Abstract

Intuitionistic fuzzy sets theory which is the generalisation of fuzzy sets theory is more suitable tool for dealing with imprecise information. The information provided by the decision maker is imprecise in many situations. So, intuitionistic fuzzy preference relation is a suitable tool for such cases. The estimation of the priority vector of the intuitionistic fuzzy preference relation is an important part in decision making. In this paper, an intuitionistic fuzzy weighted integral operator is proposed. The defined operator is employed with score functions to develop a method for the priority of an intuitionistic fuzzy preference relation to estimate the ranking of the considered alternatives. Further an experiment example has been conducted to implementation of proposed approach.

Keywords: Intuitionistic fuzzy sets, Multi–criteria Decision Making (MCDM), Intuitionistic fuzzy preference relation, Score functions, Accuracy functions.

1. INTRODUCTION:

The concept of fuzzy sets (FSs) originally introduced by Zadeh (1965) and obtained attentions in many fields. Atanassov (1986) extended concept of FSs by involving the hesitation degree and introduced the concept of intuitionistic fuzzy sets (IFSs), comprising by a membership function and a non-membership function. Due to the hesitation degree, IFSs theory is very useful in dealing with fuzziness under uncertain behaviour. Gau and Buehrer (1993) introduced the concept of vague sets, which is another generalization of fuzzy sets. But Bustince and Burillo (1996) clarify that the notion of vague sets is the same as that of IFSs.

Szmidt and Kacprzyk (1996) successfully used IFSs to solve group decision making problems. Szmidt and Kacprzyk (2002) introduced some solution concepts in group decision making with intuitionistic fuzzy preference relations, and gave a method based on fuzzy majority equated with a fuzzy linguistic quantifier, to aggregate the individual intuitionistic fuzzy preference relations into a social fuzzy preference. Gabriella, Yager, and Atanassov (2004) formed a generalized net model for multi-person multi-criteria decision making process which was based on intuitionistic fuzzy graphs. Li (2005) investigated multi-attribute decision making using intuitionistic fuzzy sets and proposed several linear programming models to calculate optimal weights for criteria.

Liu and Wang (2007) proposed a series of new score functions for the multi-attribute decision making problems based on intuitionistic fuzzy point operators and evaluation function. Xu and Yager (2006) developed some geometric aggregation operators, such as the intuitionistic fuzzy weighted geometric (IFWG) operator, the intuitionistic fuzzy ordered
weighted geometric (IFOWG) operator and the intuitionistic fuzzy hybrid geometric (IFHG) operator, and implemented the IFHG operator to multi-criteria decision making problems with intuitionistic fuzzy information. Some arithmetic aggregation operators such as the intuitionistic fuzzy weighted averaging (IFWA) operator, the intuitionistic fuzzy ordered weighted averaging (IFOWA) operator and the intuitionistic fuzzy hybrid aggregation (IFHA) operator have been presented by Xu (2007a).

Xu (2007b) defined the concepts of intuitionistic preference relation, consistent intuitionistic preference relation, incomplete intuitionistic preference relation and acceptable intuitionistic preference relation, and studied their various properties. Later on, Xu (2009) investigated the group decision making problems in which all the information provided by the decision makers was expressed as intuitionistic fuzzy decision matrices where each of the elements has been characterized by intuitionistic fuzzy number. Xu (2011) developed a series of operators for aggregating and establish various properties of these power aggregation operators under intuitionistic fuzzy environment, and then apply them to develop some approaches to multiple attribute group decision making. Recently, Xu (2012) introduced an error-analysis-based method for the priority of an intuitionistic preference relation, and then a possibility degree formula is used to derive the ranking of the considered alternatives.

In this paper, we define an intuitionistic fuzzy weighted integral operator and utilised it to introduce the priority for an intuitionistic fuzzy preference relation to estimate the ranking of the considered alternatives. Moreover, an experiment analysis on the developed approach is conducted through an illustrative example.

2. PRELIMINARIES:

2.1. Intuitionistic Fuzzy Sets

Let $U$ be a non empty set called a universe of discourse. An IFS $I$ in $U$ is defined as an object of the following form

$$I = \{ (x, t_I(x), f_I(x)) : \forall x \in U \} \text{.....(1)}$$

Here the functions $t_I : U \rightarrow [0,1]$ and $f_I : U \rightarrow [0,1]$ define the “degree of membership” and the “degree of non-membership” of the element $x \in U$ respectively, and for every element $x$ of $U$, $0 \leq t_I(x) + f_I(x) \leq 1$.

