International Journal of Modern Science and Technology *Vol. 1, Issue 2, May 2016. Page 47-51.*
http://www.ijmst.co/ *ISSN: 2456-0235*

Research Article

Interval-valued intuitionistic fuzzy transition matrices

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Abstract

Matrix theory plays a vital role in linear algebra its applications to several areas is highly remarkable. A matrix over the fuzzy algebra is called a fuzzy matrix. Transition probability matrix plays a vital role in Markov process. Transition matrix defined in fuzzy setting called as fuzzy transition matrix. Interval-valued fuzzy matrix is another generalization of fuzzy matrix. This Paper is an inspiration received from the theory of interval valed fuzzy matrices and fuzzy transition matrices. This structure namely interval valued intiutionistic fuzzy transition matrices (IVIFTM) generalizes the concept of fuzzy transition matrices. Various operations in this IVITFTMS are discussed in this paper. The complement and transpose of IVIFTM is defined and we provide some results on them. The union and intersection of two IVIFTMS are also fefoned and their commutative and associative properties are also discussed.

Keywords: Interval-valued fuzzy matrix; Interval-valued intuitionistic fuzzy matrix; Fuzzy transition matrix; Interval-valued intuitionistic fuzzy transition matrix.

Introduction

Though there are various models in Random process the role of Markov process is highly remarkable. Markov process is used to model the objects that have limited memory in the past. State transition probabilities associated with Markov chain is represented by means of a matrix called transition probability matrix or simply transition matrix. Fuzzy matrices was introduced by Thomason (Tuckwell, 1995).

Kim (1983) studied about the canonical form of an idempotent matrix. Meenakshi (2008) book gives a detailed theory about fuzzy matrices over fuzzy algebra and its applications. Vijayabalaji et al., (2016) introduced the notion of fuzzy transition matrices.

Intuitionistic fuzzy matrices (IFMs) are another interesting structure that was introduced by Pal et al., (2002). Several properties on IFMs have been studied in (Khan and Pal, 2002). The notion of interval valued intuitionistic fuzzy matrices (IVIFMs) and some basic operators on IVIFMs can be seen in (Pal and Khan, 2002).

Motivated by the above theory our aim in this paper is to introduce the notion iof interval-valued intuitionistic fuzzy transition matrix (IVIFTM) and to provide some results on it.

Preliminaries

This section recalls some basic definitions an d results which will be needed in the sequel.
Definition 2.1 (Oliver, 2005).

The homogeneous state transition probabilit y p_{ij} satisfies the following:

(1) $0 \le p_{ij} \le 1$.

(2) $\sum_j p_{ij} = 1$, $i = 1, 2, \ldots, n$

which follows from the fact that the states are mut ually exclusive and collectively exhaustive.
Example 2.2 (Oliver, 2005).

The state transition probability matrix P , where pij is the entry in the ith rowa nd jth column:

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$$

P is called the transition probability matrix.
Definition 2.3 (Meenakshi, 2008).

(1) For any two elements $A = (a_{ij})$ and $B = (b_{ij}) \in V_{m\times n}$ define $A + B = (sup\{a_{ij,}b_{ij}\}) = \vee_{ij} (a_{ij,}b_{ij})$, where $a_{ij,}b_{ij} \in F$.

(2) For any elements $A = (a_{ij}) \in V_{m\times n}$ and a scalar $k \in F$ define $kA = (inf\{k, a_{ij}\}) = \wedge_{ij} (k, a_{ij})$.

The system $V_{m\times n}$ together with these operati ons of component wise fuzzy addition and fuzzy multiplication is called fuzzy vector space over F and the scalarsre restricted to F.

Definition 2.4 (Meenakshi, 2008).

Define two operations + and on $V_{m\times n}$ as follows.

(1) $A+B = (sup\{a_{ij,}b_{ij}\}) = \vee_{ij}(a_{ij,}b_{ij})$, where $a_{ij,}b_{ij} \in F$.

(2) AB = (sup { inf { $a_{ik,}$, b_{kj} } }) = $\vee_k$ { $\wedge(a_{ik,}, b_{kj})$ }.

Definition 2.5 (Vijayabalaji et al., 2016).

Let $V_{2\times2}$ denotes set of all 2×2transition matrices over the fuzzy algebra [0,1]. The operations (+,.) are defined on $V_{2\times2}$ as follows.

For any two elements $A=(a_{ij})$ and $B=(b_{ij}) \in V_{2\times2}$

(i) $A+B = \left\{\begin{matrix} a_{ij}+b_{ij},\ if a_{ij}+b_{ij}<1 \\ a_{ij}+b_{ij}-1, if\ a_{ij}+b_{ij}>1\end{matrix}\right\}$

(ii) $A+B = \left\{\begin{matrix} 1and\ 0,\ if\ a_{ij}+b_{ij}=1\ in\ row\ 1 \\ 0\ and\ 1, if\ a_{ij}+b_{ij}=1\ in\ row\ 2\end{matrix}\right\}$

(iii) For any scalar $k\varepsilon(0,1), kA = fractional\ part\ of\ (10ka_{ij})$

The system $V_{2\times2}$ together with these operations of component wise transition addition and multiplication is called fuzzy transition matrices are restricted.

