x - 7) = \ln(x + 5)$ | Write original equation. |
| $4x - 7 = x + 5$ | Property of Equality for Logarithmic Equations |
| $3x - 7 = 5$ | Subtract $x$ from each side. |
| $3x = 12$ | Add 7 to each side. |
| $x = 4$ | Divide each side by 3. |

**Check**

$\ln(4 \cdot 4 - 7) \stackrel{?}{=} \ln(4 + 5)$

$\ln(16 - 7) \stackrel{?}{=} \ln 9$

$\ln 9 = \ln 9$ ✓

**b.**

| | |
|---|---|
| $\log_2(5x - 17) = 3$ | Write original equation. |
| $2^{\log_2(5x - 17)} = 2^3$ | Exponentiate each side using base 2. |
| $5x - 17 = 8$ | $b^{\log_b x} = x$ |
| $5x = 25$ | Add 17 to each side. |
| $x = 5$ | Divide each side by 5. |

**Check**

$\log_2(5 \cdot 5 - 17) \stackrel{?}{=} 3$

$\log_2(25 - 17) \stackrel{?}{=} 3$

$\log_2 8 \stackrel{?}{=} 3$

Because $2^3 = 8$, $\log_2 8 = 3$. ✓

Because the domain of a logarithmic function generally does not include all real numbers, be sure to check for extraneous solutions of logarithmic equations. You can do this algebraically or graphically.

## EXAMPLE 4 Solving a Logarithmic Equation

Solve $\log 2x + \log(x - 5) = 2$.

**SOLUTION**

| | |
|---|---|
| $\log 2x + \log(x - 5) = 2$ | Write original equation. |
| $\log[2x(x - 5)] = 2$ | Product Property of Logarithms |
| $10^{\log[2x(x-5)]} = 10^2$ | Exponentiate each side using base 10. |
| $2x(x - 5) = 100$ | $b^{\log_b x} = x$ |
| $2x^2 - 10x = 100$ | Distributive Property |
| $2x^2 - 10x - 100 = 0$ | Write in standard form. |
| $x^2 - 5x - 50 = 0$ | Divide each side by 2. |
| $(x - 10)(x + 5) = 0$ | Factor. |
| $x = 10$ or $x = -5$ | Zero-Product Property |

The apparent solution $x = -5$ is extraneous. So, the only solution is $x = 10$.

**Check**

$\log(2 \cdot 10) + \log(10 - 5) \stackrel{?}{=} 2$

$\log 20 + \log 5 \stackrel{?}{=} 2$

$\log 100 \stackrel{?}{=} 2$

$2 = 2$ ✓

$\log[2 \cdot (-5)] + \log(-5 - 5) \stackrel{?}{=} 2$

$\log(-10) + \log(-10) \stackrel{?}{=} 2$

Because $\log(-10)$ is not defined, $-5$ is not a solution. ✗

**Solve the equation. Check for extraneous solutions.**

5. $\ln(7x - 4) = \ln(2x + 11)$
6. $\log_2(x - 6) = 5$
7. $\log 5x + \log(x - 1) = 2$
8. $\log_4(x + 12) + \log_4 x = 3$

## EXAMPLE 5 Solving an Exponential Inequality

Solve $3^x < 20$.

## EXAMPLE 6 Solving a Logarithmic Inequality

Solve $\log x \leq 2$.

Section 6-6 Homework #5-41 odd and 47-53odd