2.2. Intuitionistic Fuzzy Number

If $A = \{ (t_a(x), f_a(x)) : \forall x \in X \}$ is an intuitionistic fuzzy set, then $a = (t_a, f_a)$ is called an intuitionistic fuzzy number (IFN), and the following relations are valid for every two intuitionistic fuzzy values $A$ and $B$:

(i) $A = B$ if and only if $t_a(x) = t_B(x)$ and $f_a(x) = f_B(x)$ for all members of $U$.

(ii) $A \leq B$ if and only if $t_a(x) \leq t_B(x)$ and $f_a(x) \geq f_B(x)$ for all members of $U$.

However the second condition is not satisfied in most of the situations. So it cannot be used to compare intuitionistic fuzzy numbers. In this case we use a score function and an accuracy function of intuitionistic fuzzy numbers for the comparisons.

**Score function:** Let $a = (t_a, f_a)$ and $b = (t_b, f_b)$ be two intuitionistic fuzzy numbers $S(a) = t_a - f_a$ and $S(b) = t_b - f_b$ be the score functions of $a$ and $b$, respectively. Bigger the score, the larger the intuitionistic fuzzy value; thus, the score function can be used as a useful way to measure intuitionistic fuzzy values.
Let motivated by these operators here we define two operational laws of intuitionistic fuzzy numbers.

From the above analysis, a method has been developed for the comparison between two intuitionistic fuzzy values, which is based on the score function and the accuracy function, defined as follows:

Let \( a = (t_a, f_a) \) and \( b = (t_b, f_b) \) be two intuitionistic fuzzy numbers and \( H(b) = t_b + f_b \) be the accuracy functions of \( a \) and \( b \), respectively.

Let \( a = (t_a, f_a) \) and \( b = (t_b, f_b) \) be two intuitionistic fuzzy numbers then

1) If \( S(a) < S(b) \) then \( a \) is smaller than \( b \) and denoted by \( a < b \);
2) If \( S(a) = S(b) \), then
   (i) If \( H(a) < H(b) \), then \( a \) is smaller \( b \) and denoted by \( a < b \);
   (ii) If \( H(a) = H(b) \), then \( a \) and \( b \) represent the same information and denoted by \( a = b \).

3. INTUITIONISTIC FUZZY WEIGHTED INTEGRAL OPERATOR:

Various operators are defined by many researchers between two intuitionistic fuzzy numbers and motivated by these operators here we define two operational laws of intuitionistic fuzzy numbers.

Let \( a = (t_a, f_a) \) and \( b = (t_b, f_b) \) be two intuitionistic fuzzy numbers, then

(i) \( a \oplus b = (t_a t_b, (1 - f_a)(1 - f_b)) \), ....(2)

(ii) \( \lambda a = \left( t_a^\lambda, f_a^\lambda \right) \), where \( \lambda > 0 \). ....(3)

Let \( c = a \oplus b \) and \( d = \lambda a \), where \( \lambda > 0 \); then both \( c \) and \( d \) are also intuitionistic fuzzy values.

Further, these two operational laws of intuitionistic fuzzy numbers are used to define an intuitionistic fuzzy weighted integral operator as follows:

Let \( a_i = (t_{a_i}, f_{a_i}) \) \((i = 1, 2, \ldots, n)\) \((n \text{ is greater than or equal to } 2)\) be a collection of \( n \) intuitionistic fuzzy numbers on \( X \). Then their aggregated value by using the IFW operator is also an intuitionistic fuzzy number, and

\[
\alpha_i = \text{IFW}(a_{i1}, a_{i2}, \ldots, a_{in}) = \left\{ \prod_{j=1}^{n} t_{a_{ij}}^{w_j}, \prod_{j=1}^{n} \left( 1 - f_{a_{ij}} \right)^{w_j} \right\} 
\]

where \( w_j \) is the weight of the alternatives \( a_i, (i = 1, 2, 3, \ldots, n) \).

**Proof:** Here we prove this theorem by using mathematical induction on the collection of intuitionistic fuzzy numbers on \( X \) i.e. \( n \).

For \( n = 2 \), we have from the operational laws defined in equations (2) and (3).

\[
\begin{align*}
(w_1)a_1 &= \left( t_{a_1}^{w_1}, f_{a_1}^{w_1} \right) \\
(w_1 + w_2)a_2 &= \left( t_{a_2}^{w_1+w_2}, f_{a_2}^{w_1+w_2} \right)
\end{align*}
\]

We have

\[
\alpha \oplus b = (t_a t_b, (1 - f_a)(1 - f_b)),
\]

So,

\[
\text{IFW}(a_1, a_2) = (w_1)a_1 \oplus (w_1 + w_2)a_2 = \left( t_{a_1}^{w_1}, t_{a_2}^{w_1+w_2}, (1 - f_{a_1})^{w_1}, (1 - f_{a_2})^{w_1+w_2} \right)
\]

It is clear that for \( n = 2 \), the result holds.

