

# Model Identification and Validation for a Nonlinear Process using Recurrent Neural Networks

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**Abstract** - System identification of non-linear process system is important and beneficial in the process industries. The main objective for the modelling task is to obtain a good and reliable tool for analysis and control system development. In this work, a model identification of a nonlinear process is performed by Nonlinear Autoregressive exogenous (NARX) Recurrent Neural Network and Elman Recurrent Neural Network (ERNN) approach. Developed models performance are analysed and best among them can be used in off-line controller design and implementation of new advanced control schemes. Here, a challenging non-linear process level control in the gravity discharge tank taken into an account, since its nonlinearity and constantly changing of cross section with respect to rise in liquid level. The model for such non-linear process were to be identified for different operating regions and are approximated to first order plus dead time model. The developed models performance is analysed for different regions and best among the two approaches, one is highlighted.

**Keywords** - System Identification, Conical Tank, NARX Recurrent Neural Network, ERNN, Nonlinear Process.

## I. INTRODUCTION

Neural networks have become a accepted tool for the modeling and identification of linear and nonlinear dynamic systems. System identification for a system, which is based on measured experimental data and artificial neural network (ANN), can be used to identify dynamic systems in order to design an ANN controller based on the system model. In many applications use static neural networks to build nonlinear input– output models of the plant. Due to its features, such as massive parallelism, flexibility, forcefulness and the inherent capability to handle nonlinear systems, this technique have been extensively used in complex nonlinear function mapping, image processing [1,2], pattern recognition and classification. The use of neural networks for dynamic system identification has been extensively researched in the past few decades. The attention of researchers was first focused on static networks such as multilayer perceptions (MLPs) [5], [6] and radial basis functions [7]. The first dynamic neural networks (DNNs) were introduced by Hopfield in the context of associative memory [8], [9] and [10], but later modifications made them capable of approximating multivariable dynamic systems. It is assured by many researchers a good model can be used in controller design and implementation of new

advanced control schemes. So identification of perfect model to a system is essential, for such cases researcher were focused to obtain an effective model [11 -14] for the linear and nonlinear systems. Normally ANNs are classified into feed forward ANNs and feedback ANNs. The feed forward ANNs can be used for static behaviour applications like pattern recognition. But ANNs cannot understand the system dynamics when it is in feed forward fashion, hence demanding an alternate approach. This is achieved by recurrent or feedback neural networks (RNNs).

This paper explores the problems of training and initialization of neural networks using NARX recurrent neural network and ERNN. At first, a classical approach has been proposed [11] to get the dynamics of the conical tank. It has non-linear structure which leads the liquid in the tank rises with respect to the inclination angle through which the tank is designed is taken up for real time analysis. Due to the non-linearity in the shape of the conical tank, a single range response cannot cover the entire range. So, full range of conical tank is sliced into different regions by introducing step change at various ranges and is divided into six operating regions. The obtained models are considered as source for network construction. In this paper, different approaches are proposed such as NARX and ERNN [3, 4] and the best one highlighted. Already some articles have been developed for nonlinear system modelling using ANNs [19, 20].

The paper is organized as follows. Section 2 process descriptions, development of mathematical model are discussed. Section 3 classical approach is discussed and models are identified from real time process. Section 4 describes different types of neural network modeling approaches. Section 5 results and discussions are discussed. Conclusions are arrived based on results and discussion in section 6.

## II. PROCESS DESCRIPTION

The conical tank system shown in Figure1 is a system with nonlinear dynamics. Its nonlinearity is described by the differential equation. It is derived according to law of conservation of mass. The proposed process is a conical tank level process. Generally this conical tank process is nonlinear in nature. In most industries like cement, sugar, etc., the cone shaped kettle is used for process action. Therefore the conical tank process is chosen as the proposed process. The conical tank of mathematical model is described in the following section.

