

## Does the Multi-Factor Model Deliver Superior Alphas? A Perspective from the Enhanced Index Fund in the Taiwan Stock Market

Yih Jeng Yu-Hsiang Hsu Shyh-Weir Tzang\*

### Abstract

The paper investigates the effectiveness of the Multi-Factor Model (MFM) by constructing an enhanced index fund (EIF) based on an ad hoc approach in weight adjustment. Based on a two-stage method maximizing the information ratio to obtain optimal alpha scores (Grinold and Kahn, 2000; Sorensen, Qian, Hua and Schoen, 2004; Qian, Hua and Sorensen, 2007), the paper finds that by adjusting the benchmark weights and smoothing optimal alpha scores using Taiwan 50 index as a benchmark, the constructed EIF can deliver higher information ratios and active returns, as well as a lower turnover rate in comparison with the capital-weighted Taiwan 50 index.

**Keywords:** alpha model, enhanced index fund, information coefficient, information ratio, multi-factor model, quantitative investment

### I. Introduction

Index funds have posted significant growth in the past few decades for institutional equity investors. For investors seeking to outperform the benchmark index by employing a more active investment strategy, enhanced index funds (EIFs) seem to have become an alternative to index funds by compromising the quantitative equity management with traditional fundamental analysis. In view of the strong growth of Taiwan's ETF market since 2003, the EIFs that pronounce to outperform the passive ETFs warrant deeper examination of their performance and risk evaluation for the investors in Taiwan.

Modern Portfolio Theory (MPT) was developed in the 1950s through the early 1970s and was considered an important advance in the finance field. Fama and Macbeth (1973) also proposed a positive relationship between the average stock returns and beta exposures in US stock markets. Instead of following the CAPM, numerous research studies have claimed that there should be more than just one factor, such as beta, that can account for stock returns. The Arbitrage Pricing Theory (APT) presented by Ross (1976) is an extension of CAPM. Inspired by the CAPM and APT, multi-factor pricing models provide alternative means to select factors that can best fully explain the returns of stocks. The influences of factors are often common characteristics across many stocks. Basu (1977, 1983), Banz (1981), Reinganum (1981), Carlton and Lakonishok (1986) and Fama and French (1992, 1993) present factor models using firm-specific factors to explain stock returns. According to Fama and French (1993), size (market capitalization), value (book-to-market or other valuation ratio such as earnings-to-price ratio) and beta factors can be used to build a three-factor model, which can account for 95% of the variability in stock market returns in the US market.

More extensive research in the framework of factor model analysis was conducted after Fama and French's three factor model. Lakonishok, Shleifer and Vishny (1994) provide evidence that sales growth and cash flow to price are significant factors in explaining stock returns in the Japanese market. Haugen and Baker (1996) find stable determinants of the cross-section of expected stock returns that contradict the efficient market hypothesis. Knez and Ready (1997) analyze the risk premium on size and book-to-market effects by implementing the robust regression analysis and find that the size effect will disappear when extreme observations are trimmed. Brennan, Chordia and Subrahmanyam (1998) examine the

relationship between stock returns and other alternative non-risk factors based on the risk-adjusted returns in the Fama-MacBeth-type regression. Their result shows that the errors-in-variables problem can be avoided without having to group securities into portfolios. Barry (2002) examines the robustness of size and the book-to-market effect in 35 emerging equity markets, where the firm size effect is found to lack robustness after removing the extreme value. Serra (2002) uses cross-sectional factor models in the emerging markets, with results indicating that most important factors in both the emerging markets and in mature markets are the same. Jun, Marathe and Shawky (2003) use turnover ratio, trading value and turnover volatility to measure liquidity and to show that liquidity factors are positively correlated with stock returns in 27 emerging markets. Morelli (2007) finds that the book-to-market ratio, rather than the size factor, is a significant variable in explaining the variability of stock returns in the UK market.

This study was motivated by the multiple factor analysis to build a rule-based stock selection model. By controlling the tracking error and turnover rate, an ad hoc approach was proposed to adjust weights in constructing EIFs that can achieve the highest possible information ratio to outperform the capital-weighted Taiwan 50 index (TAIEX 50). This paper is constructed as follows. Section II describes the methodology in constructing a quantitative alpha model. Section III contains the empirical results. Section IV presents the conclusion.

## **II. Methodology**

The right factors first have to be identified to build the MFM. According to Grinold and Kahn (2000), information is the vital input to any active management strategy which, if properly applied, allows active management to outperform passive management tracking benchmark. According to BARRA United states equity (E3) (1998), a descriptor, or called fundamental factor, can be regarded as the information that can help to analyze the potential tendency of stock returns. Therefore, it is important to understand the analysis and evaluation of information content and to know how to refine the information to build portfolios, with the process of quantitative stock selection model including three parts, as follows: (1) choice of descriptors, (2) test of the descriptor's effectiveness in return forecasting, and (3) computation of optimal alpha scores based on the two-stage model to combine the selected descriptors. Detailed descriptions are documented in the following sections.

### **II.1 Descriptors and factors**

In general, descriptors to be identified can be divided into two main categories. One category describes the firms' characteristics that are fundamental and observable from their financial statements. The other category describes overall market conditions. The paper calculates firm characteristics, i.e., descriptors, based on the approach of BARRA United states equity (E3) and Shyu et al. (2006) that use weekly and monthly frequency in order to compare portfolios that are rebalanced with different information frequency. Table 1 lists descriptors and their groups by weekly and monthly frequency. Based on the stock universe of the Taiwan 50 index (TAIEX 50), we select 5 composite factors with 29 descriptors in weekly frequency and 14 composite factors with 46 descriptors in monthly frequency for further examination. Except for Momentum, Trading Activity and Volatility that are obtained from market data, all other descriptors are taken from financial statements.

#### **Refer Table 1**

##### **II.1.1 Standardization of descriptors**

As the units of various descriptors are different, we need to scale the descriptors to

comparable the ranges. By suppressing the subscript  $t$  for time, the standardization process is as follows (BARRA, 2009):

$$\tilde{D}_{i,j} = \frac{Des_{i,j} - \mu_j}{S_j} \quad (1)$$

where  $\tilde{D}_{i,j}$  is the standardized descriptor  $j$  for company  $i$  denoted by  $Des_{i,j}$ ,  $\mu_j$  is the capital-weighted average of descriptor  $j$  across companies, and  $S_j$  is the equal-weighted standard deviation of descriptor  $j$ . For the outliers of  $\tilde{D}_{i,j}$ , we truncate the values that are greater than 3.5 to be 3.5. All filtered descriptors are again standardized by Equation (1) to compute their Z scores with a capitalization-weighted mean of zero and an equal-weighted standard deviation of 1.

### II.1.2 Selection of descriptor by its effectiveness

To identify the significant descriptors, we use the information coefficients (IC) delivered by the descriptors to measure their effectiveness in forecasting security returns. IC measures the predictive power of a descriptor and is defined as follows:

$$IC = \text{corr}(\mathbf{f}, \mathbf{r}), \quad (2)$$

where  $\mathbf{f}$  is the vector for forecasted return on security and  $\mathbf{r}$  is the vector for subsequent realized return. We use standardized descriptors as the proxy for forecasted security returns. In general, a descriptor or a factor with an IC greater than 0.1 on an annual basis has relatively good predictive power (Qian *et al.*, 2007). The  $t$ -statistics of the ICs are further computed to see if they are significantly different from zero:

$$t - \text{stat}(IC) = \frac{\text{Average}(IC)}{\text{Stdev}(IC)/\sqrt{N}} \quad (3)$$

If the IC of a descriptor is significantly different from zero, then this descriptor's predictive ability is relatively stable and consistent. Additionally, we define the success rate as the number of positive ICs divided by the total number of periods being tested. Descriptors that have a success rate higher than 55% will be selected to form composite factors for further analysis.

### II.1.3 Quintile analysis

The universe of stocks in each period is grouped into five equally weighted portfolios, according to their standardized descriptors or Z scores (Lawson & Platt, 2004). The top and bottom quintile portfolios consist of stocks with the highest and lowest 20% Z scores of the descriptors, respectively. After each quintile portfolio has been formed, we then calculate the total returns to the quintile portfolios over the following period, before the quintile portfolio is rebalanced at the end of the period.

## II.2 Converting descriptors into factors

Based on Sorensen *et al.* (2004), this paper uses a two-stage process to combine multiple alpha sources (see Appendix for detailed descriptions). The structure of constructing the two-stage model is presented in Figure 1. Using factor classifications, we divide the factors into two categories, including the "core factor" pool and "satellite factor" pool. First, we compare the predictive power of the descriptors examined during the in-sample period and select the relatively significant and effective descriptors. We also retest the effectiveness of the descriptors and include the effective descriptors in our model at each rebalancing time in the back-testing period. Descriptors that are selected in the in-sample period will be included consistently in our model. These descriptors will be known as the core descriptors.