Definition 2.6 (Oliver, 2005).

An intuitionistic fuzzy matrix (IFM) A of order n × n is defined as $A=[x_{ij},\langle a_{\mu_{ij}}, a_{\gamma_{ij}}\rangle]_{n\times n}$where$a_{\mu_{ij}}$ and $a_{\gamma_{ij}}$ are called membership and non membership values of x_{ij} in A, which maintaining the condition $0\le a_{\mu_{ij}}+a_{\gamma_{ij}}\le 1$.

For simplicity, we write $A=[x_{ij}, a_{ij}]_{n\times n}$or simply$[a_{ij}]_{n\times n}$where $a_{ij}=\langle a_{\mu_{ij}}, a_{\gamma_{ij}}\rangle$.

Definition 2.7 (Pal et al., 2002).

An interval valued intuitionistic fuzzy matrix (IVIFM) A of order n×n is defined as$A=[x_{ij},\langle a_{\mu_{ij}}, a_{\gamma_{ij}}\rangle]_{n\times n}$where $a_{\mu_{ij}}$and$a_{\gamma_{ij}}$ are both the subsets of [0, 1] which are denoted by $a_{\mu_{ij}}=[a_{\mu_{ij}L}, a_{\mu_{ij}U}]$and $a_{\gamma_{ij}}=[a_{\gamma_{ij}L}, a_{\gamma_{ij}U}]$which maintaining the condition $[a_{\mu_{ij}U}+a_{\gamma_{ij}U}]\le 1$ for $i,j=1,2,3,\dots n$.

Some operations on interval valued intuitionistic fuzzy transition matrix (IVIFTM)

Definition 3.1.

An interval valued intuitionistic fuzzy transition matrix (IVIFTM) A of order n×n is defined as $A=[x_{ij},\langle a_{\mu_{ij}}, a_{\gamma_{ij}}\rangle]_{n\times n}$where $a_{\mu_{ij}}$and$a_{\gamma_{ij}}$ are both the subsets of [0, 1] which are denoted by $a_{\mu_{ij}}=[a_{\mu_{ij}L}, a_{\mu_{ij}U}]$and $a_{\gamma_{ij}}=[a_{\gamma_{ij}L}, a_{\gamma_{ij}U}]$with the condition$[a_{\mu_{ij}U}+a_{\gamma_{ij}U}]<1$ and for each interval $a_{\mu_{ij}L}+a_{\mu_{ij}U}=1=a_{\gamma_{ij}L}+a_{\gamma_{ij}U}$for $i,j=1,2,3,\dots n$.

Definition 3.2.

Let $V_{2\times2}$ denotes set of all 2×2 interval valued intuitionistic transition matrices (IVIFTMs) over the fuzzy algebra [0, 1]. The operations (+, .) are defined on $V_{2\times2}$ as follows

For any two elements $A=\{\langle[a_{\mu_{ij}L}, a_{\mu_{ij}U}],[a_{\gamma_{ij}L}, a_{\gamma_{ij}U}]\rangle\}$ and $B=\{\langle[b_{\mu_{ij}L}, b_{\mu_{ij}U}],[b_{\gamma_{ij}L}, b_{\gamma_{ij}U}]\rangle\}\in V_{2\times2}$

$A+B=\langle[a_{\mu_{ij}L}, a_{\mu_{ij}U}],[a_{\gamma_{ij}L}, a_{\gamma_{ij}U}]\rangle+\langle[b_{\mu_{ij}L}, b_{\mu_{ij}U}],[b_{\gamma_{ij}L}, b_{\gamma_{ij}U}]\rangle$
$=\{\langle[a_{\mu_{ij}L}+b_{\mu_{ij}L}],[a_{\mu_{ij}U}+b_{\mu_{ij}U}]\rangle,\langle[a_{\gamma_{ij}L}+b_{\gamma_{ij}U}],[a_{\gamma_{ij}U},+b_{\gamma_{ij}U}]\rangle\}$

,

Since

$$\left[A+B=\left\{\begin{matrix} a_{\mu_{ij}L}+b_{\mu_{ij}U},\ if a_{\mu_{ij}L}+b_{\mu_{ij}U}<1 \\ a_{\mu_{ij}L}+b_{\mu_{ij}U}-1, if\ a_{\mu_{ij}L}+b_{\mu_{ij}U}>1\end{matrix}\right\}\right]$$

Example3.3.