A. Mathematical Model

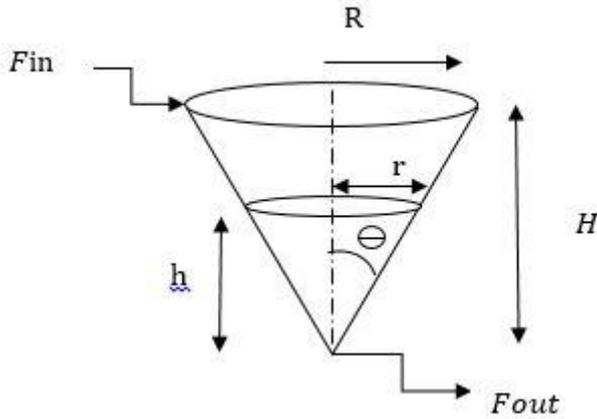


Fig.1: Schematic Diagram of Conical Tank

Here, the considered process is conical tank level process, in which the liquid level in the tank is maintained at a constant rate. It is done by adjusting the input flow rate into the tank. The block diagram of the conical tank is shown in figure 1. From the law of conservation mass,

$$F_{in} - F_{out} = \frac{A dh}{dt} \tag{1}$$

$$A = \pi r^2 \tag{2}$$

Where,  $F_{in}$  is inlet flow rate to the tank,  $F_{out}$  is the outlet flow rate from the tank,  $R$  is the radius of the tank at top level.  $H$  is the total height of the tank in cm and  $r$  is the radius at any height  $h$  in cm, then the area of the tank is written as,

$$\tan \theta = \frac{r}{h} = \frac{R}{H} \tag{3}$$

The conical tank volume in terms of height is

$$V(h) = \pi \left( \frac{R_2}{h_2} \right) h^3 \tag{4}$$

So, the relation between height and inlet flow of the tank is written as,

$$\frac{dh_3}{dt} = F_{in}(t-t_d) - K_v \sqrt{h} \tag{5}$$

When the flow is considered as gravity flow then, delay time can be calculated as,

$$t_d = (H-h) + \frac{r}{g} \tag{6}$$

The radius of the conical tank at any height  $h$  is,

$$r = \left( \frac{R}{H} \right) h \tag{7}$$

When height ( $h$ ) becomes zero, the area becomes lesser and the rate of change of volume ( $dV/dt$ ) tends to infinity. Due to the conical tank's outlet flow pipeline, the height never gets to be zero.

Considering the initial height, the final mode equation becomes,

$$d \frac{(h+H_{int})^3}{dt} = F_{in}(t-t_d) - K_v \sqrt{\frac{h+H_{in}}{\pi \left( \frac{R_2}{H_2} \right)}} \tag{8}$$

Equation (8) is the conical tank for the mathematical model. From the mathematical model the desired control action can be performed. But this mathematical model cannot certainty for all process. Therefore, the empirical approach is applied to system identification.

III. SYSTEM IDENTIFICATION

In system identification task, the conical tank identification (modelling) is done by NARX and Elman recurrent neural network based artificial neural network (ANN) architecture. The proposed model is obtained by mapping the input and output data histories. So, transfer function based classical approach is used [11] to obtain the data histories. In this approach, the system is considered as a block box or unknown system [22]. By experimental approach for random input flow rates chosen and its corresponding output is obtained as level values. From the experimental results, the process response is acquired as a nonlinear curve. The non-linear characteristic of process which leads the liquid in the tank rises with respect to the inclination angle through which the tank is designed. Due to the non-linearity in the shape of the conical tank, a single range response cannot cover the entire range. So, full range of conical tank is sliced into different regions by introducing step change at various ranges and is divided into six operating regions. From the data histories, the different network models are designed for the conical tank level process.

A. Classical Approach

Step response based method is the most commonly used method for identifying system parameters. At first, the inlet valve is at fully opened condition and outlet valve is set to a particular restriction. The open loop step response is obtained by varying the flow rate, the experimental results are noted in terms of time and height or level. The models are identified by process reaction curve method (PRC) [21] and Sunderasan Kumaraswamy (SK) method [11] method. For a change in step function the PRC method produces a response, from the response parameters like dead time ( $\tau_d$ ), the time taken for the response to change ( $\tau$ ), and the

ultimate value that the response reaches at steady state,  $\tau = 63.2\%$  of the maximum value are measured and Sunderasan and Kumaraswamy(SK) method [11] is used to develop model from the obtained response. As per the structure of the curves, the FOPTD model is given by,

