Link between Currency Swaps and Interest Rate Swap

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Abstract

We exposition the link between interest rate swaps and currency swaps. We then illustrate using numerical examples, with given term structure of the respective interest rates, on the pricing of the swaps at contract initiation. With elapse of time, we next show how to price the swaps before the next settlement date. Given changes in exchange rate, we proceed to price the currency swaps. Ensuing exercises highlight the actual cash flows among counterparties, and the netting effects in swap settlements.

Keywords: Interest rate swaps, currency swaps, initial valuation of swaps, pricing of swaps, netting effects

Introduction

Many finance textbooks present currency swaps and interest rate swaps separately in two different chapters. In this exercise, we present them together by exhibiting their link. We proceed to *price* them at the contract initiation, and to *value* each before the next payment due date. We use two currencies, USD and EUR, or symbolically \$ and €. We assume two interest rate patterns: fixed and floating. Diagrammatically, we summarize currency swaps and interest rate swaps in the following six pairs of arrows among four parties or investors, viz., A, B, C and D.

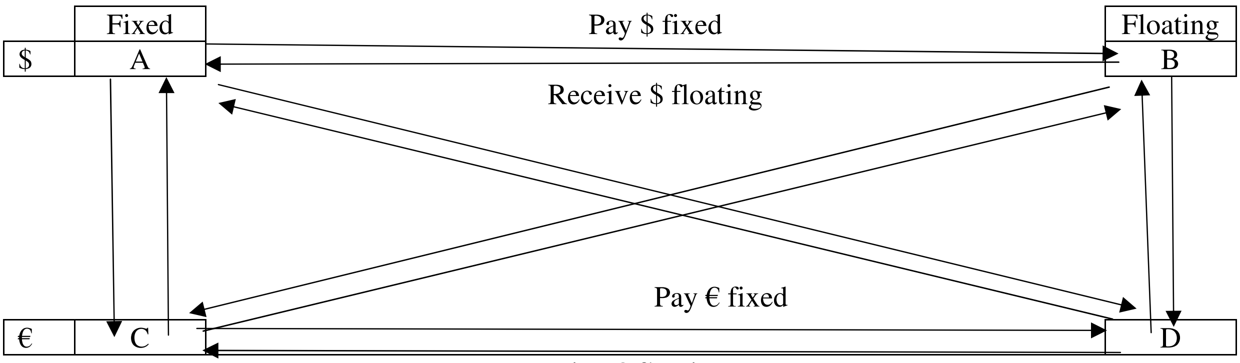

Diagram 1: Link between interest rate swaps and currency swaps

The swaps between A and B, and those between C and D are interest rate swaps. The swaps between A and C, between B and D, between A and D, and between B and C are currency swaps. Specifically, the AC swaps are fixed-fixed, the BD swaps are floating-floating, the AD and BC swaps are fixed-floating and floating-fixed respectively. More specifically, the AD swap is pay \$ fixed receive € floating from A's perspective, and it is receive \$ fixed pay € floating from D's perspective. The BD swap is pay \$ floating receive € fixed from B's perspective, and it is pay € fixed receive \$ floating from C's perspective. From the diagram, we can conclude that an interest rate swap is just a currency swap in which both currencies are the same. A currency swap is thus merely the more general instrument of the two. In other words, an interest rate swap is a special case of the currency swap in which both currencies are the same.

Pricing and Valuing of Interest Rate Swaps

In pricing the interest rate swap, we need the various periodic interest rates that run the tenure of the swap. To make the pricing exercise more practical, let's assume we are given the following *annualized* Libor USD rates. We adopt the 30-360 vis-à-vis the actual-365 convention.

Days, t	Annualized \$ rate, r_t, %	Discount factor, Z_t
90	1.2412	.996906599
180	1.2454	.993811536
270	1.2504	.990709130
360	1.2544	.987611403
	$\sum =$	3.969038667

Table 1: Annualized Libor rates at contract initiation

The fixed swap rate, FSR, is the four fixed coupons plus the \$1 par when discounted by the corresponding annualized rates will result in a \$1.00 par initially. That is:
$1.00 = FSR/(1+r_{90}¼) + FSR/(1+r_{180}½) + FSR/(1+r_{270}¾) + FSR/(1+r_{360}) + 1.00/(1+r_{360})$.
Let's call $Z_1 = 1/(1+r_{90}(90/360)) = .996906599$; $Z_2 = 1/(1+r_{180}(180/360)) = .993811536$; $Z_3 = 1/(1+r_{270}(270/360)) = .990709130$; $Z_4 = 1/(1+r_{360}(360/360)) = .987611403$.