**Refer Figure 1**

Descriptors within each factor group typically have higher correlations than those that are not within the same group. After the significant and effective descriptors have been determined based on the approach in the previous section, the first stage is to maximize the IC to find the optimal weights to linearly combine those descriptors into factors. A factor is denoted with subscript  $k$  and descriptor with subscript  $j$ , where  $k=1, \dots, K$  and  $j=1, \dots, N$ . Each factor consists of its own descriptors and  $N$  is the total number of descriptors, where  $\sum_{k=1}^K N_k = N$ . We use the following formula (Grinold & Kahn, 2000) by suppressing the subscript  $t$  for time:

$$\mathbf{IC}^* = \mathbf{IC}'\boldsymbol{\rho}^{-1} \quad (4)$$

$$Dw_{j,k} = \frac{IC_{j,k}^*}{\sum_{j=1}^{N_k} IC_{j,k}^*} \quad (5)$$

$$F_{i,k} = \sum_{j=1}^{N_k} Dw_{j,k} * \tilde{D}_{i,j,k} \quad (6)$$

where  $\mathbf{IC}$  ( $N \times 1$ ) is the cross-sectional correlation vector between the forecasted return by  $N$  descriptors and the subsequent realized return at time  $t$ ;  $N_k$  is the number of descriptors belonging to the  $k$ -th factor;  $\boldsymbol{\rho}$  ( $N \times N$ ) is the correlation matrix of descriptors at time  $t$ ;  $Dw_{j,k}$  is the optimal weight for the  $j$ -th descriptor belonging to the  $k$ -th factor at time  $t$ ;  $F_{i,k}$  is the value of the  $k$ -th factor obtained by the weighted-average standardized descriptors for the  $i$ -th company at time  $t$ ; and  $\tilde{D}_{i,j,k}$  is the value of standardized descriptor  $j$  ( $Z$  score) belonging to the  $k$ -th factor for company  $i$  at time  $t$ .

**II.3 Converting factors into an alpha score**

The second stage is to compute each stock's alpha score formed by the factors with their optimal weights obtained by maximizing the information ratio. By suppressing the subscript  $t$  for time, we calculate the factors' optimal weights as follows (Sorensen *et al.*, 2004; Qian *et al.*, 2007):

$$\mathbf{v}^* = s\boldsymbol{\Sigma}_{IC_f}^{-1} \mathbf{IC}_f \quad (7)$$

$$Fw_k = \frac{v_k^*}{\sum_{k=1}^K v_k^*} \quad (8)$$

$$alpha\_score_i = \sum_{k=1}^K Fw_k * F_{i,k} \quad (9)$$

where  $\mathbf{IC}_f$  ( $K \times 1$ ) is the cross-sectional correlation vector between the forecasted return by  $K$  factors and the subsequent realized return at time  $t$ ;  $\boldsymbol{\Sigma}_{IC}$  ( $K \times K$ ) is the covariance matrix of factors; and  $s$  is a given positive constant to make the sum of the optimal weights equal 1.  $Fw_k$  is the optimal weight of the  $k$ -th factor at time  $t$  and  $alpha\_score_i$  is the score of the  $i$ -th company required to construct an optimal multifactor alpha model. Through this two-stage approach, we can first transform the potential value-added information from the descriptors into factors to later obtain optimal factor weights to produce a set of alpha scores. In the end, we can apply these scores to construct an optimal multifactor alpha model.

**II.4 Reducing the turnover rate by smoothing the alpha score**

When the constructed portfolio is rebalanced on the weekly alpha scores, the volatility of the

weekly alpha score will induce a frequent rebalance of the constructed portfolio, thereby leading to higher turnover rates and transaction costs. Therefore, we introduce the renewal rate to smooth the alpha score, as follows:

$$\tilde{\alpha}_{i,t+1}^* = \text{renewalrate} * \tilde{\alpha}_{i,t+1} + (1 - \text{renewalrate}) * \tilde{\alpha}_{i,t} \quad (10)$$

$$\tilde{\alpha}_{i,t+2}^* = \text{renewalrate} * \tilde{\alpha}_{i,t+2} + (1 - \text{renewalrate}) * \tilde{\alpha}_{i,t+1}^* \quad (11)$$

By recursive substitution, we have:

$$\tilde{\alpha}_{i,t+T}^* = \text{renewalrate} \sum_{\tau=0}^T (1 - \text{renewalrate})^{T-\tau} * \tilde{\alpha}_{i,t+\tau} \quad (12)$$

If the renewal rate is equal to 0.5, the smoothed alpha score at time  $t$  is equal to the weighted average of the alpha score at time  $t$  and the alpha score at time  $(t-1)$  by equal weight. The higher the renewal rate, the more information is delivered by the current alpha score in computing the smoothed alpha, indicating a higher turnover rate for the portfolios.

## II.5 Combining the weekly and monthly alpha scores

To decrease the turnover rate of the constructed portfolios based on the more volatile weekly alpha scores, a combination of weekly and monthly alpha scores is adopted in this paper. Intuitively, when the weekly alpha score is combined with the monthly alpha score, the weekly alpha score may vary each week although the monthly alpha score may not. Therefore, the combined alpha score will be more stable than the weekly alpha but more volatile than the monthly alpha scores. Two methods to compute the respective weight are used to combine weekly and monthly alpha score, as follows: the equal weight and the date-decreasing weight. Date-decreasing weight means the weight of monthly alpha score will decrease as the date nears the end of the month. For example, if the current date is May 20 and the number of calendar days in May is 30, the weight of the weekly alpha score will be set to 20/30 and the weight of the monthly alpha score is 10/30. The idea behind this method is to increase the dominance of the information delivered by the weekly alpha scores as the date nears the end of the month. The minimum weight of the weekly alpha score is set at 0.5.

## II.6 Converting the alpha scores into active weights

The empirical test in this study is to build an enhanced index fund using the Taiwan 50 index as its benchmark. Based on the stock selection model mentioned above, we can create positive active holdings on certain stocks with higher forecasted alpha scores and vice versa. If the information coming from the stock selection model is correct, we can create active returns higher than the benchmark.

$$\hat{\alpha}_i = \frac{\text{AlphaScore}_i - \text{Min}(\text{AlphaScore}_i)}{\text{Max}(\text{AlphaScore}_i) - \text{Min}(\text{AlphaScore}_i)} \quad (13)$$

$$w_i^* = \frac{\hat{\alpha}_i}{\sum_{i=1}^{50} \hat{\alpha}_i} \quad (14)$$

$$w_{b,i} = \frac{MV_i}{\sum_{i=1}^{50} MV_i} \quad (15)$$

As Equation (13) shows, we can use the alpha score of stock  $i$  to calculate its relative position and to rescale the alpha score to  $\hat{\alpha}_i$ , a rescaled alpha score within between 0 and 1. The weight of each stock based on the relative size of the rescaled alpha score is computed, and stocks with higher alpha scores tend to have higher weights.

Many constraints still need to be considered in implementing the stock selection model. In the beginning of portfolio construction, stocks that are illiquid or those that have poor credit ratings are excluded to avoid portfolios with high underlying risk for individual stocks. In this

study, we do not control the industry characteristic of our enhanced index fund, as in top-down asset allocation. The paper creates an ad hoc variable, i.e., the benchmark weight, to control the degree of closeness between the enhanced index fund and the benchmark, as follows:

$$\mathbf{w}_{enhanced} = \mathbf{w}_b \cdot r + \mathbf{w}^* \cdot (1 - r), \quad (16)$$

where  $\mathbf{w}^*$  is the weight vector of the active portfolio,  $\mathbf{w}_b$  is the weight vector of the benchmark and  $r$  is the benchmark weight.

The turnover rate is computed using the following formula:

$$\text{Turnover rate} = \frac{1}{2} \sum_{i=1}^N |w_i^{new} - w_i^{old}|, \quad (17)$$

where  $w_i^{new}$  is the targeted weight and  $w_i^{old}$  is the current portfolio weight.

### III. Empirical Results

#### III.1 Data collection

As the Taiwan 50 index was first published in October 2002, this study uses data from 01/2003 to 01/2007 as the in-sample data to test the effectiveness of descriptors from which the core descriptors are further selected. The data are from quarterly and monthly financial statements held in the Taiwan Economic Journal (TEJ) financial data bank. Quarterly financial data are converted into monthly and weekly frequencies to calculate the monthly and weekly descriptors and factors. We use data from 01/2007 to 08/2011 as the out-of-sample data to backtest the portfolios<sup>1</sup>. The paper uses the most recent available financial statement to calculate the monthly and weekly factors. Additionally, the paper also adopts adjusted returns by accounting for the capitalization changes, such as cash dividends.

The following formula for adjustment is used:

$$r_{i,t} = \left( \frac{P_{i,t} + D_{i,t} + \beta_{i,t}}{P_{i,t-1} + \alpha_{i,t}} - 1 \right) \times 100\%, \quad (18)$$

where  $r_{i,t}$  and  $P_{i,t}$  are the return and price of stock  $i$  in period  $t$ , respectively;  $D_{i,t}$  is the cash dividend; and  $\alpha_{i,t}$  and  $\beta_{i,t}$  are the terms of adjustment for the capital measures.