$$G(s) = \frac{k_p e^{-\tau_d(s)}}{\tau s + 1} \tag{9}$$

Where,  $K$  is the process gain,  $\tau$  is the first order time constant and  $\tau_d$  is the delay time. The derived models are validated with real time results [8] and model obtained through SK method of identification is effectively suitable to real time response. From the response of the real time system we obtain the mentioned constants for SK method and thereby we get the FOPTD models for the real time non-linear tank process using equations 9 and 10. The process dynamics are evaluated in six regions so as to obtain effective models for different operating ranges. We are defining 6 operating regions here [11]. The first region is between 12 to 20cm, second region between 21 to 22cm, third region between 22 to 27cm, fourth region between 27 to 32cm, fifth region between 31 to 35cm, and the final region i.e. the sixth region is set between 34 to 38cm. Here, the different linearized regions information is shown in table 1.

$$\tau = 0.67(t_2 - t_1) \tag{10.a}$$

$$\tau_d = 1.3 t_1 - 0.29 t_2 \tag{10.b}$$

Table 1. Process Parameters of Conical Tank for Six Different Operating Regions.

Description	Height (cm)	Flow rate (LPH)	Gain (k)	Time constant ( $\tau$ )	Time Delay (rd)
Region - 1	12.00 to 20.00	290 to 305	0.02510	46.90	0.10
Region - 2	19.8 0 to 21.20	305 to 320	0.00490	36.85	10.90
Region - 3	21.00 to 26.80	320 to 335	0.01780	294.80	4.68
Region - 4	26.20 to 31.50	335 to 350	0.01550	274.70	3.20
Region - 5	31.50 to 34.10	370 to 385	0.00689	341.70	1.37
Region - 6	34.00 to 38.20	385 to 400	0.00994	415.40	2.00

From these six different region models of the conical tank, the necessary input and output data are obtained. By using these data histories, the NARX recurrent neural network and ERNN are designed and shown in table 2. From the data histories, for random input and output information the network is trained and the proposed process model is

designed. This network model of ERNN and NARX network is compared against transfer function based classical approach model for validation check. The main purpose of this modeling in this article is to implement a performance analysis of approach.

*B. Prediction by NARX Neural Network*

The NARX neural network (NARX 1990) is one of the recurrent network types. In this type, the network nodes are partially interconnected [18, 23]. The NARX neural network system model is designed with the help of classical approach. The classical method is a source for network construction. In this approach of modeling, there are three neurons used in hidden layer and one neuron used in output layer, since the process is first order plus a single state variable process.

Due to this neuron selection, the problem of the network can be avoided from over fitting. Finally the NARX model is designed for the proposed system by using NARX neural network for separate regions. The initial and final conditions of different regions are used to make a combined model for nonlinear conical tank level process without changing its process behaviour [24]. Finally, the planned conical tank level process model is designed by using approach. The desired NARX neural network architecture is shown in figure 2, with three hidden in addition with one input and one output neuron.

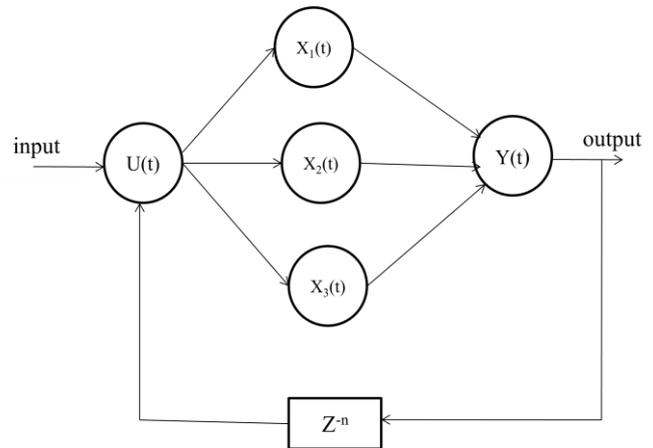


Fig.2. The NARX Network Architecture

The prediction is accomplished in NARX neural network by using the past value information of hidden node output and network input. In NARX neural network, the dynamic behaviours are observed by tracking the network states and used in future prediction. The data processing of NARX neural network is explained as, at time  $t=k$ , the input to network is the combination of the past value of the hidden node (at time  $t=k-1$ ) and the input value histories (at time  $t=k$ ) are used, where the previous value of hidden node is stored in context node. Here the weight updating is done in