Let

$$A=\begin{bmatrix}\langle[0.1,0.9],[0.1,0.9]\rangle & \langle[0.2,0.8],[0.3,0.7]\rangle \\ \langle[0.2,0.8],[0.3,0.7]\rangle & \langle[0.1,0.9],[0.1,0.9]\rangle\end{bmatrix}$$

$$B=\begin{bmatrix}\langle[0.2,0.8],[0.3,0.7]\rangle & \langle[0.3,0.7],[0.2,0.8]\rangle \\ \langle[0.2,0.8],[0.1,0.9]\rangle & \langle[0.3,0.7],[0.1,0.9]\rangle\end{bmatrix}$$

be the two IVIFTMs. Then,

$$A+B=\begin{bmatrix}\langle[0.3,0.7],[0.4,0.6]\rangle & \langle[0.5,0.5],[0.5,0.5]\rangle \\ \langle[0.4,0.6],[0.4,0.6]\rangle & \langle[0.4,0.6],[0.2,0.8]\rangle\end{bmatrix}$$

Proposition 3.4.

Let $A,B,C\varepsilon\ IVIFTM$.

(i) A+B=B+A

(ii) (A+B)+C=A+(B+C)

(iii) A+0=0+A=A

Proof.

Let $A=\{\langle[a_{\mu_{ij}L},a_{\mu_{ij}U}],[a_{\gamma_{ij}L},a_{\gamma_{ij}U}]\rangle\}$, $B=\{\langle[b_{\mu_{ij}L},b_{\mu_{ij}U}],[b_{\gamma_{ij}L},b_{\gamma_{ij}U}]\rangle\}$ and $C=\{\langle[c_{\mu_{ij}L},c_{\mu_{ij}U}],[c_{\gamma_{ij}L},c_{\gamma_{ij}U}]\rangle\}$

(i) $A+B=\langle[a_{\mu_{ij}L},a_{\mu_{ij}U}],[a_{\gamma_{ij}L},a_{\gamma_{ij}U}]\rangle+\langle[b_{\mu_{ij}L},b_{\mu_{ij}U}],[b_{\gamma_{ij}L},b_{\gamma_{ij}U}]\rangle$

$=\{\langle[a_{\mu_{ij}L}+b_{\mu_{ij}L}],[a_{\mu_{ij}U}+b_{\mu_{ij}U}]\rangle,\langle[a_{\gamma_{ij}L}+b_{\gamma_{ij}U}],[a_{\gamma_{ij}U}+b_{\gamma_{ij}U}]\rangle\}$

$=\{\langle[b_{\mu_{ij}L}+c_{\mu_{ij}L}],[b_{\mu_{ij}U}+a_{\mu_{ij}U}]\rangle,\langle[b_{\gamma_{ij}L}+a_{\gamma_{ij}U}],[b_{\gamma_{ij}U}+a_{\gamma_{ij}U}]\rangle\}$

$=\langle[b_{\mu_{ij}L},b_{\mu_{ij}U}],[b_{\gamma_{ij}L},b_{\gamma_{ij}U}]\rangle+\langle[a_{\mu_{ij}L},a_{\mu_{ij}U}],[a_{\gamma_{ij}L},a_{\gamma_{ij}U}]\rangle$

A+B$=B+A$

(ii) $(A+B)+C=\left\{\langle[a_{\mu_{ij}L},a_{\mu_{ij}U}],[a_{\gamma_{ij}L},a_{\gamma_{ij}U}]\rangle+\langle[b_{\mu_{ij}L},b_{\mu_{ij}U}],[b_{\gamma_{ij}L},b_{\gamma_{ij}U}]\rangle\right\}+\langle[c_{\mu_{ij}L},c_{\mu_{ij}U}],[c_{\gamma_{ij}L},c_{\gamma_{ij}U}]\rangle$

$=\{\{\langle[a_{\mu_{ij}L}+b_{\mu_{ij}L}],[a_{\mu_{ij}U}+b_{\mu_{ij}U}]\rangle,\langle[a_{\gamma_{ij}L}+b_{\gamma_{ij}U}],[a_{\gamma_{ij}U}+b_{\gamma_{ij}U}]\rangle\}\}$
$+\langle[c_{\mu_{ij}L},c_{\mu_{ij}U}],[c_{\gamma_{ij}L},c_{\gamma_{ij}U}]\rangle$

$=\{\langle[(a_{\mu_{ij}L}+b_{\mu_{ij}L})+c_{\mu_{ij}L}],[(a_{\mu_{ij}U}+b_{\mu_{ij}U})+c_{\mu_{ij}U}]\rangle,\langle[(a_{\gamma_{ij}L}+b_{\gamma_{ij}U})+c_{\gamma_{ij}L}],[(a_{\gamma_{ij}U}+b_{\gamma_{ij}U})+c_{\gamma_{ij}U}]\rangle\}$