#### III.2 In-sample descriptors selection

As we know, some key descriptors work stably in the long term whereas some descriptors may work in the short term. In this study, our quantitative model includes some factors that work stably within the in-sample period, which we refer to as the core factors. The model then uses a rolling window to capture other factors that work in the short term for the out-of-sample period. Descriptors will be defined as the core descriptors if the top quintile cumulative returns are substantially different from the bottom quintile cumulative returns. The results of these three descriptors, *CashTurnover*, *RelTradingIntensityIMIY* and *VolumeSensity*, are presented for illustration purposes.

#### Refer Figure 2

The quintile cumulative returns for the descriptors of *CashTurnover* and

---

<sup>1</sup> Taiwan Securities and Exchange Law (TSEL) requires that listed firms publish their first quarterly financial statements by the end of March, second quarterly statements by the end of August, third quarterly statements by the end of October, and fourth quarterly statements by the end of the following March. On the other hand, listed firms in Taiwan are also required to publish monthly sales revenue reports before the 10<sup>th</sup> of the following month.



*RelTradingIntensity1MIY* show that their top quintile portfolios are consistently better than are bottom quintile portfolios. However, the descriptor *VolumeSensity* shows a reverse order in portfolio quintiles, of which the portfolio of the last quintile performs better than does the fourth quintile, the fourth quintile better than the third quintile and so on. Therefore, this descriptor consistently has better predictive power, suggesting that a short strategy can be implemented.

### III.3 Core descriptors in the quantitative alpha model

After analyzing the 12-month rolling average IC and quintile cumulative returns with the in-sample data, we select sixteen monthly descriptors from Table 1 as the core descriptors listed in Table 2.

#### Refer Table 2

Table 2 shows the summary statistics of monthly performance by core descriptors based on the in-sample data from 01/2003 to 01/2007 for a total of 48 months. The average IC is calculated using all ICs from the in-sample data. An active return in the top represents the active returns for the portfolio in the first quintile, while an active return in the bottom represents the active returns for the portfolio in the fifth quintile. In this section, the active returns are calculated by equally weighted portfolios based on composites of the benchmark.

From Table 2, we find that the average IC for *CapitalVar*, *RelTradingIntensity3MIY*, *SalesQoQ*, *ShareTurnoverM*, *ShareTurnoverQ*, *ShareTurnover1Y* and *ShareTurnover3Y* are negative and that all of their active returns in the top are smaller than the active returns in the bottom. This indicates that a reverse strategy for these descriptors may be more appropriate. The average IC for *CapExpToDepr*, *CashTurnover*, *ROAIQ*, *LNEarning*, *LNCashFlow*, *PayoutToPrice*, *TCFToPrice*, *RelTradingIntensity1MIY* and *ValueToGrowth* are positive. The spread of active returns between the top and the bottom was positive, suggesting a long strategy where investment in stocks with a high factor Z-score may be more profitable.

#### Refer Table 3

Table 3 shows the summary statistics of weekly performance by core descriptors based on the in-sample data from 01/2003 to 01/2007, for a total of 208 weeks. This study includes eleven weekly core descriptors in the model, of which *CumRet1W*, *CumRet4W* and *ShareTurnover24W* exhibit negative ICs. Their active returns in the top quintile are smaller than their active returns in the bottom. This means that a reverse strategy may be considered for these descriptors. After the core descriptors are selected by the process mentioned above, a rolling window test can be implemented to determine the effectiveness and stability of the descriptors examined, while satellite descriptors can also be selected in the following backtesting period.

### III.4 Building the quantitative stock selection model

This study uses a two-stage model to build a quantitative stock selection model. After combining factors with optimal weights, we can calculate the alpha score. If the alpha score is effective, we can make the investment decision based on the alpha score. An effective alpha strategy means that investments in stocks with higher alpha scores are expected to achieve higher profits. Additionally, the weights of the component stocks of the benchmark are adjusted using their respective alpha scores. We calculate quintile cumulative returns to

test the effectiveness of the alpha-score-based strategy. Figures 3 and 4 present the quintile cumulative returns for the monthly and weekly alpha scores. In Figure 3, the cumulative returns of the portfolio in the first quintile are found to be the highest while the cumulative returns of the portfolio in the fourth quintile are the lowest. The cumulative returns of the portfolios in the fifth quintile are consistently higher than the portfolios in the third and fourth quintiles after 2009.

#### **Refer Figures 3 & 4**

Figure 4 shows that the cumulative returns of the portfolios in five quintiles for the weekly alpha score can be more easily recognized than in Figure 3. If we continuously invest in stocks with high alpha scores in the long run, we tend to beat the benchmark and achieve superior profits. However, as the portfolios are rebalanced each week, increasing transaction costs may undermine the performance of the portfolios.

#### **III.5 Constructing the enhanced index fund**

In this section, we use the alpha scores for each stock to construct the portfolio at each portfolio rebalancing date. In consideration of the weekly rebalancing of a portfolio that leads to higher transaction costs, the smoothed alpha score is adopted and the turnover rate is also computed in the performance analysis.

According to equation (14), a zero benchmark weight implies that only the optimal weights delivered by the smoothed alpha scores will be adopted to construct the portfolio and that the tracking error will be the highest in comparison with other portfolios constructed using the non-zero benchmark weight. The active returns of the portfolio based on the zero benchmark weight are also found to be higher than with other portfolios.

#### **Refer Table 4**

Table 4 shows the performances of different portfolios that are constructed using different benchmark weights based on weekly and monthly rebalancing. In general, a tracking error of the constructed EIF is acceptable within the range of three to six percent by practitioners. The benchmark weight can be used to control the tracking error in portfolio rebalancing. In panel A of Table 4, the tracking error is less than 4% when the benchmark weight is greater than 0.2. In panel B of Table 4, however, the active return of the weekly rebalanced EIF is lower than that of the monthly rebalanced EIF. This is mainly due to the higher transaction costs derived from a high turnover rate. The transaction costs did not significantly affect the tracking error in the weekly rebalancing portfolios. However, the abnormal active returns sharply decrease and lead to lower IRs of average 0.5 compared with 0.7 in monthly rebalancing portfolios.

To reduce the turnover rate, the paper adopts the alpha-smoothing method to reduce the turnover rate of the constructed portfolios. For example, when the renewal rate is set at 0.5, a weighted average of alpha score is calculated using the weighting the alpha score in prior period with the current alpha score. Various renewal rates across different benchmark weights are used to construct the monthly rebalancing portfolios, with the empirical results presented in Table 5.

#### **Refer Table 5 & Figure 5**



According to Table 5 and Figure 6, a significant positive relationship is found between the renewal rate and the turnover rate of the portfolios. The tracking error is not significantly affected by the different levels of the renewal rate across the benchmark weights. For a given level of the renewal rate, the benchmark weight is negatively related to the tracking error. However, an optimal renewal rate can be determined by observing the largest active return as well as the IR and the smallest tracker error. When the renewal rate is 0.6 for a given benchmark weight, a higher information ratio and active return can be achieved. However, the tracking error also increases. We therefore use 0.5 to proxy the renewal rate in order to achieve a lower tracking error, although at a relatively higher active return and IR.

#### Refer Table 6

Table 6 shows the performance of the portfolios constructed with the combined alpha scores based on the two methods for a given renewal rate of 0.5. The results show that the turnover rates substantially decrease when the portfolios are constructed using an equal-weighted combination of the alpha scores. The result is intuitive as weekly alpha score may vary each week within a month, whereas the monthly alpha score does not result in a substantial decrease in the turnover rate based on the equal weight method. However, portfolios using the date-decreasing method have turnover rates that are slightly higher than do portfolios using the weekly alpha score. Despite a barely improved turnover rate, the portfolio returns based on the date-decreasing method are found to consistently have higher active returns and information ratios than do portfolio returns using the equal weight method. Additionally, the combined alpha score based on the date-decreasing method also generates higher active returns and information ratios than does the weekly alpha score with almost the same level of tracking error. Therefore, portfolio returns constructed using the combined alpha score based on the date-decreasing method outperforms other portfolios using other methods in terms of active returns and IR.

Figure 6 show that the cumulative returns of the EIF constructed using weekly, monthly and combined alpha scores with a benchmark weight of 0 and a renewal rate of 0.5. Here, we use the date-decreasing method to compute the combined alpha score. For comparison purposes, the tracking error is controlled at 6%. The performance of the EIF based on the combined alpha score is found to consistently outperform portfolios based on two other methods, which also exhibit higher cumulative returns than does the benchmark of the Taiwan 50 index.

#### IV. Conclusion

There are many different factors that cause variability in stock price. The multi-factor quantitative model, however, is a useful approach that systematically captures the multi-dimensional drivers of stock price. In this study, we use multiple factors to build the stock selection model to filter factors that can be used in constructing the portfolios. The paper tests the effectiveness of 48 monthly descriptors and 29 weekly descriptors. We calculate the average IC,  $t$  statistics, success rate and quintile analysis to measure the “skill” for each descriptor.