Hence, $1.00 = FSR(Z_1 + Z_2 + Z_3 + Z_4) + 1.00{*}Z_4$.
Solving for FSR, we get $FSR = (1 - Z_T)/\sum Z_i$.

Hence, the quarterly fixed swap rate for the USD, $FSR_\$ = (1 - .987611403) / 3.969038667 = 0.00312130933$ = .312130933% per quarter or 1.248523733% per year.

The means the pay \$ fixed side promises to pay .00312130933 per \$1 notional value for each of the next quarter and pays \$1.00 on the fourth quarter.

Now, let's price the \$ floating side. To avoid paying each other initially, the \$ floating side will also need to price its present value to be at \$1.00. So, at t=0, $1.00 = (\$.012412/4 + \$1)/(1 + r_{90}(90/360))$. The \$1 in the numerator is because of the fact that the floating side resets to \$1 at each of the settlement day. In this case, reset takes place at t=90, 180, and 270. The \$.012412/4 is the floating side's coupon payment to the fixed side set at t=0, but to be made at t=90 days later. Apparently, numerator and denominator are the same, and therefore the floating side's present value is also \$1.00.

Since the initial present value is set to \$1.00 for both sides, no ***net*** payment from either side is needed at t=0.

In valuing the interest rate swap, we try to determine which side wins and which side loses with the progress of time. Winning and losing depend entirely on the movement of the interest rates.

Let's continue with the previous example. Sixty (60) days after the interest rate swap's initiation, let's assume that the Libor USD rates for 30, 120, 210 and 300 days are as shown below.

Day, t	Annualized Libor \$ rate, r_t, %	Discount factor, Z_t
30	1.4525	0.998791047
120	1.4572	0.995166146
210	1.4618	0.991544931
300	1.4688	0.987908006
	∑ =	3.973410130

Table 2: Annualized Libor rates 60 days after contract initiation

The pay \$ fixed's present value, PV = .003121309(3.973410130) + 1(.987908006) = 1.000310247.

The receive \$ floating's present value, PV = (.012412/4)(.998791047) + 1(.998791047) = 1.001890296.

Therefore, the pay fixed receive-floating net position = 1.001890296 – 1.000310247 = .001580049.

Given the notional value to be \$10m, then the pay fixed receive floating party A's net position on day 60 = .001580049*10m = \$15,800.49. This is party A's gain. The net position for the receive fixed-pay floating of party B will be –\$15,800.49. It is also intuitive that the pay-fixed side gains because the interest rate term structure has shifted upward, as we can see from the two given tables of data. Such upward shit favors the pay-fixed side at the expense of the pay-floating side.

If the two sides of this swap engage in mark-to-the-market, then A gains \$15,800.49 gain and B loses \$15,800.49 at t=60 days. The two sides can then reprice the swap. The new fixed swap rate will be:

FSR_{new} = (1 - .987908006)/3.973410130 = .003043228.

The new FSR will be .003043228 for the remaining tenure of the swap after the mark-to-market.

Pricing and Valuing of Currency Swaps

In pricing the currency swaps, we need the interest rates of the two currencies plus the spot exchange rate. Two additional differences (when compared with interest rate swaps) are: (1) the \$1 numeraire notional value of home currency needs to be converted at the spot exchange rate to the corresponding amount of the foreign currency, and; (2) there is no fixed rate determination in the pay floating-receive floating swap. This corresponds to the pair of arrows between parties B and D in Diagram 1. Let's proceed with a numerical example using annualized Euribor rates as shown below, and assume the spot exchange rate, S_0, to be .8900 €/\$ or 1.1236 \$/€.

Days, t	Annualized Euribor rates, r_p, %	Discount factor, Z_t
90	1.6864	0.995801700
180	1.6922	0.991609999
270	1.7082	0.987350558
360	1.7364	0.982932362
	∑=	3.957694609

Table 3: Annualized Euribor rates at contract initiation

Using the same model as in the case of Libor, the quarterly fixed swap rate for the EUR, $FSR_€$ = (1 - .982932362)/3.957694609 = .0043125201= .43125201% per quarter = 1.725008039% p.a.

To summarize, our currency swaps involving USD and EUR would have a quarterly rate of .312130933% or an annual rate of 1.248523733% in USD, and a quarterly rate of .43125201% or an annual rate of 1.725008039% in EUR. The notional principal in USD is $1.00 while the notional principal in EUR is 1/1.1236 = €0.89. Specifically, referring to Diagram 1, we have the four currency swaps (from the first party's perspective):

Swap AC: pay $ fixed at 1.2485%, receive € fixed at 1.725%;
Swap AD: pay $ fixed at 1.2485%, received € floating;
Swap BC: pay $ floating, receive € fixed at 1.725%, and;
Swap BD: pay $ floating, receive € floating.

