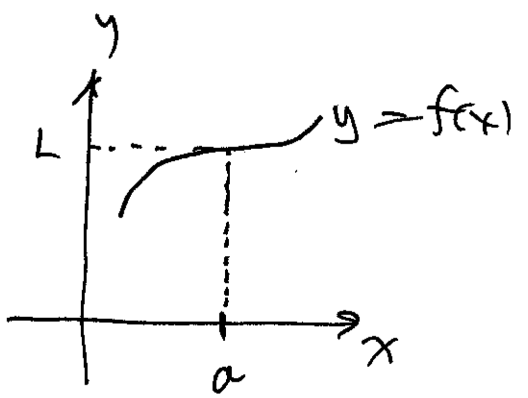
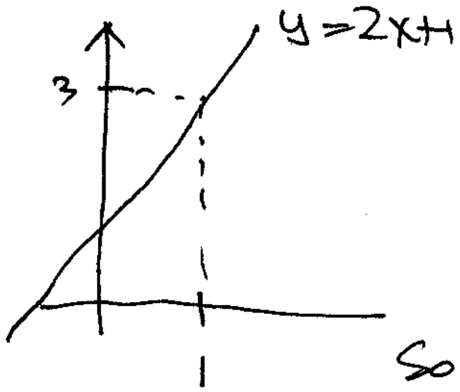
So now we are getting an idea of what $\lim_{x \to a} f(x) = L$ means.



The closer we get to $x=a$ the close $f(x)$ gets to $y=L$.

For example

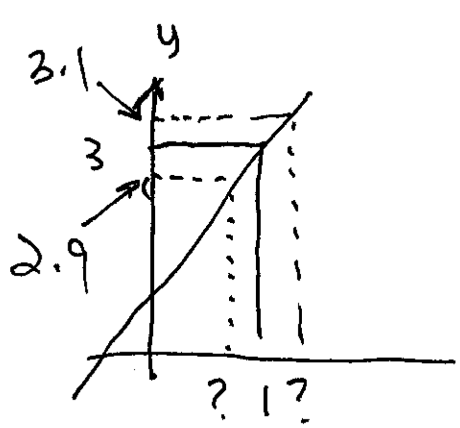
$$\lim_{x \to 1} 2x+1 = 3$$



so how close. For example suppose we want to be within .1 of 3. what values of x would we need.

So if $y=3.1$ then $2x+1=3.1$

$2x=2.1 \Rightarrow x=1.05$



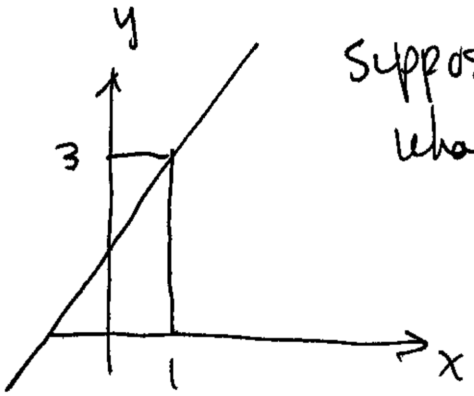
if $y=2.9$ then $2x+1=2.9$

$2x=1.9$

$x=.95$

so as long as x is

$$.95 < x < 1.05 \text{ then } 2.9 < f(x) < 3.1$$



suppose I want to be within .005 of 3
what values of x would I need?

$$2x+1 = 3.005$$

$$2x = 2.005 \Rightarrow x = 1.0025$$

$$2x+1 = 2.995 \text{ so } 2x = 1.995 \Rightarrow x = .9975$$

so if $.9975 < x < 1.0025 \Rightarrow 2.995 < f(x) < 3.005$

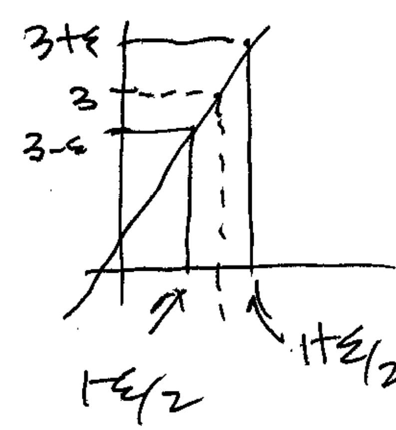
or $-.0025 < x-1 < .0025 \Rightarrow -.005 < f(x)-3 < .005$