The selected descriptors are further combined using a two-stage model to form factors that are classified into different categories, while descriptors within the same category are combined as a single factor. In combining descriptors, the set of the optimal weights are computed using two approaches. The first stage uses the approach proposed by Grinold & Kahn (2000). The second stage uses the approach proposed by Sorensen *et al.* (2004). The combined factors are further used to compute their respective alpha scores that can

differentiate good stocks from bad ones in the stock universe. If a stock has a higher alpha score, it is expected to deliver a higher return in the next period.

The empirical result suggests the alpha score calculated using our quantitative model can create various portfolios with different returns. If we invest in portfolios with higher alpha scores, the profit is higher than when investing in portfolios with lower alpha scores. Based on the alpha score obtained from the models, an EIF is constructed based on the composite stocks of the Taiwan 50 index. In other words, the EIF is an active management of the Taiwan 50 index that puts more weight on stocks with higher alpha scores.

To control the tracking error and turnover rate, the original benchmark weight and renewal rate are constructed in our portfolios. The empirical results indicate that by setting the benchmark weight and renewal rate at 0.4 and 0.5, respectively, the active returns and turnover rate of the EIF based on the weekly alpha score are 2.37% and 285.32%, respectively, with the IR at 0.67. Moreover, the strategy based on the alpha score by combining the weekly and monthly alpha scores results in a higher information ratio (0.83) and higher active return (2.94%), although it delivers only a slightly higher turnover rate (296.26%) than does the weekly alpha score strategy. According to Grinold and Kahn (2000), an information ratio (IR) of 0.5 is good, while 1.0 is exceptional. Therefore, we conclude that the quantitative alpha stock selection model is useful in the construction of EIF, which outperforms the benchmark with an acceptable range of tracking error.

**Appendix: Optimal alpha model**

This study utilizes multi-factor model to estimate and forecast alphas based on a broad universe of stocks. By Grinold’s (1989) Fundamental Law of Active Management (FLAM), the information ratio can be expressed as follows:

$$IR = IC\sqrt{BR} \tag{A.1}$$

where

$$IR = \frac{E(\alpha)}{\sigma_\alpha} \text{ (expected active return } \alpha \text{ divided by estimated active risk } \sigma_\alpha)$$

$IC$  = correlation of security (normalized z-scores) with the subsequent rank of security performance

$BR$  = size of the opportunity set

In his framework, the manager’s skills are measured by the information coefficient ( $IC$ ). Consistency in forecasting is the key to successful investment. The forecasting quality depends on the manager’s skill and strategy, while repeatability is breadth.

Grinold and Kahn (2000) introduce the refined forecasts by the following formula:

$$\begin{aligned} E(\mathbf{r}|\mathbf{g}) &= E(\mathbf{r}) + Cov(\mathbf{r},\mathbf{g}) \cdot Var^{-1}(\mathbf{g}) \cdot (\mathbf{g} - E(\mathbf{g})) \\ \phi &= Cov(\mathbf{r},\mathbf{g}) \cdot Var^{-1}(\mathbf{g}) \cdot (\mathbf{g} - E(\mathbf{g})) \end{aligned} \tag{A.2}$$

where

$\phi = E(\mathbf{r}|\mathbf{g}) - E(\mathbf{r})$  ;  $\mathbf{r}$  = vector of excess returns for N assets;

$\mathbf{g}$  = raw forecast vector (K forecasts) ;

$E(\mathbf{r})$  = naive (consensus) forecast;

$E(\mathbf{g})$  = expected forecast ;

$E(\mathbf{r}|\mathbf{g})$  = the expected return conditional on  $\mathbf{g}$

When we consider more than one forecast, we can combine forecasts with a set of optimal weights. The optimal weights are calculated as follows:

$$\mathbf{g} = [g_1, g_2, \dots, g_k]$$

$$\text{Var}(\mathbf{g}) = \begin{bmatrix} \text{Std}(g_1) & & 0 \\ & \ddots & \\ 0 & & \text{Std}(g_k) \end{bmatrix} \cdot \boldsymbol{\rho}_g \cdot \begin{bmatrix} \text{Std}(g_1) & & 0 \\ & \ddots & \\ 0 & & \text{Std}(g_k) \end{bmatrix} \quad (\text{A.3})$$

$\mathbf{g}$  is the vector of forecasts and  $\boldsymbol{\rho}_g$  is the correlation matrix of the forecasts. Now the covariance matrix between the returns and  $K$  signals will involve  $K$  information coefficients:

$$\text{Cov}(\mathbf{r}, \mathbf{g}) = \omega \cdot [IC_1, IC_2, \dots, IC_k] \cdot \begin{bmatrix} \text{Std}(g_1) & & 0 \\ & \ddots & \\ 0 & & \text{Std}(g_k) \end{bmatrix} \quad (\text{A.4})$$

We can substitute equation (A.3) and equation (A.4) into the forecasting formula equation (A.2) to find the following:

$$\begin{aligned} \phi &= \omega \cdot [IC_1, IC_2, \dots, IC_k] \cdot \boldsymbol{\rho}_g^{-1} \cdot \begin{bmatrix} \frac{1}{\text{Std}(g_1)} & & 0 \\ & \ddots & \\ 0 & & \frac{1}{\text{Std}(g_k)} \end{bmatrix} \cdot \begin{bmatrix} g_1 - E(g_1) \\ \vdots \\ g_k - E(g_k) \end{bmatrix} \\ &= \omega \cdot \mathbf{IC}^T \cdot \boldsymbol{\rho}_g^{-1} \cdot \mathbf{z} \end{aligned} \quad (\text{A.5})$$

where  $z_j = \frac{g_j - E(g_j)}{\text{Std}(g_j)}$

The  $\mathbf{z}$  is the standardized forecast vector.

Qian and Hua (2004) provide an extended research on active management. They model information ratio in another way:

$$IR = \frac{\alpha}{\sigma_\alpha} = \frac{\overline{IC}_t}{\text{Std}(IC_t)} \quad (\text{A.6})$$

Equation (A.6) follows from the estimated  $\alpha$  divided by the estimated  $\sigma_\alpha$ :

$$\begin{aligned} \alpha &= \overline{IC}_t \sqrt{N} \sigma_{\text{target}} \overline{\text{dis}(R_t)} \\ \sigma_\alpha &= \text{Std}(IC_t) \sqrt{N} \sigma_{\text{target}} \overline{\text{dis}(R_t)} \end{aligned} \quad (\text{A.7})$$

By Qian *et al.* (2007), when other things being equal, the higher the average IC for a factor, the better the reward-to-risk ratio. In addition, the more stable the IC is over time, the better the result. They argue that the assumption of FLAM by Grinold (1989) which considers a single period skill and assumes the  $\text{Std}(IC)$  is zero is not realistic. Ye (2008) also proposed that investors should be aware of the impact that IC volatility has on active strategies. After Qian and Hua (2004) provide new approaches to model information ratio, Sorensen *et al.* (2004) and Qian *et al.* (2007) provide an extended method to combine multiple alpha sources as follows:

$$IC_{c,t} = \frac{\mathbf{1}}{\sqrt{\mathbf{v}' \boldsymbol{\Phi} \mathbf{v}}} \sum_{i=1}^m v_i IC_{i,t} \quad (\text{A.8})$$

$$\overline{IC}_c = \frac{\mathbf{1}}{\sqrt{\mathbf{v}' \boldsymbol{\Phi} \mathbf{v}}} \sum_{i=1}^m v_i \overline{IC}_i = \frac{\mathbf{1}}{\sqrt{\mathbf{v}' \boldsymbol{\Phi} \mathbf{v}}} \mathbf{v}' \overline{\mathbf{IC}} \quad (\text{A.9})$$

$$\text{Std}(IC_{c,t}) = \frac{\mathbf{1}}{\sqrt{\mathbf{v}' \boldsymbol{\Phi} \mathbf{v}}} \sqrt{\sum_{i=1}^m \sum_{j=1}^m v_i v_j \rho_{ij,IC} \sigma_{IC_i} \sigma_{IC_j}} = \frac{\mathbf{1}}{\sqrt{\mathbf{v}' \boldsymbol{\Phi} \mathbf{v}}} \sqrt{\mathbf{v}' \boldsymbol{\Sigma}_{IC} \mathbf{v}} \quad (\text{A.10})$$

where

$\mathbf{v} = (v_1, v_2, v_3, \dots, v_k)$  is a set of weight vector which is a linear combination of  $M$  factor

$\Phi$  = the covariance matrix of factors

$\overline{\mathbf{IC}} = (\overline{IC_1}, \overline{IC_2}, \dots, \overline{IC_m})$  is the average IC vector

$\Sigma_{IC}$  = covariance matrix of IC

We can substitute Equation (A.9) and Equation (A.10) into the Equation (A.6) and obtained the following:

$$IR = \frac{\alpha}{\sigma_\alpha} = \frac{\overline{IC}_t}{Std(IC_t)} = \frac{\mathbf{v}' \overline{\mathbf{IC}}}{\sqrt{\mathbf{v}' \Sigma_{IC} \mathbf{v}}} \quad (\text{A.11})$$