usual manner by comparing the actual and desired output. In this network, Bayesian regulation learning algorithm is used instead of Levenberg–Marquardt learning algorithm for network adoption. By this adoption mechanism, the observation of system dynamics is acquired well. Let consider, input to the network is  $U(k)$  and the network output by  $Y(k)$ . The total input to the hidden unit is denoted as  $Z(k)$ . The output of the hidden unit is denoted as  $X(k)$  then  $W^{UX}$  and  $W^{XY}$  are the weights from input to hidden unit and hidden to output unit respectively. Now, the network prediction is mathematically calculated as,

$$X(k) = f\{Z(k)\} \tag{11}$$

$$Y(k) = W^{xy}(k)X(k) \tag{13}$$

$$Z(k) = W^{ux1}(k)U(k) + W^{ux2}(k)U(k) + W^{ux3}(k)U(k) \tag{14}$$

When consider  $n$  state, it is in the following form,

$$X(k) = \sum_{k=0}^n W^{ux}(k)X(k) \tag{15}$$

$$Y(k) = W^{xy}(k)X(k)$$

At  $k=1$ , the output node value is feedback to the input node.

So  $U(k) = Y(k - 1)$  in (6) and then, consider the hidden unit as linear then,

$$X(k) = \sum_{k=0}^n W^{ux}(k) Y(k-1)U(k) \tag{17}$$

And then,

$$Y(k) = g\{X(k)\}$$

$$Y(k) = W^{xy}(k)X(k)$$

$$Y(k) = W^{xy}(k) \left\{ \sum_{k=0}^n W^{ux}(k) Y(k) - 1 \right\} U(k) \tag{18}$$

This  $y(k)$  can be expanded in the following format,

$$Y(k) = A_1(k - 1) + A_2Y(k - 2) + \dots + A_nY(k - 2) + B_1U(k - 1) + B_2U(k - 2) + \dots + B_nU(k - n) \tag{19}$$

Hence, from the equation (18) and (19), it is clear that by using NARX neural network, any higher order dynamic system can be designed. By training at a repeatable fashion, its dynamics behaviour could be observed [15, 17]. From these mathematical derivations, it is clear that the data mapping is carried out. Identification of system by Elman

Neural Network for conical tank level process is done for the purpose of comparative analysis and to show that the NARX is better than Elman network model.

#### IV. RESULTS OF THE EXPERIMENT

Thus, NARX neural network based model is designed for the conical tank. Here the validation of based model is done by comparing the model with classical approach based transfer function model.

The best model is obtained by comparing the NARX net and ERNN model. The figure 5 shows the comparison responses, which displays the comparison between NARX net and ERNN model responses in the sub figure 5a, 5b, 5c, 5d, 5e and 5f respectively for the different six regions of the conical tank. From these following responses, it can be proved that the NARX neural network based modelling approach is superior than ERNN approach. From the response of NARX and ERNN, it is proved that the NARX network model gives the est model. The ERNN neural network model gives the indecent model to the desired process. Hence NARX network model is considered as a proposed network model and this network model are also used for further processing work such as region combination and controller design.