or $|x-1| < .0025$ then $|f(x)-3| < .005$

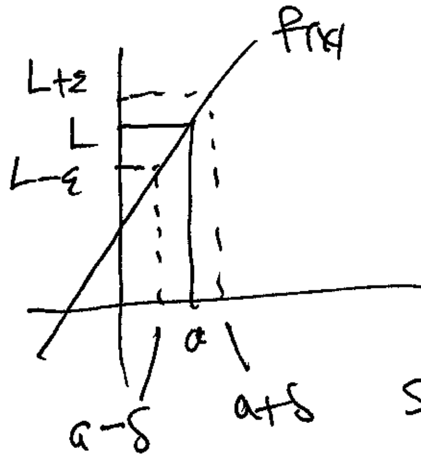
Now suppose I want to be within ε of 3
How close to 1 do I need to be

$2x+1 = 3+\varepsilon \qquad 2x = 2+\varepsilon \qquad x = 1+\varepsilon/2$

$2x+1 = 3-\varepsilon \qquad 2x = 2-\varepsilon \qquad x = 1-\varepsilon/2$



Formal Def^n of a Limit



$$\lim_{x \to a} f(x) = L$$

For every $\varepsilon > 0$ there exists $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

Ex 1 $\displaystyle\lim_{x \to 1} 2x+1 = 3$

Aside	Proof
$\|2x+1-3\| < \varepsilon$	if $\|x-1\| < \varepsilon/2$
$\|f-L\| < \varepsilon$	$2\|x-1\| < \varepsilon$
$\|2x-2\| < \varepsilon$	$\|2x-2\| < \varepsilon$
$2\|x-1\| < \varepsilon$	$\|2x+1-3\| < \varepsilon$
$\|x-1\| < \varepsilon/2$	$\|f(x)-L\| < \varepsilon$ ✓

choose $\delta = \varepsilon/2$

Ex 2 $\lim_{x \to -2} 3x+2 = -4$

For every $\varepsilon > 0$ there exist $\delta > 0$ such that

$|3x+2-(-4)| < \varepsilon$ whenever $|x-(-2)| < \delta$

Aside	Proof
$\lvert 3x+2+4 \rvert < \varepsilon$	if $\lvert x+2 \rvert < \varepsilon/3$
$\lvert 3x+6 \rvert < \varepsilon$	$3\lvert x+2 \rvert < \varepsilon$
$3\lvert x+2 \rvert < \varepsilon$	$\lvert 3x+6 \rvert < \varepsilon$
$\lvert x+2 \rvert < \varepsilon/3$	$\lvert 3x+2-(-4) \rvert < \varepsilon$
choose $\delta = \varepsilon/3$	$\lvert f(x) - L \rvert < \varepsilon$ ✓

Ex 3 $\lim_{x \to 0} -5x+2 = 2$

For every $\varepsilon > 0$ there exists a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $|x-a| < \delta$

Aside

$|-5x+2-2|<\varepsilon$

$|-5x|<\varepsilon$

$5|x|<\varepsilon$

$|x|<\varepsilon/5$

choose $\delta=\varepsilon/5$

Proof

if $|x|<\varepsilon/5$

$5|x|<\varepsilon$

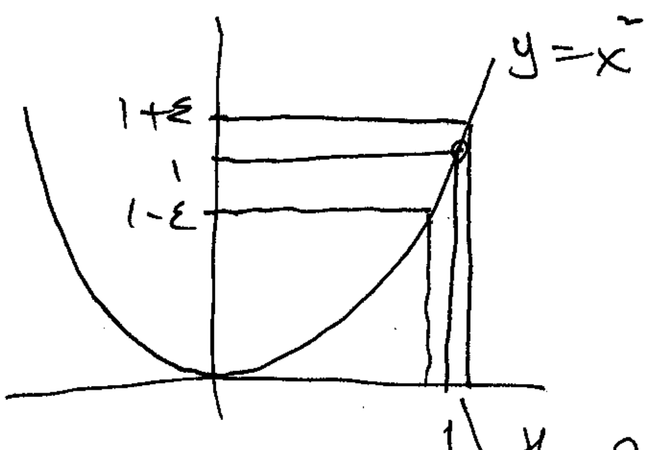
$|5x|<\varepsilon$ } same

$|-5x|<\varepsilon$

$|-5x+2-2|<\varepsilon$

$|f(x)-L|<\varepsilon$ ✓

Ex4 $\lim_{x\to 1} x^2=1$ This problem is much harder



the 2 δ are different so you'll need to pick the smaller of the two. In this class we will stick with linear functions f(x).