In order to value the currency swaps, we need the new term structure in EUR with the remaining time until maturity of the swaps. Sixty (60) days later, we have the following 30-, 150-, 210, and 300-day term structure for the annualized Euribor rates, and the EUR spot exchange rate, S_{60}, has appreciated to .8800 €/$ or 1.1364 $/€

Days, t	Annualized Euribor rate, r_p, %	Discount factor, Z_t
30	1.8246	0.998481808
120	1.8292	0.993939619
210	1.9346	0.988840767
300	1.9398	0.984092150
	∑ =	3.965354345

Table 4: Annualized Euribor rates 60 days after contract initiation

To recap, at t=60 days later:

PV of fixed $ = $1.000310248, and; PV of floating $ = $1.001891295.

Now, let's find PV of fixed € = .0043125201*3.965354345 + 1(.98409215) = €1.00119282.

PV of floating € = (.016864/4)*.99848181 + 1(.99848181) = €1.002691408

Next, we need to convert the last two PV's in €1.00 notional principal loan using the initial exchange rate of .89 €/$ into actual € amount to match up with the $1.00 notional principal for the currency swap to work.

Actual PV of € fixed = 1.00119282 * .89 = €.89106161.

Actual PV of € floating = 1.002691408 * .89 = €.892395353.

Finally, we proceed to convert the PV's in € using spot exchange rate at 1.1363 $/€ to $.

PV of € fixed in $ = .89106161*(1/.88) = $1.012570011, and

PV of € floating in $ = .892395353*(1/.88) = $1.014085629.

Hence, we can value the four currency swaps:
1) Value of swap AC (pay $ fixed, receive € fixed) = $1.012570011 - $1.000310248 = $.012259763.

2) Value of swap AD (pay $ fixed, receive € floating) = $1.014085629 - $1.000310248 = $.013775381.

3) Value of swap BC (pay $ floating, receive € fixed) = $1.012570011 - $1.001891295 = $.010678716.

4) Value of swap BD (pay $ floating, receive € floating) = $1.014085629 - $1.001891295 = $.012194334.

Note that all of these numbers are positive. Therefore, all 4 currency swaps show gains as a result of a combination of the interest-rate changes in the two economies as well as the exchange-rate change. To the respective counterparties, their swaps are worth the same numerical amounts, but with a negative sign each. For example, the counterparty for swap AC is an investor who pays € fixed, and receives $ fixed, and his swap will experience a loss of $.012259763 per $1 notional value or €.010788591 per €1. Of course, the last two numbers are connected via the current exchange rate at .8800 €/$ or 1.1363636 $/€.

Problems

After reading the above primer on how to price and value swaps, answer the following questions. Pay special attention to which side of the swap's position you are supposed to evaluate.

Q1: Based on the data given in Table 4, and the $FSR_€$ calculated, what is the net position of Party D in an interest rate swap (see Diagram 1) given that the notional principal is €8.9 million?

Q2: On t=90 days later, spot exchange rate = .8700 €/$. What are the six *bilateral* cash flows experienced by each of the four parties in Diagram 1? Use netting, and from the perspective of the first party relative to the second party, i.e., for swap AB, use A's perspective relative to B's perspective. The notional principal is $10m or €8.9m. Leave all final answers in $.

Q3: On t=180 days, immediately after they have settled their quarterly payments, the following term structure exist for annualized Libor and annualized Euribor rates. Determine who wins and who loses among the four parties. Further, assume the spot exchange rate has returned to 1.1236 $/€ or .8900 €/$.

Assume notional principal of $10.0m or €8.9m.

Days, t	Annualized Libor rate, $r_\$$, %	Discount factor, $Z_{t,\$}$	Annualized Euribor rate, $r_€$, %	Discount factor, $Z_{t,€}$
90	1.1612	.997105404	1.4886	.996292298
180	1.1654	.994206757	1.4924	.992593296

Q4: On t=270 days, immediately after they have settled their quarterly payments, the following term structure exist for annualized Libor and annualized Euribor rates. Determine who wins and who loses among the four parties. Further, assume the spot exchange rate has returned to 1.0989 $/€ or .91 €/$. Assume notional principal of $10.0m or €8.9m.

Days, t	Annualized Libor rate, $r_{\$}$, %	Discount factor, $Z_{t,\$}$	Annualized Euribor rate, $r_{€}$, %	Discount factor, $Z_{t,€}$
90	1.2412	.996906599	1.6864	.995801700

Q5: Using the annualized Libor and Euribor rate in Q4, describe the cash flows for all six swaps at t=360 when all swaps mature. Assume spot exchange rate = .8900 €/$. Be explicit on who pays whom.

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