Let us take for \( n = k \), the result is true i.e.

\[
\text{IFW}(a_1, a_2, \ldots, a_k) = \left\{ \prod_{j=1}^{k} t_{a_{j1}}^{w_j}, \prod_{j=1}^{k} \left( 1 - f_{a_{j1}} \right)^{w_j} \right\}
\]

Then for \( n = k + 1 \), we have

\[
\begin{align*}
\text{IFW}(a_1, a_2, \ldots, a_{k+1}) &= \left\{ \prod_{j=1}^{k} t_{a_{j1}}^{w_j}, \prod_{j=1}^{k} \left( 1 - f_{a_{j1}} \right)^{w_j}, \prod_{j=1}^{k+1} \left( 1 - f_{a_{j1+1}} \right)^{w_j+1} \right\} \\
&= \left\{ \prod_{j=1}^{k+1} t_{a_{j1}}^{w_j}, \prod_{j=1}^{k+1} \left( 1 - f_{a_{j1+1}} \right)^{w_j+1} \right\}
\end{align*}
\]

That is for \( n = k + 1 \), the result still holds. Therefore, by mathematical induction the result will be hold for every value of \( n \).
4. AN APPROACH TO PRIORITY OF AN INTUITIONISTIC FUZZY PREFERENCE RELATION:

Let us consider a finite set of \( m \) alternatives \( A = \{a_1, a_2, a_3, \ldots, a_m\} \) in a decision making problem. Suppose an expert is invited to provide his/her preference information numerically over the alternatives in the set \( A \). The expert compares each pair of alternatives, \((x_i, x_j)\), and gives the numerical value as:

\[ a(x_i, x_j) = (t(x_i, x_j), f(x_i, x_j)) \]

where \( t(x_i, x_j) \) denotes the certainty degree to which the alternative \( x_i \) is preferred to \( x_j \), and \( f(x_i, x_j) \) indicates the certainty degree to which the alternative \( x_j \) is not preferred to \( x_i \), satisfying the conditions:

\[ t(x_i, x_j) \geq 0, \quad f(x_i, x_j) \geq 0, \quad t(x_i, x_j) + f(x_i, x_j) \leq 1, \quad \pi(x_i, x_j) = 1 - t(x_i, x_j) - f(x_i, x_j). \]

Step 1. Obtain the intuitionistic fuzzy preference relation matrix by the information provided by experts. The decision maker characterised the factors as numerical values in percentage by using the expert knowledge and available data. This information is placed in the following intuitionistic preference relation

\[ R = (a_{ij})_{m \times m} = \begin{bmatrix}
(t_{11}, f_{11}) & (t_{12}, f_{12}) & (t_{13}, f_{13}) & \cdots & (t_{1m}, f_{1m}) \\
(t_{21}, f_{21}) & (t_{22}, f_{22}) & (t_{23}, f_{23}) & \cdots & (t_{2m}, f_{2m}) \\
(t_{31}, f_{31}) & (t_{32}, f_{32}) & (t_{33}, f_{33}) & \cdots & (t_{3m}, f_{3m}) \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
(t_{m1}, f_{m1}) & (t_{m2}, f_{m2}) & (t_{m3}, f_{m3}) & \cdots & (t_{mm}, f_{mm})
\end{bmatrix}, \]

and transform the intuitionistic fuzzy values decision making matrix \( R \) into its equivalent intuitionistic fuzzy preference relation \( R' = (b_{ij})_{n \times n} \) (where, \( b_{ij} = (t_{ij}, t_{ij} + \pi_{ij}) \)).

Step 2. Solving the model (M-2) proposed by Xu (2009), obtained the weight \( w_i \) of each alternative \( a_i \).

Step 3. Evaluate the overall values of the alternative \( a_i (i = 1, 2, 3, \ldots, m) \), by aggregating in the \( r^{th} \) line of the expected preference relation matrix by utilising the defined intuitionistic fuzzy Choquet integral operator:

\[ a_i = IFW(a_{i1}, a_{i2}, \ldots, a_{im}) = \left( \sum_{i=1}^{n} t_{ij} \sum_{i=1}^{n} (1 - f_{ij}) \sum_{i=1}^{n} w_i \right) \]

Step 5. Thus the overall values \( a_i \) of the alternatives \( a_i (i = 1, 2, 3, \ldots, m) \) are used to calculate the score functions or the accuracy functions to rank the alternatives \( a_i (i = 1, 2, 3, \ldots, m) \) and select the best one.