Therefore, a set of optimal model weight that can be derived from the maximized IR is as follows:

$$\mathbf{v} = s \Sigma_{IC}^{-1} \overline{\mathbf{IC}} \quad (\text{A.12})$$

## References

- Banz, R. 1981. The relationship between return and market value of common stocks. *Journal of Financial Economics*, 9(1): 3-18.
- BARRA Inc. 1998. *United States equity (E3) risk model handbook*.
- BARRA, 2001. EUE2 - A new equity model for Europe, Research notes.
- BARRA, 2009. The Barra Europe equity model (EUE3).
- Barry, C. B., Goldreyer, E., Lockwood, L., & Rodriguez, M. 2002. Robustness of size and value affects in emerging equity markets, 1985-2000. *Emerging Markets Review*, 3(1): 1-30.
- Basu, S. 1977. Investment performance of common stocks in relation to their price-earnings ratios: a test of the efficient market hypothesis. *Journal of Finance*, 32(3): 663-682.
- Basu, S. 1983. The relationship between earnings yield, market value, and return for NYSE common stocks. *Journal of Financial Economics*, 12(1): 129-156.
- Brennan, M., Chordia, T., & Subrahmanyam, A. 1998. Alternative factor specifications, security characteristics, and the cross-section of expected stock returns. *Journal of Financial Economics*, 49(3): 345-73.
- Carlton, W., & Lakonishok, J. 1986. The size anomaly: does industry group matter? *Journal of Portfolio Management*, 12: 36-40.
- Fama, E. F. & French, K. R. 1992. The cross-section of expected stock returns. *Journal of Finance*, 47(2): 427-465.
- \_\_\_\_\_. 1993. Common risk factors in the returns to stocks and bonds. *Journal of Financial Economics*, 33(1): 3-56.
- Fama, E. F. & MacBeth, J. D. 1973. Risk, return, and equilibrium: empirical tests. *Journal of Political Economy*, 81(3): 607-636.
- Grinold, R. C., & Kahn, R. N. 2000. *Active portfolio management*. New York, NY: McGraw-Hill.
- Grinold, R. C. 1989. The fundamental law of active management. *Journal of Portfolio Management*, 15(3): 30-37.
- Haugen, R. A. and Baker, N. L. 1996. Commonality in the determinants of expected stock returns. *Journal of Financial Economics*, 41:401-439.
- Jun, S., Marathe, A., & Shawky, H. A. 2003. Liquidity and stock returns in emerging equity markets. *Emerging Market Review*, 4(1): 1-24.

- Knez, P. J., & Ready, M. J. 1997. On the robustness of size and book-to-market in cross-sectional regressions. *Journal of Finance*, 52(4): 1355-1382.
- Lakonishok, J., Shleifer, A., & Vishny, R. W. 1994. Contrarian investment, extrapolation and risk. *Journal of Finance*, 49(5): 1541-1578.
- Lawson, R., & Platt, G. 2004. *The A-Z of quant.* Macquarie Group. <http://www.macquarie.com/uk/research/index.html> Accessed Jan. 3, 2010.
- Morelli, D. 2007. Beta, size, book-to-market equity and returns: a study based on UK data. *Journal of Multinational Financial Management*, 17(3): 257-272.
- Qian, E. E., & Hua, R. H. 2004. Active risk and information ratio. *Journal of Investment Management*. 2(3): 20-34.
- Qian, E. E., Hua, R. H., & Sorensen, E. H. 2007. *Quantitative equity portfolio management: modern techniques and applications.* Boston, MA: Chapman & Hall/CRC Press.
- Reinganum, M. 1981. Misspecification of capital asset pricing: empirical anomalies based on earnings yields and market values. *Journal of Financial Economics*, 9(1): 19-46.
- Ross, S. A. 1976. The arbitrage theory of capital asset pricing. *Journal of Finance*, 13(3): 341-360.
- Serra, A. P. 2002. The cross-sectional determinants of returns—evidence from emerging markets' stocks. *Journal of Emerging Market Finance*, 2(2): 123-162.
- Shyu, S. D., Jeng, Y., Ton, W. H., Lee, K. J., & Chuang, H. M. 2006. Taiwan multi-factor model construction: equity market neutral strategies application. *Managerial Finance*, 32(11): 915-947.
- Sorensen, E. H., Qian, E. E., Hua, R. and Schoen, R. 2004. Multiple alpha sources and active management. *Journal of Portfolio Management*, 30(2): 39-45.
- Stephan, T. G., Maurer, R., & Dürr, M. 2000. A multiple factor model for European stocks. Working paper no.57, Goethe University Frankfurt am Maim.

Table 1: List of factors and descriptors by weekly and monthly frequency

<b>Factor</b>	<b>Descriptor</b>
<b>Weekly frequency</b>	
<i>Momentum</i>	CumRet1W, CumRet4W, CumRet12W, CumRet24W, CumRet36W, CumRet48W
<i>Size</i>	LNCap, LNSales
<i>Trading Activity</i>	ShareTurnover1W, ShareTurnover4W, ShareTurnover12W, ShareTurnover24W, RelTradingIntensity1W1M, RelTradingIntensity4W3M
<i>Value</i>	BookToPrice, PayoutToPrice, PayoutToPrice3Y, CashDivToPrice, CashDivToPrice3Y, FCFToPrice, OCFToPrice, EPSToPrice, EPSToPrice1Y, SalesToPrice1M, SalesToPrice1Y, ValueToGrowth
<i>Volatility</i>	CumulativeRange, LNPrice, PriceHighToLow
<b>Monthly frequency</b>	
<i>Capital Spending</i>	CapExpToDepr, FCFToMarketCap
<i>Currency Sensitivity</i>	CurrencySensitivity
<i>Earnings Quality</i>	ROA1Q, ROE1Q, GrossMargin, NetIncomeMargin, EBITMargin
<i>Earnings Variability</i>	EarningVar, CashFlowVar
<i>Efficiency</i>	CashTurnover, FixAssetTurnover, TotalAssetTurnover
<i>Growth</i>	CapitalVar, AssetGrowth, EarningGrowth, EarningQoQ, EarningYoY, SalesQoQ
<i>Leverage</i>	BookLeverage, DebtAssetR
<i>Momentum</i>	HistoricalAlpha, CumRet6M, CumRet1Y
<i>Size</i>	LNCap, LNEarning, LNCashFlow
<i>Solvency</i>	InterestCovRatio
<i>Trading Activity</i>	ShareTurnoverM, ShareTurnoverQ, ShareTurnoverY, ShareTurnover3Y, RelTradingIntensity1M1Y, RelTradingIntensity3M1Y
<i>Value</i>	BookToPrice, PayoutToPrice, PayoutToPrice3Y, FCFToPrice, OCFToPrice, TCFToPrice, SalesToPrice1M, SalesToPrice1Y
<i>Value to growth ratio</i>	ValueToGrowth
<i>Volatility</i>	BetaTimesSigma, LNPrice, VolumeSensitivity

\* There are 5 composite factors with 29 descriptors in the weekly frequency and 14 composite factors with 46 descriptors in the monthly frequency. Except for *Momentum*, *Trading Activity* and *Volatility* are obtained from market data, while all other descriptors are taken from financial statements.



Table 2: Summary of monthly performance by core descriptors\*

Factor	Descriptor	AvgIC	t-stat	Success Rate	Active Return		
					Top	Bottom	Top - Bottom
<i>Capital Spending</i>	CapExpToDepr	0.02	1.03	0.62	1%	-6%	6%
<i>Efficiency</i>	CashTurnover	0.04	1.77	0.55	9%	-7%	16%
<i>Earnings Quality</i>	ROA1Q	0.05	1.48	0.51	5%	-8%	13%
<i>Growth</i>	CapitalVar	0.01	0.34	0.51	0%	1%	<b>-1%</b>
<i>Growth</i>	SalesQoQ	-0.05	-2.50	0.36	-4%	10%	<b>-13%</b>
<i>Size</i>	LN Earning	0.06	2.13	0.60	4%	-4%	9%
<i>Size</i>	LNCashFlow	0.06	2.73	0.66	3%	-8%	11%
<i>Trading Activity</i>	ShareTurnoverM	-0.01	-0.15	0.49	-2%	-1%	<b>-1%</b>
<i>Trading Activity</i>	ShareTurnoverQ	-0.01	-0.26	0.47	-5%	-1%	<b>-4%</b>
<i>Trading Activity</i>	ShareTurnoverY	-0.02	-0.52	0.45	-8%	1%	<b>-10%</b>
<i>Trading Activity</i>	ShareTurnover3Y	-0.06	-1.40	0.45	-10%	3%	<b>-13%</b>
<i>Trading Activity</i>	RelTradingIntensity1M1Y	0.05	1.64	0.66	10%	-7%	17%
<i>Trading Activity</i>	RelTradingIntensity3M1Y	-0.01	-0.25	0.49	-3%	-1%	<b>-1%</b>
<i>Value</i>	PayoutToPrice	0.05	1.46	0.62	2%	-5%	7%
<i>Value</i>	TCFToPrice	0.03	1.22	0.55	5%	-8%	13%
<i>Value to growth</i>	ValueToGrowth	-0.02	-0.54	0.47	1%	0%	1%
<i>Volatility</i>	VolumeSensity	-0.09	-2.65	0.34	-11%	11%	<b>-22%</b>

\* In-sample period starts from 01/2003 to 01/2007, for a total of 48 months. The universe of stocks in each month is grouped into five equally weighted portfolios according to their standardized descriptors or Z scores. Top and bottom quintile portfolios consist of stocks with the highest and lowest 20% Z scores of the descriptors, respectively. After each quintile portfolio has been formed, the total returns to the quintile portfolios over the following month are computed before the quintile portfolio is rebalanced at the end of each month. The t-stat is computed using Equation (3) in the paper. The success rate is defined as the number of positive ICs divided by the total number of periods being tested.

