Math 1497 - Calc 2

Last class $\int \sin^m x \cos^n x \, dx$

(1) $u = \sin x$, (2) $u = \cos x$ (3) $\sin^2 x = \frac{1-\cos 2x}{2}$

$\cos^2 x = \frac{1+\cos 2x}{2}$

Today we consider

$$\int \tan^n x \sec^n x \, dx$$

and we look at the following cases:

(i) n odd $\sec x$ present

(ii) n even

(iii) $\tan x$ only

(iv) $\sec x$ only

ex 1 $\int \tan^3 x \sec^3 x \, dx$

$\int \tan^2 x \sec^2 x \sec x \tan x \, dx$

$u = \sec x \quad du = \sec x \tan x \, dx$

$1 + \tan^2 x = \sec^2 x$

so $\int (u^2 - 1) u^2 \, du = \int u^4 - u^2 \, du = \frac{u^5}{5} - \frac{u^3}{3} + C$

$= \frac{1}{5} \sec^5 x - \frac{1}{3} \sec^3 x + C$

ex 2 n even

$\int \tan^3 x \sec^4 x \, dx$ let $u = \tan x$

$du = \sec^2 x \, dx$

$\int \tan^3 x \sec^2 x \sec^2 x \, dx$

$\int u^3 (1 + u^2) \, du = \int u^3 + u^5 \, du = \frac{1}{4} u^4 + \frac{u^6}{6} + C$

$= \frac{1}{4} \tan^4 x + \frac{1}{6} \tan^6 x + C$

Ex3 $\tan x$ only

$$\int \tan x\,dx = -\ln|\cos x| + C$$

$$\int \tan^2 x\,dx = \int \sec^2 x - 1\,dx = \tan x - x + C$$

$$\int \tan^3 x\,dx = \int \tan x \tan^2 x\,dx$$

$$= \int \tan x (\sec^2 x - 1)\,dx$$

$$= \int \tan x \sec^2 x\,dx - \int \tan x\,dx$$

$u = \tan x$

$$= \frac{\tan^2 x}{2} - (-\ln|\cos x|) + C$$

$$\int \tan^4 x\,dx = \int \tan^2 x \tan^2 x\,dx = \int \tan^2 x(\sec^2 x - 1)\,dx$$

$$= \int \tan^2 x \sec^2 x\,dx - \int \tan^2 x\,dx$$

$u = \tan x$ ↑ done earlier

(iv) $\int \sec^n x\,dx$ only

$$\int \sec x\,dx = \ln|\sec x + \tan x| + c$$

$$\int \sec^2 x\,dx = \tan x + c$$

$$\int \sec^3 x\,dx = \int \sec x \sec^2 x\,dx \Leftarrow \text{int-parts}$$

$u = \sec x \qquad v = \tan x$

$du = \sec x \tan x\,dx \qquad dv = \sec^2 x\,dx$

$$= \sec x \tan x - \int \sec x \tan^2 x\,dx$$

$$= \sec x \tan x - \int \sec x(\sec^2 x - 1)\,dx$$

$$= \sec x \tan x - \int \sec^3 x\,dx + \int \sec x\,dx$$

$$2\int \sec^3 x\,dx = \sec x \tan x + \ln|\sec x + \tan x|$$

$$\int \sec^3 x\,dx = \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| + c$$

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$$\int_0^{\pi/3} \sec^{3/2} x \tan x \, dx$$

$$\int_0^{\pi/3} \sec^{1/2} x \sec x \tan x \, dx$$

let $u = \sec x \quad du = \sec x \tan x \, dx$

$x = 0 \quad u = \sec 0 = \frac{1}{\cos 0} = 1$

$x = \pi/3 \quad u = \sec \pi/3 = \frac{1}{\cos \pi/3} = \frac{1}{1/2} = 2$

$$\int_1^2 u^{1/2} du = \frac{2u^{3/2}}{3} \Big|_1^2 = \frac{2}{3} 2^{3/2} - \frac{2}{3}$$

$$= \frac{4\sqrt{2}}{3} - \frac{2}{3}$$