5. AN ILLUSTRATIVE NUMERICAL EXAMPLE:

Consider a priority problem of the intuitionistic fuzzy preference relation, which is taken from Xu (2012):

Step 1. The decision maker characterised the factors as numerical values in percentage by using the expert knowledge and available data. This information is placed in the following intuitionistic preference relation matrix, which is taken from Xu (2012):

\[ R = (a_{ij})_{6 \times 6} = \begin{bmatrix}
(0.5,0.5) & (0.6,0.4) & (0.7,0.2) & (0.5,0.3) & (0.4,0.5) & (0.6,0.2) \\
(0.4,0.6) & (0.5,0.5) & (0.3,0.4) & (0.2,0.5) & (0.6,0.3) & (0.7,0.3) \\
(0.2,0.7) & (0.4,0.3) & (0.5,0.5) & (0.6,0.2) & (0.3,0.6) & (0.8,0.2) \\
(0.3,0.5) & (0.5,0.2) & (0.2,0.6) & (0.5,0.5) & (0.4,0.4) & (0.7,0.1) \\
(0.5,0.4) & (0.3,0.6) & (0.6,0.3) & (0.4,0.4) & (0.5,0.5) & (0.6,0.4) \\
(0.2,0.6) & (0.3,0.7) & (0.2,0.8) & (0.1,0.7) & (0.4,0.6) & (0.5,0.5)
\end{bmatrix} \]

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Step 4. Similarly, we find

\[ \mathbf{b}_{ij} = \begin{pmatrix} (0.5, 0.5) & (0.6, 0.6) & (0.7, 0.8) & (0.5, 0.7) & (0.4, 0.5) & (0.6, 0.8) \\ (0.4, 0.4) & (0.5, 0.5) & (0.3, 0.6) & (0.2, 0.5) & (0.6, 0.7) & (0.7, 0.7) \\ (0.2, 0.3) & (0.4, 0.7) & (0.5, 0.5) & (0.6, 0.8) & (0.3, 0.4) & (0.8, 0.8) \\ (0.3, 0.5) & (0.5, 0.8) & (0.2, 0.4) & (0.5, 0.5) & (0.4, 0.6) & (0.7, 0.9) \\ (0.5, 0.6) & (0.3, 0.4) & (0.6, 0.7) & (0.4, 0.6) & (0.5, 0.5) & (0.6, 0.6) \\ (0.2, 0.4) & (0.3, 0.3) & (0.2, 0.2) & (0.1, 0.3) & (0.4, 0.4) & (0.5, 0.5) \end{pmatrix} \]

Using the model (M-2) proposed by Xu (2009), we get the following linear programming problem

\[ \begin{align*}
0.5w_1 - 0.5w_2 + d_{10} & \geq 0.100, \\
0.5w_1 - 0.5w_3 - d_{13} & \geq 0.200, \\
0.5w_1 - 0.5w_4 + d_{14} & \geq 0.000, \\
0.5w_2 - 0.5w_5 + d_{15} & \geq 0.100, \\
0.5w_2 - 0.5w_6 - d_{16} & \geq 0.200, \\
0.5w_3 - 0.5w_7 + d_{25} & \geq 0.300, \\
0.5w_3 - 0.5w_8 - d_{26} & \geq 0.200, \\
0.5w_4 - 0.5w_9 + d_{34} & \geq 0.100, \\
0.5w_4 - 0.5w_10 - d_{35} & \geq 0.200, \\
0.5w_5 - 0.5w_{11} + d_{45} & \geq 0.300, \\
0.5w_5 - 0.5w_{12} - d_{46} & \geq 0.200, \\
0.5w_6 - 0.5w_{13} + d_{56} & \geq 0.100, \\
0.5w_6 - 0.5w_{14} - d_{56} & \geq 0.200,
\end{align*} \]

and each \( d_{ij} \geq 0 \), each \( w_i \geq 0 \),

and solving the linear programming problem, we found the optimal priority vector \( w = (0.2413, 0.0413, 0.4133, 0.0947, 0.2078, 0.0016)^T \), i.e., the weights of the alternative \( a_i \) (\( i = 1, 2, \ldots, 6 \)) are \( w_1 = 0.2413, w_2 = 0.0413, w_3 = 0.4133, w_4 = 0.0947, w_5 = 0.2078 \) and \( w_6 = 0.0016 \), respectively.