$=\{\langle[a_{\mu_{ij}L}+(b_{\mu_{ij}L}+c_{\mu_{ij}L})],[a_{\mu_{ij}U}+(b_{\mu_{ij}U}+c_{\mu_{ij}U})]\rangle,\langle[a_{\gamma_{ij}L}+(b_{\gamma_{ij}U}+c_{\gamma_{ij}L})],[a_{\gamma_{ij}U}+(b_{\gamma_{ij}U}+c_{\gamma_{ij}U})]\rangle\}$

$=\langle[a_{\mu_{ij}L},a_{\mu_{ij}U}],[a_{\gamma_{ij}L},a_{\gamma_{ij}U}]\rangle+$

$\{\{\langle[b_{\mu_{ij}L}+c_{\mu_{ij}L}],[b_{\mu_{ij}U}+c_{\mu_{ij}U}]\rangle,\langle[b_{\gamma_{ij}L}+c_{\gamma_{ij}U}],[b_{\gamma_{ij}U}+c_{\gamma_{ij}U}]\rangle\}\}$

$=A+(B+C)$

Since $0.1\le lower\ value\le 0.5, 0.5\le upper\ value\le 1$.

Therefore $(A+B)+C=A+(B+C)$

(iii) $A+0=\left\{\langle[a_{\mu_{ij}L},a_{\mu_{ij}U}],[a_{\gamma_{ij}L},a_{\gamma_{ij}U}]\rangle\right\}+\{\langle[0,0],[0,0]\rangle\}$

$=\{\langle[a_{\mu_{ij}L}+0],[a_{\mu_{ij}U}+0]\rangle,\langle[a_{\gamma_{ij}L}+0],[a_{\gamma_{ij}U},+0]\rangle\}$
$=\{\langle[a_{\mu_{ij}L},a_{\mu_{ij}U}]\rangle,\langle[a_{\gamma_{ij}L},a_{\gamma_{ij}U}]\rangle\}$
$=A$

Hence $A+0=0+A=A$.

Theorem 3.5.

Let A be an IVIFTM of any order then, $A+A\neq A$.

Proof.

Let $A=\left\{\langle[a_{\mu_{ij}L},a_{\mu_{ij}U}],[a_{\gamma_{ij}L},a_{\gamma_{ij}U}]\rangle\right\}$

$A+A=\left\{\langle[a_{\mu_{ij}L},a_{\mu_{ij}U}],[a_{\gamma_{ij}L},a_{\gamma_{ij}U}]\rangle\right\}+$
$=\{\langle[a_{\mu_{ij}L}+a_{\mu_{ij}L}],[a_{\mu_{ij}U}+c_{\mu_{ij}U}],[a_{\gamma_{ij}L}+a_{\gamma_{ij}L}],[a_{\gamma_{ij}U}+a_{\gamma_{ij}U}]\rangle\}$
$=\left\{\langle[b_{\mu_{ij}L},b_{\mu_{ij}U}],[b_{\gamma_{ij}L},b_{\gamma_{ij}U}]\rangle\right\}$
$+A$

Definition 3.5.

Let $A=\{\langle[a_{\mu_{ij}L},a_{\mu_{ij}U}],[a_{\gamma_{ij}L},a_{\gamma_{ij}U}]\rangle\}$ be the IVIFTM. Then complement of A and transpose of A is defined as follows:

(i) $\bar{A}=\{\langle[a_{\gamma_{ij}L},a_{\gamma_{ij}U}],[a_{\mu_{ij}L},c_{\mu_{ij}U}]\rangle\}$.

$A^T=\{\langle[a_{\mu_{ji}L},a_{\mu_{ji}U}],[a_{\gamma_{ji}L},a_{\gamma_{ji}U}]\rangle\}_{n\times n}$

Example3.6.

Let

$$A=\begin{bmatrix}\langle[0.2,0.8],[0.5,0.5]\rangle & \langle[0.4,0.6],[0.3,0.7]\rangle\\ \langle[0.1,0.9],[0.3,0.7]\rangle & \langle[0.2,0.8],[0.5,0.5]\rangle\end{bmatrix}$$

(i)
$$\bar{A}=\begin{bmatrix}\langle[0.5,0.5],[0.2,0.8]\rangle & \langle[0.3,0.7],[0.4,0.6]\rangle\\ \langle[0.3,0.7],[0.1,0.9]\rangle & \langle[0.5,0.5],[0.2,0.8]\rangle\end{bmatrix}$$

(ii)
$$A^T=\begin{bmatrix}\langle[0.2,0.8],[0.5,0.5]\rangle & \langle[0.1,0.9],[0.3,0.7]\rangle\\ \langle[0.4,0.6],[0.3,0.7]\rangle & \langle[0.2,0.8],[0.5,0.5]\rangle\end{bmatrix}$$

Proposition 3.7.

Let $A,B\epsilon IVIFTM$. Then the following results hold.

(i) $(A^T)^T=A$.

(ii) $(A+B)^T=A^T+B^T$.

Proof.

