Class - XI

MATHMATICS

Sequences and Series

A. Write the first five terms of each of the sequences whose nth terms are:

1. $a_n = n (n + 2)$

Ans:

On substituting *n* = 1, 2, 3, 4, and 5, we get the first five terms

$a_1 = 1(1 + 2) = 3$

$a_2 = 2(2 + 2) = 8$

$a_3 = 3(3 + 2) = 15$

$a_4 = 4(4 + 2) = 24$

$a_5 = 5(5 + 2) = 35$

Hence, the required terms are 3, 8, 15, 24, and 35.

2. $a_n = n/n+1$

Ans:

On substituting *n* = 1, 2, 3, 4, 5, we get

$$a_1 = \frac{1}{1+1} = \frac{1}{2},\ a_2 = \frac{2}{2+1} = \frac{2}{3},\ a_3 = \frac{3}{3+1} = \frac{3}{4},\ a_4 = \frac{4}{4+1} = \frac{4}{5},\ a_5 = \frac{5}{5+1} = \frac{5}{6}$$

Hence, the required terms are 1/2, 2/3, 3/4, 4/5 and 5/6.

3. $a_n = 2^n$

Ans:

On substituting *n* = 1, 2, 3, 4, 5, we get

$a_1 = 2^1 = 2$

$a_2 = 2^2 = 4$

$a_3 = 2^3 = 8$

$a_4 = 2^4 = 16$

$a_5 = 2^5 = 32$

Hence, the required terms are 2, 4, 8, 16, and 32.

4. **$a_n = (2n – 3)/6$**

Ans:

On substituting *n* = 1, 2, 3, 4, 5, we get

$$a_1 = \frac{2\times1-3}{6} = \frac{-1}{6}$$

$$a_2 = \frac{2\times2-3}{6} = \frac{1}{6}$$

$$a_3 = \frac{2\times3-3}{6} = \frac{3}{6} = \frac{1}{2}$$

$$a_4 = \frac{2\times4-3}{6} = \frac{5}{6}$$

$$a_5 = \frac{2\times5-3}{6} = \frac{7}{6}$$

Hence, the required terms are -1/6, 1/6, 1/2, 5/6 and 7/6..

5. $a_n = (-1)^{n-1}\ 5^{n+1}$

Ans:

On substituting *n* = 1, 2, 3, 4, 5, we get

$$a_1 = (-1)^{1-1} 5^{1+1} = 5^2 = 25$$
$$a_2 = (-1)^{2-1} 5^{2+1} = -5^3 = -125$$
$$a_3 = (-1)^{3-1} 5^{3+1} = 5^4 = 625$$
$$a_4 = (-1)^{4-1} 5^{4+1} = -5^5 = -3125$$
$$a^5 = (-1)^{5-1} 5^{5+1} = 5^6 = 15625$$
Hence, the required terms are 25, –125, 625, –3125, and 15625.

6.

$$a_n = n\frac{n^2+5}{4}$$

Solution:

On substituting *n* = 1, 2, 3, 4, 5, we get first 5 terms

$$a_1 = 1\cdot\frac{1^2+5}{4} = \frac{6}{4} = \frac{3}{2}$$
$$a_2 = 2\cdot\frac{2^2+5}{4} = 2\cdot\frac{9}{4} = \frac{9}{2}$$
$$a_3 = 3\cdot\frac{3^2+5}{4} = 3\cdot\frac{14}{4} = \frac{21}{2}$$
$$a_4 = 4\cdot\frac{4^2+5}{4} = 21$$
$$a_5 = 5\cdot\frac{5^2+5}{4} = 5\cdot\frac{30}{4} = \frac{75}{2}$$
Hence, the required terms are 3/2, 9/2, 21/2, 21 and 75/2.