Table 3: Summary of weekly performance by core descriptors\*

Factor	Descriptor	AvgIC	t-stat	Success Rate	Active Return		
					Top	Bottom	Top - Bottom
<i>Momentum</i>	CumRet1W	-0.09	-5.89	0.37	-20%	26%	<b>-46%</b>
<i>Momentum</i>	CumRet4W	-0.02	-1.27	0.47	-2%	3%	<b>-5%</b>
<i>Size</i>	LnSales	0.01	0.66	0.54	7%	-3%	10%
<i>Trading Activity</i>	ShareTurnover24W	-0.01	-0.37	0.47	-6%	0%	<b>-6%</b>
<i>Trading Activity</i>	RelTradingIntensity4W3M	0.03	2.34	0.58	7%	-8%	16%
<i>Value</i>	PayoutToPrice	0.01	0.67	0.50	0%	-4%	4%
<i>Value</i>	PayoutToPrice3Y	0.01	0.65	0.51	6%	2%	4%
<i>Value</i>	CashDivToPrice	0.01	0.98	0.51	1%	-3%	3%
<i>Value</i>	CashDivToPrice3Y	0.01	0.66	0.50	5%	2%	3%
<i>Value</i>	EPSToPrice1Y	0.02	1.15	0.55	2%	-10%	12%
<i>Volatility</i>	PriceHighToLow	0.01	0.76	0.51	8%	0%	8%

\* In-sample data starts from 01/2003 to 01/2007, for a total of 208 weeks. The universe of stocks in each week is grouped into five equally weighted portfolios according to their standardized descriptors or Z scores. Top and bottom quintile portfolios consist of stocks with the highest and lowest 20% Z scores of descriptors, respectively. After each quintile portfolio has been formed, the total returns to the quintile portfolios over the following week are computed before the quintile portfolio is rebalanced at the end of week. The t-stat is referred to equation (3) in the paper. The success rate is defined as the number of positive ICs divided by the total number of periods being tested.

Table 4: Performance of portfolios constructed using different benchmark weights \*

Benchmark weight	Return (net of cost)	Active return (net of cost)	Tracking error (net of cost)	Turnover rate	IR	maxDrawdown
<i>Panel A: monthly rebalance</i>						
0	7.35%	3.08%	4.27%	144.63%	0.72	-50.68%
0.2	6.94%	2.66%	3.69%	117.77%	0.72	-50.86%
0.4	6.52%	2.25%	3.14%	92.00%	0.72	-51.05%
0.6	6.09%	1.82%	2.65%	67.68%	0.69	-51.24%
0.8	5.65%	1.37%	2.25%	46.63%	0.61	-51.43%
<i>Panel B: weekly rebalance</i>						
0	8.31%	2.74%	5.37%	976.52%	0.51	-48.16%
0.2	7.92%	2.35%	4.47%	777.95%	0.53	-48.28%
0.4	7.53%	1.96%	3.64%	580.22%	0.54	-49.00%
0.6	7.12%	1.55%	2.92%	384.70%	0.53	-49.72%
0.8	6.67%	1.09%	2.43%	197.24%	0.45	-50.44%

\* The renewal rate is not considered in this table.

Table 5: Performance of portfolios constructed by various renewal rates using the weekly alpha score

Renewal rate	Benchmark weight	Return (net of cost)	Active return (net of cost)	Tracking error (net of cost)	Turnover rate	IR	Max Drawdown
<b>0.05</b>	0	7.59%	2.18%	5.02%	155.66%	0.43	-53.05%
	0.2	7.27%	1.87%	4.23%	126.34%	0.44	-52.66%
	0.4	6.93%	1.52%	3.50%	101.91%	0.43	-52.28%
	0.6	6.55%	1.14%	2.88%	83.33%	0.40	-51.90%
	0.8	6.13%	0.72%	2.45%	72.49%	0.29	-51.53%
<b>0.1</b>	0	7.88%	2.47%	4.99%	189.37%	0.49	-52.54%
	0.2	7.51%	2.10%	4.20%	151.83%	0.50	-52.25%
	0.4	7.12%	1.71%	3.48%	118.72%	0.49	-51.97%
	0.6	6.70%	1.29%	2.86%	91.51%	0.45	-51.69%
	0.8	6.21%	0.81%	2.43%	73.98%	0.33	-51.43%
<b>0.2</b>	0	8.23%	2.82%	4.98%	258.60%	0.57	-51.80%
	0.2	7.81%	2.40%	4.19%	205.58%	0.57	-51.65%
	0.4	7.36%	1.95%	3.46%	156.15%	0.56	-51.52%
	0.6	6.88%	1.47%	2.84%	112.27%	0.52	-51.39%
	0.8	6.33%	0.93%	2.42%	79.33%	0.38	-51.27%
<b>0.4</b>	0	8.72%	3.31%	5.06%	403.03%	0.65	-50.47%
	0.2	8.20%	2.79%	4.24%	319.67%	0.66	-50.59%
	0.4	7.67%	2.27%	3.49%	238.65%	0.65	-50.72%
	0.6	7.12%	1.71%	2.84%	162.50%	0.60	-50.86%
	0.8	6.50%	1.09%	2.41%	97.26%	0.45	-51.01%
<b>0.5</b>	0	8.88%	3.47%	5.12%	482.66%	0.68	-49.86%
	0.2	8.33%	2.92%	4.28%	382.94%	0.68	-50.10%
	0.4	7.77%	2.37%	3.51%	285.32%	0.67	-50.35%
	0.6	7.19%	1.78%	2.85%	191.92%	0.63	-50.62%
	0.8	6.55%	1.14%	2.41%	109.33%	0.47	-50.89%
<b>0.6</b>	0	8.95%	3.54%	5.17%	570.82%	0.68	-49.28%
	0.2	8.39%	2.98%	4.32%	453.24%	0.69	-49.64%
	0.4	7.82%	2.41%	3.54%	337.46%	0.68	-50.01%
	0.6	7.23%	1.82%	2.87%	225.44%	0.64	-50.39%
	0.8	6.58%	1.17%	2.41%	123.79%	0.49	-50.77%
<b>0.8</b>	0	8.77%	3.36%	5.31%	781.42%	0.63	-48.23%
	0.2	8.25%	2.84%	4.43%	621.63%	0.64	-48.78%
	0.4	7.72%	2.31%	3.60%	463.11%	0.64	-49.37%
	0.6	7.17%	1.77%	2.90%	307.53%	0.61	-49.97%
	0.8	6.57%	1.17%	2.42%	161.05%	0.48	-50.56%

Table 6: Performance of portfolios based on the combined weekly and monthly alpha score with a renewal rate of 0.5

Combination method	Benchmark weight	Return (net of cost)	Active return (net of cost)	Tracking error (net of cost)	Turnover rate	IR	Max Drawdown
equal weight *							
	0	9.67%	4.09%	4.92%	328.37%	0.83	-49.21%
	0.2	9.00%	3.43%	4.21%	260.63%	0.81	-49.59%
	0.4	8.32%	2.74%	3.54%	195.83%	0.77	-49.98%
	0.6	7.60%	2.02%	2.96%	136.54%	0.68	-50.37%
	0.8	6.82%	1.24%	2.53%	88.22%	0.49	-50.77%
date-decreasing weight *							
	0	9.99%	4.41%	5.06%	499.53%	0.87	-48.18%
	0.2	9.26%	3.68%	4.27%	396.90%	0.86	-48.77%
	0.4	8.52%	2.94%	3.55%	296.26%	0.83	-49.37%
	0.6	7.76%	2.18%	2.92%	199.84%	0.75	-49.96%
	0.8	6.93%	1.36%	2.48%	113.85%	0.55	-50.56%

\* Two following methods are used to combine the weekly and monthly alpha scores: the equal weight and the date-decreasing weight. Date-decreasing weight means the weight of the monthly alpha score will decrease as the date nears the end of the month. For example, if the current date is May 14 and number of days in May is 30, the weight of the weekly alpha score will be equal to 14/30 while the weight of monthly alpha score is 16/30. At the end of May, the weight of the monthly alpha score will become zero. The idea behind this method is that the share of the weekly alpha score will increase as the date increases in the month.