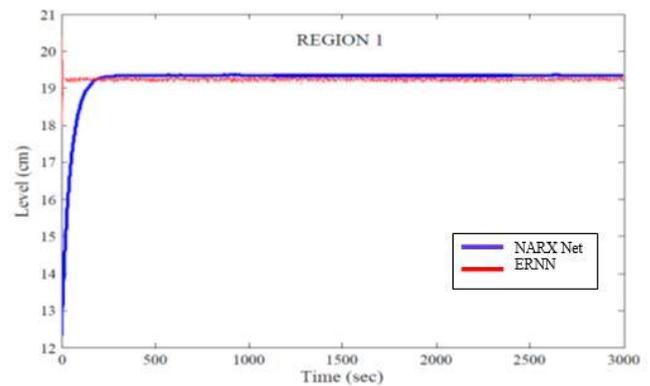


Fig.5a: For Region 1

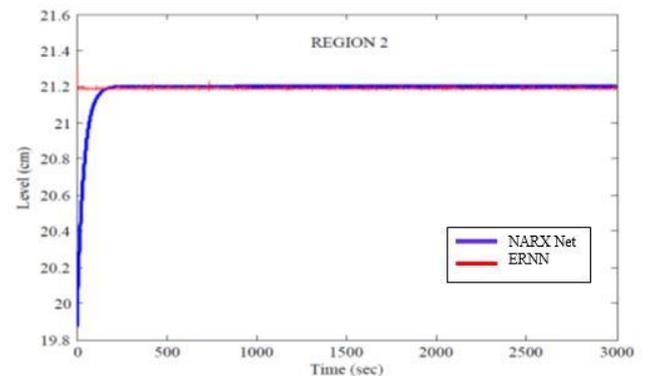


Fig.5b: For Region 2

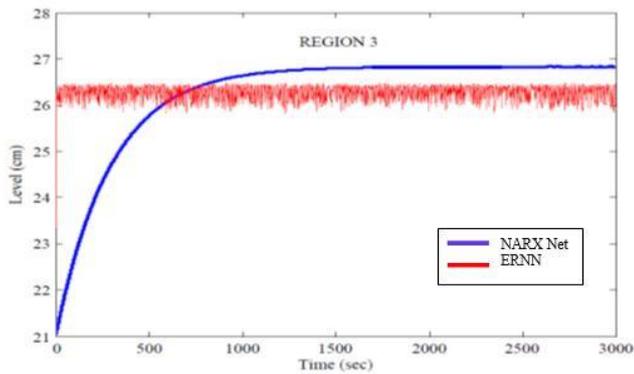


Fig.5c: For Region 3

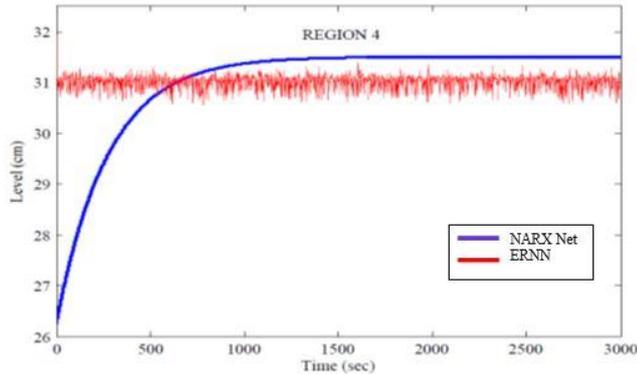


Fig.5d: For Region 4

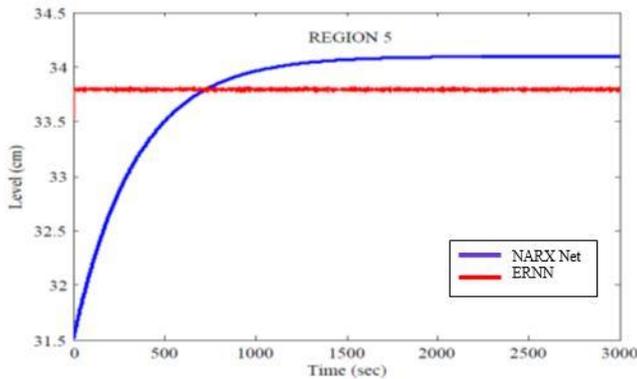


Fig.5e: For Region 5

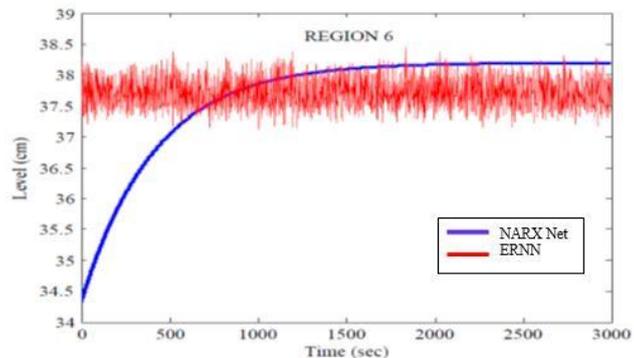


Fig.5f: For Region 6

Fig.5: NARX and ERNN Model's Comparison Responses

After obtaining the model for all six regions, the region combination is performed, where the combination of all six regions is achieved with respect to initial and final conditions of process (shown in table 1). The combination process provides a single combined model of six linearized regions to the nonlinear conical tank process. The grouping model response is shown in figure 6.