Step 3. Using the model (M-2) proposed by Xu (2009), we get the following linear programming problem

\[ \begin{align*}
0.5w_1 - 0.5w_2 + d_{10} & \geq 0.100, \\
0.5w_1 - 0.5w_3 - d_{13} & \geq 0.300, \\
0.5w_1 - 0.5w_4 - d_{14} & \geq 0.200, \\
0.5w_2 - 0.5w_5 + d_{15} & \geq 0.000, \\
0.5w_2 - 0.5w_6 - d_{16} & \geq 0.000, \\
0.5w_3 - 0.5w_7 + d_{25} & \geq 0.000, \\
0.5w_3 - 0.5w_8 - d_{26} & \geq 0.000, \\
0.5w_4 - 0.5w_9 + d_{34} & \geq 0.000, \\
0.5w_4 - 0.5w_{10} - d_{35} & \geq 0.000, \\
0.5w_5 - 0.5w_{11} + d_{45} & \geq 0.000, \\
0.5w_5 - 0.5w_{12} - d_{46} & \geq 0.000, \\
0.5w_6 - 0.5w_{13} + d_{56} & \geq 0.000, \\
0.5w_6 - 0.5w_{14} - d_{56} & \geq 0.000,
\end{align*} \]

and each \( d_{ij} \geq 0 \), each \( w_i \geq 0 \),

and solving the linear programming problem, we found the optimal priority vector \( w = (0.2413, 0.0413, 0.4133, 0.0947, 0.2078, 0.0016)^T \), i.e., the weights of the alternative \( a_i \) (\( i = 1, 2, \ldots, 6 \)) are \( w_1 = 0.2413, w_2 = 0.0413, w_3 = 0.4133, w_4 = 0.0947, w_5 = 0.2078 \) and \( w_6 = 0.0016 \), respectively.

Step 4. Using the defined intuitionistic fuzzy weighted integral operator in equation (5), we evaluate the overall value \( a_i \) (\( i = 1, 2, 3, 4, 5, 6 \)). For \( i = 1 \), the overall value is calculate as:

\[ a_1 = IFC(a_{11}, a_{12}, \ldots, a_{16}) = \left( \prod_{i=1}^{6} \sum_{j=1}^{6} \prod_{j=1}^{6} (1 - f_{ij}) \right) \frac{\left( \sum_{i=1}^{6} w_i \right)}{\left( \sum_{j=1}^{6} \left( \frac{1}{w_i} \right) \right)} \]

\[ a_1 = \frac{1}{\left(1 - f_{12}\right) \left(1 - f_{13}\right) \left(1 - f_{14}\right) \left(1 - f_{15}\right) \left(1 - f_{16}\right)} \]

\[ = \left(0.5\right)^{0.2413} \left(0.6\right)^{0.2826} \left(0.7\right)^{0.6959} \left(0.5\right)^{0.7906} \left(0.4\right)^{0.9984} \left(0.6\right)^{1} \]

\[ = 0.080434, 0.190932 \]

Similarly, we find

\[ a_2 = (0.035101, 0.133400), a_3 = (0.053229, 0.115038), \]
\[ a_4 = (0.034509, 0.135670), a_5 = (0.062573, 0.108177), a_6 = (0.005420, 0.015271). \]

Step 5. The scores of the alternatives are calculated by utilizing the overall values of the alternatives \( a_i \) (\( i = 1, 2, 3, \ldots, 6 \)) and placed below:
We obtain the ranking of the mentioned six factors as \( a_5 > a_4 > a_3 > a_2 > a_1 > a_6 \). Hence from the six factors \( (a_1, a_2, a_3, a_4, a_5, a_6) \), we conclude that \( a_5 \) is the best option from the mentioned alternatives.

6. CONCLUSION:

The estimation of the priority vector of the intuitionistic fuzzy preference relation is an important part in decision making with an intuitionistic fuzzy preference relation matrix. Further, the study is an effort in the same direction which involves a weighted integral operator under the intuitionistic fuzzy environment to aggregate the values of the alternatives. Then we have employed the score functions of the aggregated values obtained by defined intuitionistic fuzzy weighted integral operator, from which the best alternative can be selected from the considered alternatives. Finally, an example is given to illustrate the multi-criteria decision making process for priority in intuitionistic fuzzy preference relation. The novelty is that the proposed method is using only two exact and simple concepts: defined intuitionistic fuzzy weighted integral operator and scores function, which can save much more time. Therefore, the developed method is of considerable practicality in actual applications.

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