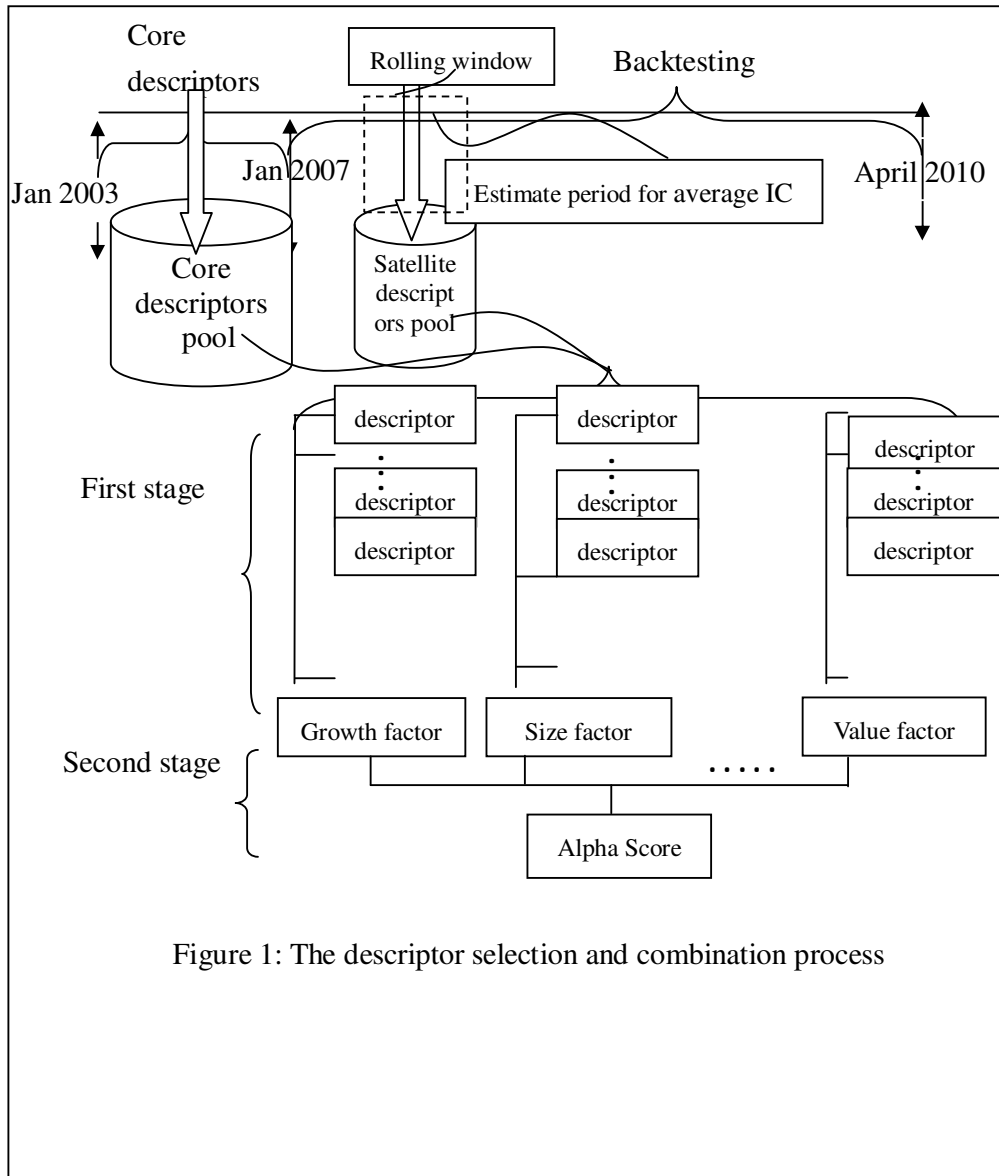
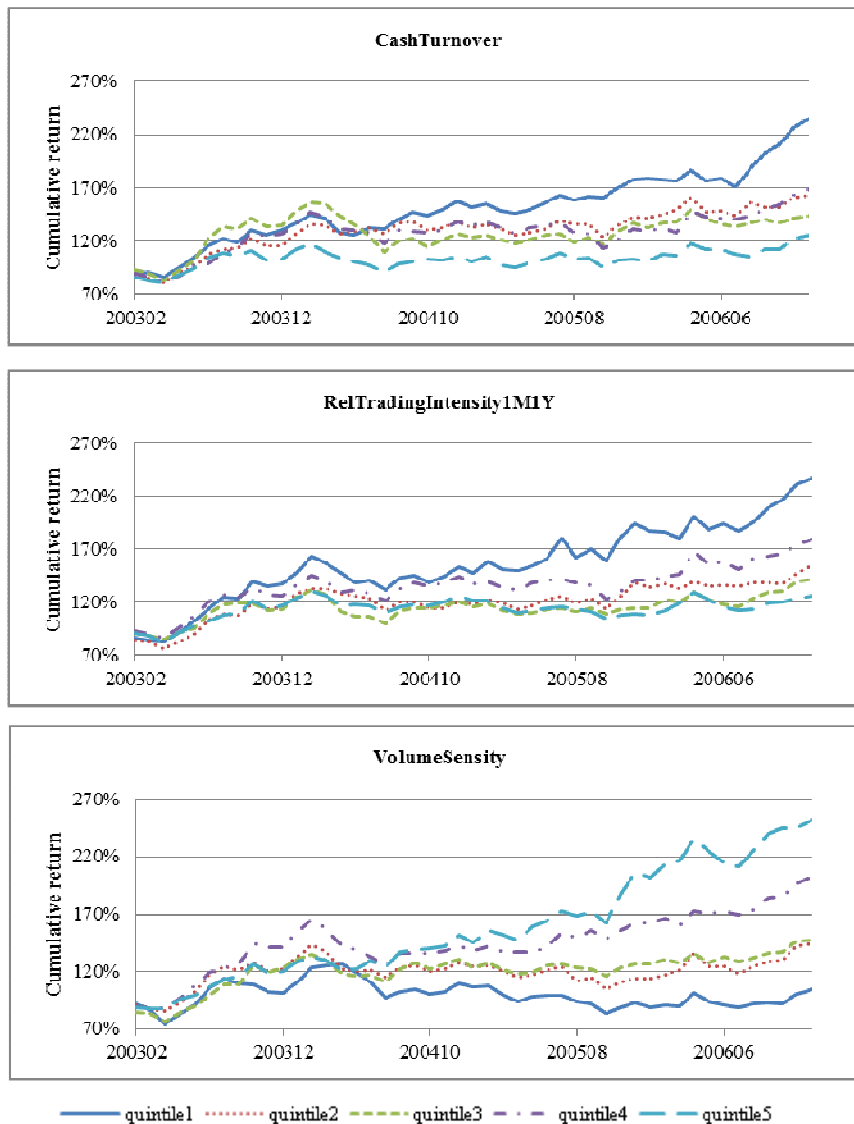


Figure 1: The descriptor selection and combination process





**Figure 2: Quintile cumulative returns of three monthly descriptors**  
 The quintile cumulative returns of three monthly descriptors from 01/2003 to 01/2007 are *CashTurnover* , *RelTradingIntensity1MIY* and *VolumeSensity*, as shown in the top, center and bottom panels. The quintile cumulative returns for *CashTurnover* and *RelTradingIntensity1MIY* show that their top quintile portfolios are consistently better than are bottom quintile portfolios. The descriptor *VolumeSensity* shows a reverse order in the portfolio quintiles, of which the portfolio of the last quintile performs better than does the fourth quintile, the fourth quintile better than the third quintile and so on. This indicates that this descriptor also has a consistently better predictive power and that a short strategy can be implemented.

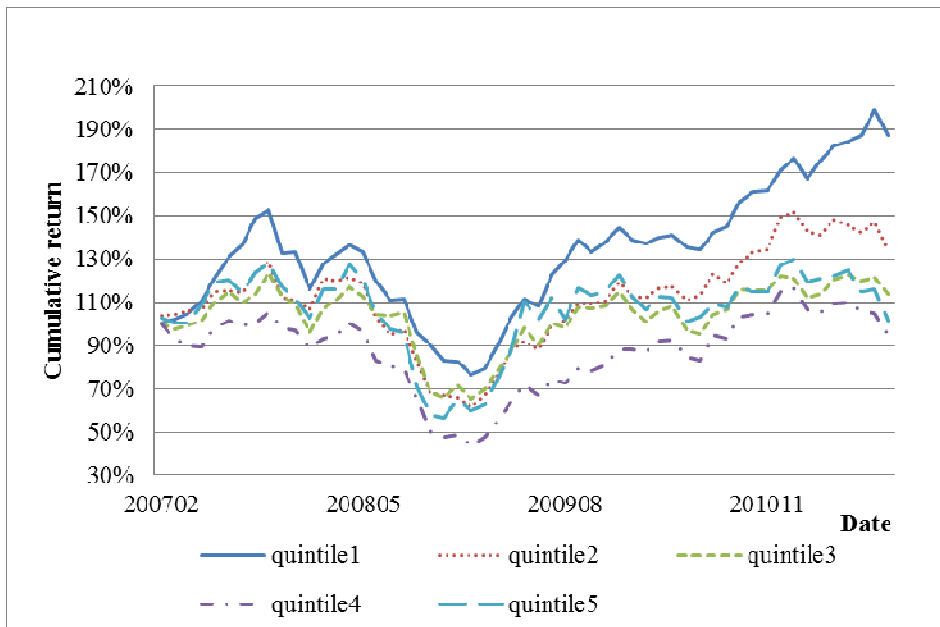


Figure 3: Quintile cumulative returns for the monthly alpha score (Sample period: Jan. 5, 2007 to August 26, 2011)