(i) Let $A=\left\{\langle[a_{\mu_{ij}L},a_{\mu_{ij}U}],[a_{\gamma_{ij}L},a_{\gamma_{ij}U}]\rangle\right\}$

Now, $A^T=\{\langle[a_{\mu_{ij}L},a_{\mu_{ij}U}],[a_{\gamma_{ij}L},a_{\gamma_{ij}U}]\rangle\}^T$
$=\left\{\langle[a_{\mu_{ji}L},a_{\mu_{ji}U}],[a_{\gamma_{ji}L},a_{\gamma_{ji}U}]\rangle\right\}$

$(A^T)^T=\left\{\langle[a_{\mu_{ji}L},a_{\mu_{ji}U}],[a_{\gamma_{ji}L},a_{\gamma_{ji}U}]\rangle\right\}^T$
$-\left\{\langle[a_{\mu_{ij}L},a_{\mu_{ij}U}],[a_{\gamma_{ij}L},a_{\gamma_{ij}U}]\rangle\right\}-A$

(ii) Let $A=\left\{\langle[a_{\mu_{ij}L},a_{\mu_{ij}U}],[a_{\gamma_{ij}L},a_{\gamma_{ij}U}]\rangle\right\}$,
$B=\left\{\langle[b_{\mu_{ij}L},b_{\mu_{ij}U}],[b_{\gamma_{ij}L},b_{\gamma_{ij}U}]\rangle\right\}$

$A+B=\langle[a_{\mu_{ij}L},a_{\mu_{ij}U}],[a_{\gamma_{ij}L},a_{\gamma_{ij}U}]\rangle+\langle[b_{\mu_{ij}L},b_{\mu_{ij}U}],[b_{\gamma_{ij}L},b_{\gamma_{ij}U}]\rangle$

$= \{\langle [a_{\mu_{ij}L} + b_{\mu_{ij}L}], [a_{\mu_{ij}U} + b_{\mu_{ij}U}] \rangle, \langle [a_{\gamma_{ij}L} + b_{\gamma_{ij}U}], [a_{\gamma_{ij}U} + b_{\gamma_{ij}U}] \rangle\}$

$$(A+B)^T = \{\langle [a_{\mu_{ij}L} + b_{\mu_{ij}L}], [a_{\mu_{ij}U} + b_{\mu_{ij}U}] \rangle, \langle [a_{\gamma_{ij}L} + b_{\gamma_{ij}U}], [a_{\gamma_{ij}U}, + b_{\gamma_{ij}U}] \rangle\}^T$$

$$= \{\langle [a_{\mu_{ji}L} + b_{\mu_{ji}L}], [a_{\mu_{ji}U} + b_{\mu_{ji}U}] \rangle, \langle [a_{\gamma_{ji}L} + b_{\gamma_{ji}U}], [a_{\gamma_{ji}U} + b_{\gamma_{ji}U}] \rangle\}$$

$$= \{\langle [a_{\mu_{ji}L}, a_{\mu_{ji}U}], [a_{\gamma_{ji}L}, a_{\gamma_{ji}U}] \rangle\} + \{\langle [b_{\mu_{ji}L}, b_{\mu_{ji}U}], [b_{\gamma_{ji}L}, b_{\gamma_{ji}U}] \rangle\}$$

$$= \{\langle [a_{\mu_{ij}L}, a_{\mu_{ij}U}], [a_{\gamma_{ij}L}, a_{\gamma_{ij}U}] \rangle\}^T + \{\langle [b_{\mu_{ij}L}, b_{\mu_{ij}U}], [b_{\gamma_{ij}L}, b_{\gamma_{ij}U}] \rangle\}^T$$

$$= A^T + B^T$$

Proposition 3.8.

Let $A, B \epsilon IVIFTM$. Then the following results hold.

(i) $\overline{(\overline{A})} = A$.

(ii) $\overline{(A+B)} = \overline{A} + \overline{B}$.

(iii) $\overline{(A)}^T = \overline{(A^T)}$.

Proof.

It follows from the definition.

Definition 3.9.

Let A and B be two IVIFTMS then we define the union and intersection as follows.

(i) The union of A and B is denoted by $A \cup B$ and defined as

$$A \cup B = \left\{ \langle x, [\sup(a_{\mu_{ij}L}, b_{\mu_{ij}L}), \inf(a_{\mu_{ij}U}, b_{\mu_{ij}U})], [\inf(a_{\gamma_{ij}L}, b_{\gamma_{ij}U}), \sup(a_{\gamma_{ij}U}, b_{\gamma_{ij}U})] \rangle / x \epsilon X \right\}$$

(ii) The intersection of A and B is denoted by $A \cap B$ and defined as

$$A \cap B = \left\{ \langle x, [\inf(a_{\mu_{ij}L}, b_{\mu_{ij}L}), \sup(a_{\mu_{ij}U}, b_{\mu_{ij}U})], [\sup(a_{\gamma_{ij}L}, b_{\gamma_{ij}U}), \inf(a_{\gamma_{ij}U}, b_{\gamma_{ij}U})] \rangle / x \epsilon X \right\}$$