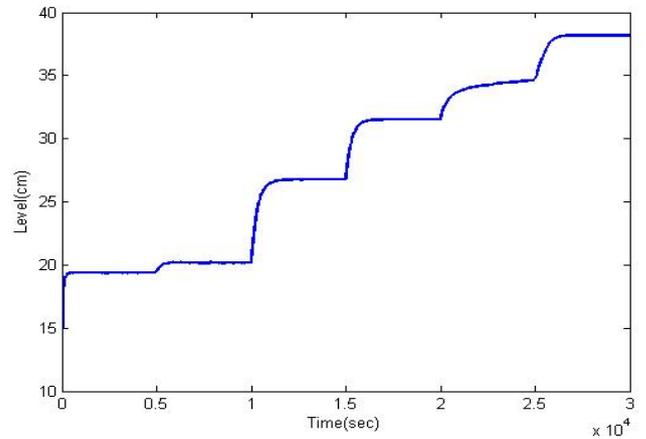


Fig.6: Combined model for Different Six Regions

Table 2. Different model comparison with MSE

Description	Network type	Number of Iterations	Mean Square Error
Region 1	NARX NET	52	$6.3 \cdot 10^{-6}$
	ERNN	104	0.0025
Region 2	NARX NET	65	$3.2 \cdot 10^{-6}$
	ERNN	112	0.056
Region 3	NARX NET	43	$2.2 \cdot 10^{-6}$
	ERNN	125	0.0089
Region 4	NARX NET	56	$1.2 \cdot 10^{-6}$
	ERNN	126	0.002
Region 5	NARX NET	73	$5.2 \cdot 10^{-6}$
	ERNN	125	$1.7 \cdot 10^{-2}$
Region 6	NARX NET	67	$4.6 \cdot 10^{-5}$
	ERNN	137	$2.7 \cdot 10^{-2}$

Now, to show the NARX neural network performance as the best, the performance measurement is carried out with ERNN. The performance measurement of is analysed based on its mean square error (MSE) .These performance measurement analysis is shown in the table 2.

From the table 2, the performance measurement is analysed. The NARX network model provided the desired response in minimum epochs with small value of MSE. But at the same number of epochs the other network that ERNN do not give a desired result and also those mean square

errors are huge when compared with NARX net. From this performance analysis the NARX model gives a better result than Elman neural network model. Hence, the NARX model is better for nonlinear dynamic process modelling.

#### V. CONCLUSION

Thus, the system identification is successfully carried out for the conical tank level process using NARX and ERNN approach. In nonlinear process modelling, the major problems are to ensure the stability factors and its dynamic behaviours. These problems were well recognised using NARX and ERNN approach in addition with classical approach. The internal states repetition enables investigation of the dynamic behaviour as well as maintains the stability factor of the process. The validation of the network model is done by comparing it region wise against the model obtained by transfer function based model. The six different models and its corresponding initial and final conditions are used to form one equivalent model. Finally the NARX based model's performance is analysed with ERNN based on its mean square error (MSE). From the performance analysis, it is proved that the NARX model based model provides better results. Hence the NARX model is validated and well suited for design the conical tank model.

