Math 1496 Calculus I – Final Name

This test consists of 2 parts. You are required to do all of part A #1-6 and any 4 of part B. The point value for each question is in the parenthesis. Please show all of your work for full credit!

1. (20) Calculate the following limits.

(i) $\lim_{x\to 0} \dfrac{\sqrt{x+1}-1}{x}$

(ii) $\lim_{x\to 0} \dfrac{\sin 5x}{\sin x}$

(iii) $\lim_{x\to \infty} \dfrac{x^2-4x-1}{2x^2-x+5}$

(v) $\lim_{x\to -2} \dfrac{2x^2+5x+2}{x+2}$

(vi) $\lim_{x\to 1} \dfrac{\ln x}{x-1}$

2. (20) Calculate the first derivative (either $f'(x)$ or y') of the following. Do not simplify your answer

(i) $y = (x^2 + 1) \ln x$

(ii) $y = x^x$

(iii) $x^2y - xy^2 = 3x - y + 1$

(iv) $y = \dfrac{e^{5x}}{\tan x}$

3. (20) Integrate the following.

(i) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$

(ii) $\int \frac{x}{\sqrt{x+1}} dx$

(iii) $\int_0^4 \frac{x}{x^2+1}\, dx$

(iv) $\int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin^2 x} dx$

4. (7) Find the equation of the tangent to $y = \sqrt{x+3}$ at $P(1,2)$.

5. (6) Find the absolute minimum and absolute maximum of $f(x) = x^3 - 3x^2 - 9x$ on $[-2,2]$.

6. (7) Find the area between $y = x^2$ and $y = x + 2$. Sketch the region.

Part B - Choose any 4 of the following 5

7. (10) Consider the function

$$f(x) = \begin{cases} x^2, & x \leq 2 \\ 4, & x > 2 \end{cases}$$

Is $f(x)$ continuous at $x = 2$? Is $f(x)$ differentiable at $x = 2$. Provide a sketch of $f(x)$

8. (10) Consider the function

$$f(x) = x^3 - 3x + 2$$

Determine:

(i) The critical points

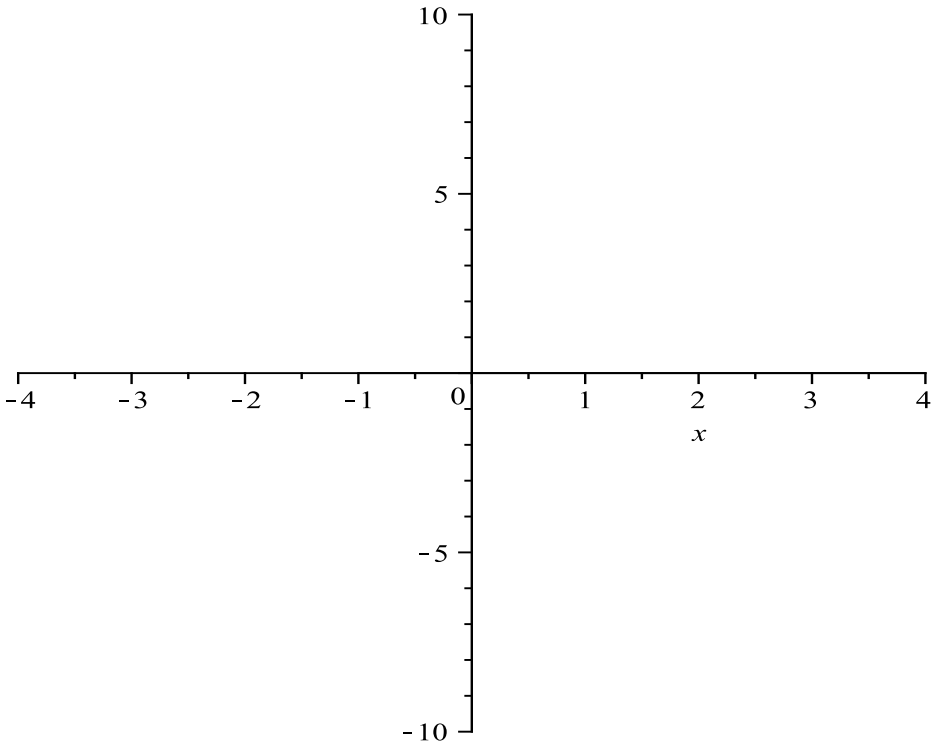
(ii) When y is increasing and decreasing

(iii) Determine whether any of the critical points are minimum or maximum

(iv) Inflection points

(iv) When y is concave up and down

(v) Then sketch the curve and label.



9. (10) A 13-foot ladder is leaning against a vertical wall when Jack begins pulling the foot of the ladder away from the wall at a rate of 0.5 ft/s. How fast is the top of the ladder sliding down the wall when the foot of the ladder is 5 ft from the wall?

10. (10) A rectangular pen is built with one side against a barn. Two hundred meters of fencing are used for the other three sides of the pen. What dimensions maximize the area of the pen?

11. (10) Find the volumes of revolution when the region bound by the following curve

$$y = x^2, \quad y = 1, \quad x = 0, \quad \text{in } Q1$$

is revolved about the x and y axis.