Let

$$A = \begin{bmatrix} \langle [0.4, 0.6], [0.1, 0.9] \rangle & \langle [0.3, 0.7], [0.2, 0.8] \rangle \\ \langle [0.3, 0.7], [0.5, 0.5] \rangle & \langle [0.4, 0.6], [0.3, 0.7] \rangle \end{bmatrix}$$

$$B = \begin{bmatrix} \langle [0.5, 0.5], [0.3, 0.7] \rangle & \langle [0.4, 0.6], [0.2, 0.8] \rangle \\ \langle [0.1, 0.9], [0.2, 0.8] \rangle & \langle [0.3, 0.7], [0.1, 0.9] \rangle \end{bmatrix}$$

be two IVIFTMs. Then,

(i)

$$A \cup B = \begin{bmatrix} \langle [0.4, 0.6], [0.1, 0.9] \rangle & \langle [0.3, 0.7], [0.2, 0.8] \rangle \\ \langle [0.3, 0.7], [0.5, 0.5] \rangle & \langle [0.4, 0.6], [0.3, 0.7] \rangle \end{bmatrix} \cup \begin{bmatrix} \langle [0.5, 0.5], [0.3, 0.7] \rangle & \langle [0.4, 0.6], [0.2, 0.8] \rangle \\ \langle [0.1, 0.9], [0.2, 0.8] \rangle & \langle [0.3, 0.7], [0.1, 0.9] \rangle \end{bmatrix}$$

$$A \cup B = \begin{bmatrix} \langle [0.5, 0.5], [0.1, 0.9] \rangle & \langle [0.4, 0.6], [0.2, 0.8] \rangle \\ \langle [0.3, 0.7], [0.2, 0.8] \rangle & \langle [0.4, 0.6], [0.1, 0.9] \rangle \end{bmatrix}$$

(ii)

$$A \cap B = \begin{bmatrix} \langle [0.4, 0.6], [0.1, 0.9] \rangle & \langle [0.3, 0.7], [0.2, 0.8] \rangle \\ \langle [0.3, 0.7], [0.5, 0.5] \rangle & \langle [0.4, 0.6], [0.3, 0.7] \rangle \end{bmatrix} \cap \begin{bmatrix} \langle [0.5, 0.5], [0.3, 0.7] \rangle & \langle [0.4, 0.6], [0.2, 0.8] \rangle \\ \langle [0.1, 0.9], [0.2, 0.8] \rangle & \langle [0.3, 0.7], [0.1, 0.9] \rangle \end{bmatrix}$$

$$A \cap B = \begin{bmatrix} \langle [0.4, 0.6], [0.3, 0.7] \rangle & \langle [0.4, 0.6], [0.2, 0.8] \rangle \\ \langle [0.1, 0.9], [0.3, 0.7] \rangle & \langle [0.3, 0.7], [0.2, 0.8] \rangle \end{bmatrix}$$

Theorem 3.11.

Let A, B and C be the three IVIFTMs , then the following are true.

(i) $A \cup B = B \cup A$

(ii) $A \cap B = A \cap B$

(iii) $A \cup (B \cup C) = (A \cup B) \cup C$

(iv) $A \cap (B \cap C) = (A \cap B) \cap C$

Proof.

Let $A = \{\langle [a_{\mu_{ij}L}, a_{\mu_{ij}U}], [a_{\gamma_{ij}L}, a_{\gamma_{ij}U}] \rangle\}$, $B = \{\langle [b_{\mu_{ij}L}, b_{\mu_{ij}U}], [b_{\gamma_{ij}L}, b_{\gamma_{ij}U}] \rangle\}$ and $C = \{\langle [c_{\mu_{ij}L}, c_{\mu_{ij}U}], [c_{\gamma_{ij}L}, c_{\gamma_{ij}U}] \rangle\}$

(i)

$$A \cup B = \langle [a_{\mu_{ij}L}, a_{\mu_{ij}U}], [a_{\gamma_{ij}L}, a_{\gamma_{ij}U}] \rangle \cup \langle [b_{\mu_{ij}L}, b_{\mu_{ij}U}], [b_{\gamma_{ij}L}, b_{\gamma_{ij}U}] \rangle$$

$$= \{\langle \sup(a_{\mu_{ij}L}, b_{\mu_{ij}L}), \inf(a_{\mu_{ij}U}, b_{\mu_{ij}U}) \rangle, \langle \inf(a_{\gamma_{ij}L}, b_{\gamma_{ij}U}), \sup(a_{\gamma_{ij}U}, b_{\gamma_{ij}U}) \rangle\}$$