#### VI. REFERENCES

- [1] Chu, S. R., Shoureshi, R., and Tenorio, M. (1990), "Neural networks for system identification", *IEEE Control Systems Magazine*, Apr.,31-34.
- [2] Masri, S.F., Chassiakos, A. G., and Caughey, T. K. (1993). "Identification of nonlinear dynamic systems using neural networks." *J. Appl. Mech.*, 60, 123-133.
- [3] D.T.Pham & X.Liu, (1996), "Training of NARX networks and dynamic system modelling," *International Journal of Systems Science*, Vol. 27, pp. 221-226.
- [4] Olivera Jovanovic, (1997), "Identification of dynamic system using neural network", *the scientific journal facta universitates in Architecture and Civil Engineering*, Vol.1, No 4, pp.525-532.
- [5] Rumelhart, D. E. and J. L. McClelland (1986). *Parallel distributed processing : explorations in the microstructure of cognition*. MIT Press. Cambridge, Mass.
- [6] Churchland, P. S., T.J. Sejnowski and T.J. Seynowski (1992). *The computational brain*. MIT Press. Cambridge, Mass.
- [7] Broomhead, D. S. and D. Lowe (1988). Multi-variable functional interpolation and adaptive networks. *Complex Systems* 2, 321-355.
- [8] Hopfield, J. J. (1984). Neurons with graded response have collective computational properties like those of 2-state neurons. *Proceedings of the National Academy of Sciences of the United States of America-Biological Sciences* 81(10), 3088- 3092.
- [9] Hopfield, J. J. and D. W. Tank (1985). Neural computation of decisions in optimization problems. *Biological Cybernetics* 52(3), 141-152.
- [10] Hopfield, J. J. and D. W. Tank (1986). Computing with neural circuits - a model. *Science* 233(4764), 625-633.
- [11] Sukanya R. Warier & Sivanandam Venkatesh, (2012) "Design of Controllers based on MPC for a Conical Tank System", *International Conference on Advances in Engineering, Science and Management (ICAESM -2012)*, pp. 309-313.
- [12] T.Pushpaveni, S.SrinivasuluRaju, N.Archana & M.Chandana, (2013) "Modelling and Controlling of Conical tank system using adaptive controllers and performance comparison with conventional PID", *International Journal of Scientific & Engineering Research*, Volume 4.
- [13] Swati Mohanty, (2009), "Artificial neural network based system Identification and model predictive control of a flotation Column", *Journal of Process Control*, pp 991-999.
- [14] P.Aravind, M.Valluvan & S.Ranganathan, (2013) "Modelling and Simulation of Non Linear Tank", *International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering*, Vol. 2.
- [15] N.S. Bhuvanewari, G. Uma & T.R. Rangaswamy, (2009), "Adaptive and optimal control of a non-linear process using Intelligent controllers", *International journal of control and intelligent system*, Applied Soft Computing 9, pp 182-190.
- [16] Moonyong Lee & Sunwon Park, (2000), "Process Control Using a Neural Network Combined with the Conventional PID Controllers", *The Institute of Control, Automation and Systems Engineers*, KOREA, Vol. 2, No. 3.
- [17] Olivera Jovanovic, (1997), "Identification of dynamic system using neural network", *the scientific journal facta universitates in Architecture and Civil Engineering*, Vol.1, No 4, pp.525-532.
- [18] Hwan Kim, Stanley Fok, Kingsley Fregene, Dong-Hoon Lee, Tae-Seok Oh & David W.L.Wang, (2004), "Neural Network Based System Identification and Controller Synthesis for an Industrial Sewing Machine," *International Journal of Control, Automation, and Systems*, Vol. 2, No. 1
- [19] M.Norgaard, O.Ravn, N.K.Poulsen & L.K.Hansen, (2009), "Neural Networks for Modelling and Control of Dynamic Systems", SPRINGER publisher.
- [20] R.Ramadevi, B.Sheela Rani & V.Prakash, (2012), "Role of Hidden Neurons in NARX Recurrent Network in Classification of Cavitations Signals", *International Journal of Computer Applications*, Vol.37, No.7, pp.9-13.
- [21] D. R. Coughanowar (1991), *Process Systems Analysis and Control*, Tata McGraw Hill.
- [22] Sjobergetal, Automatica, "Nonlinear black-box modelling in system identification", SPRINGER publisher.
- [23] D.T.Pham & X.Liu, (1996), "Training of NARX networks and dynamic system modelling," *International Journal of Systems Science*, Vol. 27, pp. 221-226.
- [24] M.Norgaard, O.Ravn, N.K.Poulsen & L.K.Hansen, (2009), "Neural Networks for Modelling and Control of Dynamic Systems", SPRINGER publisher.