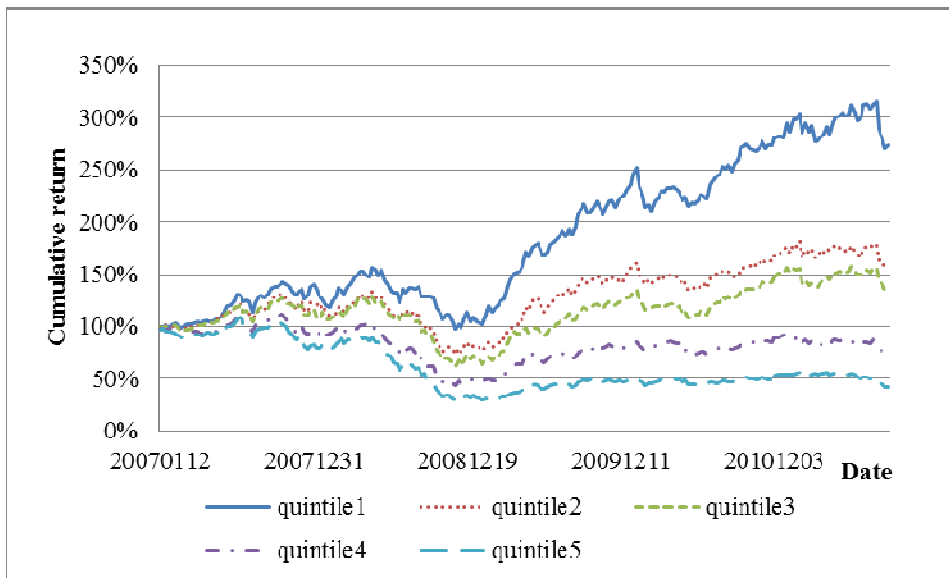


Figure 4: Quintile cumulative returns for the weekly alpha score (Sample period: Jan. 5, 2007 to August 26, 2011)

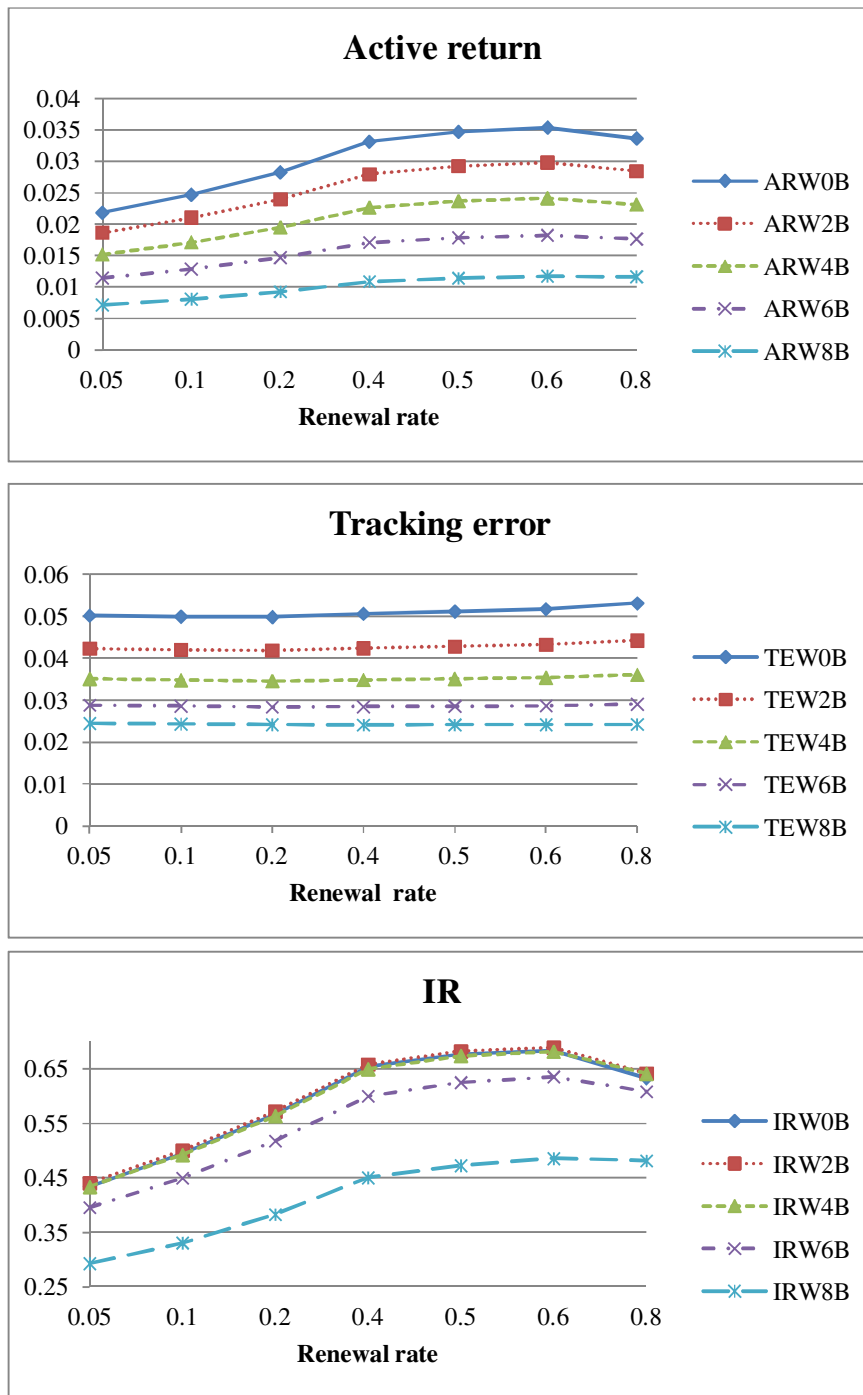


Figure 5: Active returns, tracking errors and IRs of the portfolio returns based on the weekly alpha score across all benchmark weights for different renewal rates. W0B represents the zero benchmark weight and W2B the 0.2 benchmark weight. The rest applies.

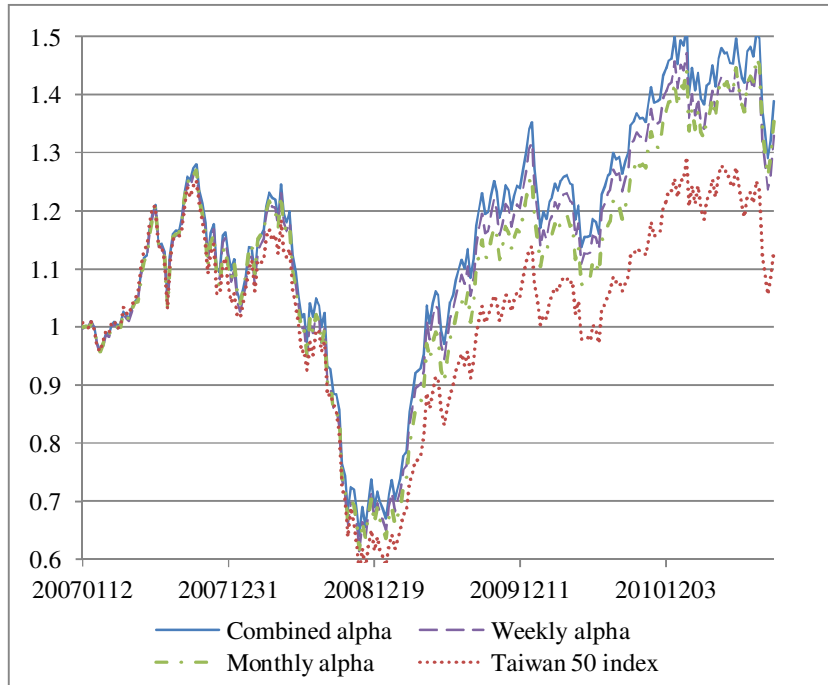


Figure 6: Cumulative return of the enhanced index fund

For comparison purposes, we set the benchmark weight and renewal rate to be 0 and 0.5, respectively, with a controlled tracking error of 6%. The cumulative returns of the portfolios are computed by using the combined alpha score based on the date-decreasing method and the weekly alpha score.

#### Authors

##### Yih Jeng

Associate Professor, Department of Finance, National Sun-Yat Sen University,  
[yihjeng2@gmail.com](mailto:yihjeng2@gmail.com)

##### Yu-Hsiang Hsu

Fund Manager, Quantitative Index Investment Department, Fubon Asset Management Co. Ltd.  
[fly\\_fly88@hotmail.com](mailto:fly_fly88@hotmail.com)

##### Shyh-Weir Tzang\*

Associate Professor, Department of Finance, Asia University. [swtzang@gmail.com](mailto:swtzang@gmail.com).

\*Corresponding Author