$$= \{\langle \sup(b_{\mu_{ij}L}, a_{\mu_{ij}L}), \inf(b_{\mu_{ij}U}, a_{\mu_{ij}U}) \rangle, \langle \inf(b_{\gamma_{ij}L}, a_{\gamma_{ij}U}), \sup(b_{\gamma_{ij}U}, a_{\gamma_{ij}U}) \rangle\}$$

$$= \langle [b_{\mu_{ij}L}, b_{\mu_{ij}U}], [b_{\gamma_{ij}L}, b_{\gamma_{ij}U}] \rangle \cup \langle [a_{\mu_{ij}L}, a_{\mu_{ij}U}], [a_{\gamma_{ij}L}, a_{\gamma_{ij}U}] \rangle$$

$$A \cup B = B \cup A$$

(ii)

$$A \cap B = \langle [a_{\mu_{ij}L}, a_{\mu_{ij}U}], [a_{\gamma_{ij}L}, a_{\gamma_{ij}U}] \rangle \cap \langle [b_{\mu_{ij}L}, b_{\mu_{ij}U}], [b_{\gamma_{ij}L}, b_{\gamma_{ij}U}] \rangle$$

$$= \{\langle \inf(a_{\mu_{ij}L}, b_{\mu_{ij}L}), \sup(a_{\mu_{ij}U}, b_{\mu_{ij}U}) \rangle, \langle \sup(a_{\gamma_{ij}L}, b_{\gamma_{ij}U}), \inf(a_{\gamma_{ij}U}, b_{\gamma_{ij}U}) \rangle\}$$

$$= \{\langle \inf(b_{\mu_{ij}L}, a_{\mu_{ij}L}), \sup(b_{\mu_{ij}U}, a_{\mu_{ij}U}) \rangle, \langle \sup(b_{\gamma_{ij}L}, a_{\gamma_{ij}U}), \inf(b_{\gamma_{ij}U}, a_{\gamma_{ij}U}) \rangle\}$$

$$= \langle [b_{\mu_{ij}L}, b_{\mu_{ij}U}], [b_{\gamma_{ij}L}, b_{\gamma_{ij}U}] \rangle \cap \langle [a_{\mu_{ij}L}, a_{\mu_{ij}U}], [a_{\gamma_{ij}L}, a_{\gamma_{ij}U}] \rangle$$

$$A \cap B = B \cap A$$

(iii) $(A \cup B) \cup C = \{\langle [a_{\mu_{ij}L}, a_{\mu_{ij}U}], [a_{\gamma_{ij}L}, a_{\gamma_{ij}U}] \rangle \cup \langle [b_{\mu_{ij}L}, b_{\mu_{ij}U}], [b_{\gamma_{ij}L}, b_{\gamma_{ij}U}] \rangle\} \cup$

$\langle [c_{\mu_{ij}L}, c_{\mu_{ij}U}], [c_{\gamma_{ij}L}, c_{\gamma_{ij}U}] \rangle$

$= \{\langle \sup(a_{\mu_{ij}L}, b_{\mu_{ij}L}), \inf(a_{\mu_{ij}U}, b_{\mu_{ij}U}) \rangle, \langle \inf(a_{\gamma_{ij}L}, b_{\gamma_{ij}L}), \sup(a_{\gamma_{ij}U}, b_{\gamma_{ij}U}) \rangle\}$

$\cup \langle [c_{\mu_{ij}L}, c_{\mu_{ij}U}], [c_{\gamma_{ij}L}, c_{\gamma_{ij}U}] \rangle$

$= \{\langle \sup[(a_{\mu_{ij}L}, b_{\mu_{ij}L}), c_{\mu_{ij}L}], \inf[(a_{\mu_{ij}U}, b_{\mu_{ij}U}), c_{\mu_{ij}U}] \rangle,$

$\langle \inf[(a_{\gamma_{ij}L}, b_{\gamma_{ij}U}), c_{\gamma_{ij}L}], \sup[(a_{\gamma_{ij}U}, b_{\gamma_{ij}U})], c_{\gamma_{ij}U}] \rangle\}$

$= \{\langle \sup[a_{\mu_{ij}L}, (b_{\mu_{ij}L}, c_{\mu_{ij}L})], \inf[a_{\mu_{ij}U}, (b_{\mu_{ij}U}, c_{\mu_{ij}U})] \rangle,$

$\langle \inf[a_{\gamma_{ij}L}, (b_{\gamma_{ij}U}, c_{\gamma_{ij}L})], \sup[a_{\gamma_{ij}U}, (b_{\gamma_{ij}U}, c_{\gamma_{ij}U})] \rangle\}$

$= \langle [a_{\mu_{ij}L}, a_{\mu_{ij}U}], [a_{\gamma_{ij}L}, a_{\gamma_{ij}U}] \rangle \cup$

