

# **Point of View of the Statistician**

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**Workshop on Impact of  
Dose Uncertainties on the  
Dose Response  
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# Point of View of the Statistician

- **Characterize different types of dosimetry uncertainties**
- **Indicate how dosimetry uncertainties distort dose-response analyses**
- **Discuss methods for correcting the distorting effects of dosimetry uncertainties**

# Some preliminaries

- **Most work addressing dosimetry uncertainties based on the assumption that errors are independent of disease status (non-differential)**
- **Linear dose-response function plays important role in radiation epidemiology**
- **Relative risk =  $1 + \beta$  dose where  $\beta$  is excess relative risk per unit of dose**

# Some important distinctions

- **Classical errors versus Berkson errors**
- **Shared errors versus Errors that are independent for different subjects**
- **Impact on dose-response analyses depends on these distinctions**

# **Classical Error (Measurement Error)**

- **Error is independent of true dose**
- **Can be thought of as error that arises from an imprecise measuring device**
- **Variance of estimated doses larger than variance of true doses**
- **Adjustment needed to avoid attenuation of the true dose-response**

# Examples of Classical Errors

- **Errors in readings of film badge dosimeters**
- **Errors in questionnaire data used in estimating doses**

# **Berkson Error (Grouping Error)**

- **Error is independent of observed dose**
- **Error that results when**
  - **Single mean dose used to represent group**
  - **Same model is used to estimate doses for a group**
- **Variance of true doses larger than variance of measured doses**
- **Little distortion in linear dose-response**

# Shared Errors

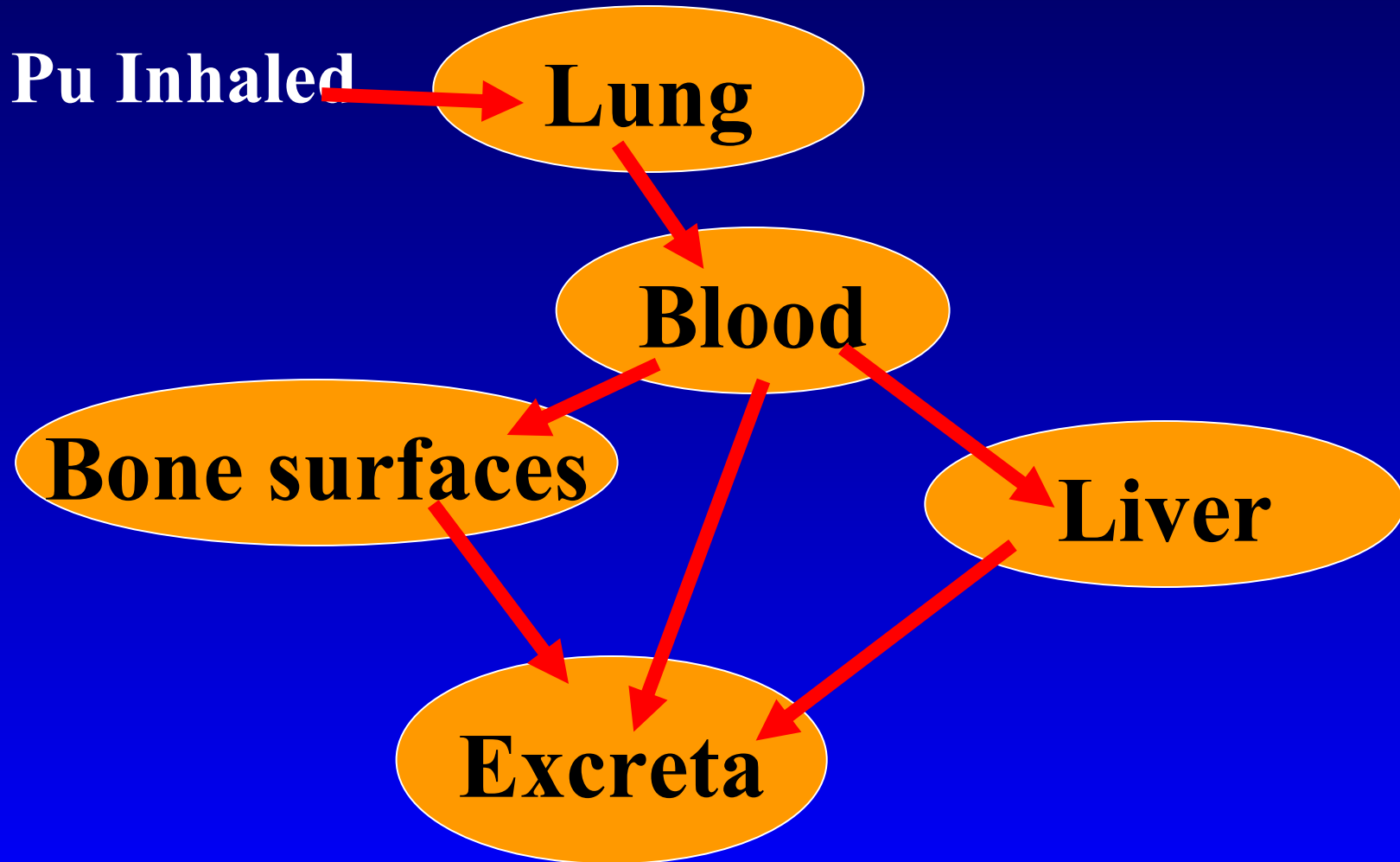
- **Also known as systematic errors**
- **Examples**
  - **Errors in the source term for an environmental exposure**
  - **Errors in doses assigned to groups of subjects**
  - **Errors in parameters of models used to convert measurements to doses**