$\left\{\langle \sup(b_{\mu_{ij}L}, c_{\mu_{ij}L}), \inf(b_{\mu_{ij}U}, c_{\mu_{ij}U}) \rangle, \langle \begin{matrix} \inf(b_{\gamma_{ij}L}, c_{\gamma_{ij}U}), \\ \sup(b_{\gamma_{ij}U}, c_{\gamma_{ij}U}) \end{matrix} \rangle\right\}$

$$(A \cup B) \cup C = A \cup (B \cup C)$$

(iv) $(A \cap B) \cap C = \{\langle [a_{\mu_{ij}L}, a_{\mu_{ij}U}], [a_{\gamma_{ij}L}, a_{\gamma_{ij}U}] \rangle \cap \langle [b_{\mu_{ij}L}, b_{\mu_{ij}U}], [b_{\gamma_{ij}L}, b_{\gamma_{ij}U}] \rangle\} \cap$

$\langle [c_{\mu_{ij}L}, c_{\mu_{ij}U}], [c_{\gamma_{ij}L}, c_{\gamma_{ij}U}] \rangle$

$= \left\{\langle \inf(a_{\mu_{ij}L}, b_{\mu_{ij}L}), \sup(a_{\mu_{ij}U}, b_{\mu_{ij}U}) \rangle, \langle \begin{matrix} \sup(a_{\gamma_{ij}L}, b_{\gamma_{ij}U}), \\ \inf(a_{\gamma_{ij}U}, b_{\gamma_{ij}U}) \end{matrix} \rangle\right\}$

$\cap \langle [c_{\mu_{ij}L}, c_{\mu_{ij}U}], [c_{\gamma_{ij}L}, c_{\gamma_{ij}U}] \rangle$

$= \{\langle \inf[(a_{\mu_{ij}L}, b_{\mu_{ij}L}), c_{\mu_{ij}L}], \sup[(a_{\mu_{ij}U}, b_{\mu_{ij}U}), c_{\mu_{ij}U}] \rangle,$

$\langle \sup[(a_{\gamma_{ij}L}, b_{\gamma_{ij}U}), c_{\gamma_{ij}L}], \inf[(a_{\gamma_{ij}U}, b_{\gamma_{ij}U})], c_{\gamma_{ij}U}] \rangle\}$

$= \{\langle \inf[a_{\mu_{ij}L}, (b_{\mu_{ij}L}, c_{\mu_{ij}L})], \sup[a_{\mu_{ij}U}, (b_{\mu_{ij}U}, c_{\mu_{ij}U})] \rangle,$

$\langle \sup[a_{\gamma_{ij}L}, (b_{\gamma_{ij}U}, c_{\gamma_{ij}L})], \inf[a_{\gamma_{ij}U}, (b_{\gamma_{ij}U}, c_{\gamma_{ij}U})] \rangle\}$

$= \langle [a_{\mu_{ij}L}, a_{\mu_{ij}U}], [a_{\gamma_{ij}L}, a_{\gamma_{ij}U}] \rangle \cap$

$\left\{\langle \inf(b_{\mu_{ij}L}, c_{\mu_{ij}L}), \sup(b_{\mu_{ij}U}, c_{\mu_{ij}U}) \rangle, \langle \begin{matrix} \sup(b_{\gamma_{ij}L}, c_{\gamma_{ij}U}), \\ \inf(b_{\gamma_{ij}U}, c_{\gamma_{ij}U}) \end{matrix} \rangle\right\}$

$$(A \cap B) \cap C = A \cap (B \cap C)$$

Conclusions

In this paper we have generalized the concept of fuzzy transition matrices in interval valued fuzzy settings and provided some interesting results on it. As a further plan we intend to introduce the notion of fuzzy soft transition matrices and to provide a decision theory on it.

Conflict of Interest

Authors declare there are no conflicts of interest.

References

1. Khan SK and Pal M. Some operations on intuitionistic fuzzy matrices, Presented in International Conference on Analysis and Discrete Structures, Indian Institute of Technology, Kharagpur, India. (2002) Dec.22-24
2. Kim JB. Idempotent and inverses in fuzzy matrices, Malaysian Math, 6(2) (1983) 57-61.
3. Meenakshi AR. Fuzzy Matrix Theory and Applications, MJP Publishers, 2008.
4. Oliver CI. Fundamentals of applied probability and random processes, Elsevier Academic Press (2005).
5. Pal M and Khan SK. Interval-Valued Intuitionistic Fuzzy Matrices, NIFS 11 (2005) 1, 16-27.
6. Pal M, Khan SK and Shyamal AK. Intuitionistic fuzzy matrices, Notes on Intuitionistic Fuzzy Sets, 8(2) (2002) 51-62.
7. Rudin W. Functional Analysis, McGraw Hill Science (1991).
8. Tuckwell HC. Elementary Applications of Probability Theory, 2nd Ed., Chapman and Hall, London (1995).
9. Vijayabalaji S, Shyamsundar G, Parthiban J and Narayanan T, Fuzzy operations on transition matrices, Global Journal of Pure and Applied Mathematics, (Accepted), 2016.