# Error Structure

- **For many epidemiological studies, error structure is a complicated mixture of different types of errors**
- **Illustration: Plutonium dosimetry used in nuclear worker studies exposed to inhaled plutonium**
  - **Example: Mayak plant**

# Inhaled Plutonium Dynamics



# Plutonium Dosimetry

- **Dose estimates based on**
  - measurements of radioactivity in urine
  - occupational histories
- **Use models for rates of clearance and transfer to estimate initial body burden and doses to lung, liver, bone surfaces and other organs**

# **Some uncertainties in Plutonium Dosimetry**

- **Imprecision in urine measurements**
  - Unshared classical error
- **Uncertainties in biokinetic models and parameter values used to estimate deposition and clearance in organs of the body**
  - Shared classical error
- **Models can only approximate behavior of plutonium in a given individual**
  - Berkson error

# Possible Effects of Errors in Dose Estimates

- **Bias in estimates of linear risk coefficients**
- **Distortion of the shape of the dose-response function**
- **Underestimation of uncertainty**
- **Reduction in statistical power for detecting dose-response relationships**

# Statistical approaches for accounting for dosimetry uncertainties

## What they can't do

- Improve power and precision of estimated risk coefficients

## What they can do

- Avoid misleading results
- Correct biases in risk coefficients
- Widen confidence intervals to reflect dosimetry uncertainties

# Error Structure

- **Identify sources of error**
- **Nature of the error from each source**
  - Classical or Berkson?
  - Shared or unshared?
- **Describe the magnitude and distribution error from each source**
  - Subjective judgments usually required

# Simple approaches that may help

- **Restrict analyses to subjects with higher quality dose estimates**
- **Conduct analyses based on doses calculated from alternative models**



# Statistical approaches for accounting for dosimetry uncertainties

- **Regression calibration**
- **Full maximum likelihood**
- **Multiple imputations**
- **Likelihood averaging**

# Regression Calibration

- Replace the estimated doses with  $E(\text{true dose}|\text{estimated dose}) = E(x|z)$
- Easy to apply once have the  $E(x|z)$
- Leads to unbiased estimates of linear risk coefficients.
- Limitations
  - An approximation for non-linear models
  - Uncertainty in risk estimates may be underestimated

# Regression Calibration

- How do we obtain  $E(x|z)$  ?

$$E(x|z) = \frac{\int xf(z|x)f(x) dx}{\int f(z|x)f(x) dx}.$$

- Need information on
  - distribution of estimated doses  $z$  conditional on the true doses  $x$ :  $f(z|x)$
  - distribution of the true doses  $x$ :  $f(x)$   
(Can be a challenging task)

# Regression Calibration

Let  $x$  =true dose;  $z$ =estimated dose

Errors ( $z-x$ ) are independent of  $x$  (classical errors)

$$z|x \sim N(x, \kappa^2); \quad x \sim N(\mu, \sigma^2)$$

$$\text{Then } z \sim N(\mu, \sigma^2 + \kappa^2)$$

$$E(x |z) = (1-c) \mu + c z$$

$$\text{where } c = \sigma^2 / (\sigma^2 + \kappa^2)$$

Cochran 1968

# Regression Calibration

$\log z|x \sim N(\log x, \kappa^2); \log x \sim N(\mu, \sigma^2);$

$$E(x|z) = [E(\log z)]^{(1-c)} z^c$$

where  $c = \sigma^2 / (\sigma^2 + \kappa^2)$

# Regression Calibration Examples

- **A-bomb survivors** (Pierce et al. 1990; 2009)
  - Increased slope by 10%
- **European residential radon case-control studies**  
(Reeves et al. 1998; Darby et al. 1998)
  - Increased slope by 100%
- **Colorado uranium miners** (Stram et al. 1999)
  - Decreased magnitude of inverse exposure-rate effect

# Full maximum likelihood

- **Regression model** : Relates disease to true dose
  - Linear relative risk model a common choice
- **Measurement model**: Relates estimated doses ( $z$ ) to true doses ( $x$ )
- **Exposure model**: Specifies distribution of true doses ( $x$ )

# Study of leukemia and thyroid cancer in relation to Utah fallout

- **Lognormal distribution for classical (measurement) errors**
- **Non-parametric distribution for true doses**
- **Used Markov chain Monte Carlo (MCMC) methods to implement**
- **Increased estimates for leukemia and thyroid neoplasms**
- **Increased uncertainty in both estimates**



# Study of leukemia and thyroid cancer in relation to Utah fallout

- **Uncertainty information consisted of standard error for each subject**
  - Included both unshared classical error and shared error
  - Thomas et al. assumed all was unshared classical error
  - Shared error later considered by Mallick et al. 2002
- **Study revisited with dosimetry (and other) improvements**
  - Lyon et al. 2006; Li et al. Biometrics 2007

# Multiple imputations

- **Use Monte Carlo methods to generate N replications of the doses based on the error structure**
- **Challenging to set up simulations appropriately**
- **Take account of correlations (shared errors)**
- **Assume unshared classical error has been addressed**
  - **Stram and Kopecky (2003)**
  - **Stayner et al. (2008)**

**“We take as our starting point a Berkson model ...”**

# Multiple imputations

How do we use the N samples?

**A simple approach:**

- Estimate  $\beta$  and its variance for each sample
- Average over samples
- Estimate total variance as sum of sampling variance and variance due to dosimetry uncertainties

**A better approach: Likelihood averaging**

# **Likelihood Averaging**

## **(Monte Carlo Maximum Likelihood)**

- **Use Monte Carlo methods to generate N replications of the doses based on the error structure**
- **Estimate the likelihood function for each data set and average over samples**
- **Use average likelihood to estimate parameter and uncertainties**

# Likelihood Averaging: ORNL nuclear worker study

- ~ 5345 Oak Ridge National Laboratory workers exposed to external radiation and monitored with personal dosimeters
- Addressed uncertainty in factors used to convert recorded doses to organ dose
  - Same factor used for all dosimeter readings in a given time period (1943, 1944-52, 1953-79, 1980-97)
  - Shared error

# ORNL nuclear worker study

Estimated excess relative risk (ERR) per Gy  
for solid cancers

- **Conventional analysis estimate:**  
5.38 (90% CI: 0.54 to 12.58)
- **Based on likelihood averaging**  
4.82 (90% CI: 0.41 to 13.31)

# Examples where dose estimation errors have been taken into account

- **A-bomb survivors** (Pierce et al. 1996; 2008)
- **Residential radon exposure** (Reeves et al. 1998)
- **Utah fallout study** (Thomas et al. 1999; Mallick et al. 2002; Li et al. 2007)
- **Underground miners** (Stram et al. 1999)
- **ORNL nuclear workers** (Stayner et al. 2007)
- **Hanford fallout study** (Stram and Kopecky 2003; Hoffman et al. 2007)
- **Tinea capitis patients** (Schafer et al. 2001; Lubin et al. 2004)
- **Chornobyl thyroid study** (Kopecky et al. 2006)

# Summary

- **Methods for adjusting for dosimetry error are available**
- **Require detailed understanding of error structure**
  - **Lots of communication between dosimetrists and statisticians**
- **Methods are often computationally intensive**
- **Increasingly, errors are being evaluated and considered in radiation dose-response analyses**