B. Find the indicated terms in each of the sequences whose n^{th} terms are:

1. $a_n = 4n - 3$; a_{17}, a_{24}

Ans:

On substituting *n* = 17, we get

$a_{17} = 4(17) - 3 = 68 - 3 = 65$

Next, on substituting *n* = 24, we get

$a_{24} = 4(24) - 3 = 96 - 3 = 93$

2. $a_n = n^2/2^n$; a^7

Ans:

Now, on substituting *n* = 7, we get

$a_7 = 7^2/2^7 = 49/ 128$

3. $a_n = (-1)^{n-1} n^3$; a_9

Ans:

On substituting *n* = 9, we get

$a_9 = (-1)^{9-1} (9)^3 = 1 \times 729 = 729$

C. Write the first five terms of each of the sequences in Exercises 1 to 3 and obtain the corresponding series:

1. $a_1 = 3$, $a_n = 3a_{n-1} + 2$ for all $n > 1$

Solution:

Given, $a_n = 3a_{n-1} + 2$ and $a_1 = 3$

Then,

$a_2 = 3a_1 + 2 = 3(3) + 2 = 11$

$a_3 = 3a_2 + 2 = 3(11) + 2 = 35$

$a_4 = 3a_3 + 2 = 3(35) + 2 = 107$

$a_5 = 3a_4 + 2 = 3(107) + 2 = 323$

Thus, the first 5 terms of the sequence are 3, 11, 35, 107 and 323.

Hence, the corresponding series is

3 + 11 + 35 + 107 + 323 …….

2. $a_1 = -1$, $a_n = a_{n-1}/n$, $n \geq 2$

Solution:

Given,

$a_n = a_{n-1}/n$ and $a_1 = -1$

Then,

$a_2 = a_1/2 = -1/2$

$a_3 = a_2/3 = -1/6$

$a_4 = a_3/4 = -1/24$

$a_5 = a_4/5 = -1/120$

Thus, the first 5 terms of the sequence are -1, -1/2, -1/6, -1/24 and -1/120.

Hence, the corresponding series is

-1 + (-1/2) + (-1/6) + (-1/24) + (-1/120) + …….

3. $a_1 = a_2 = 2$, $a_n = a_{n-1} – 1$, $n > 2$

Solution:

Given,

$a_1 = a_2$, $a_n = a_{n-1} – 1$

Then,

$a_3 = a_2 – 1 = 2 – 1 = 1$

$a_4 = a_3 – 1 = 1 – 1 = 0$

$a_5 = a_4 – 1 = 0 – 1 = -1$

Thus, the first 5 terms of the sequence are 2, 2, 1, 0 and -1.

The corresponding series is

2 + 2 + 1 + 0 + (-1) + ……

D.

1. Find the sum of odd integers from 1 to 2001.

Solution:

The odd integers from 1 to 2001 are 1, 3, 5, . . . 1999, 2001.

It clearly forms a sequence in A.P.

Where, the first term, $a = 1$

Common difference, $d = 2$

Now,

a + (n -1)d = 2001

1 + (n-1)(2) = 2001

2n – 2 = 2000

2n = 2000 + 2 = 2002

n = 1001

We know,

$S_n = n/2 [2a + (n-1)d]$

$$S_n = \frac{1001}{2}\left[2\times1+\left(1001-1\right)\times2\right]$$

$$=\frac{1001}{2}\left[2+1000\times2\right]$$

$$=\frac{1001}{2}\times2002$$

$$=1001\times1001$$

$$=1002001$$

Therefore, the sum of odd numbers from 1 to 2001 is 1002001.

2. Find the sum of all natural numbers lying between 100 and 1000, which are multiples of 5.

Solution:

The natural numbers lying between 100 and 1000, which are multiples of 5, are 105, 110, … 995.

It clearly forms a sequence in A.P.

Where, the first term, $a = 105$

Common difference, $d = 5$
Now,
$a + (n - 1)d = 995$
$105 + (n - 1)(5) = 995$
$105 + 5n - 5 = 995$
$5n = 995 - 105 + 5 = 895$
$n = 895/5$
$n = 179$
We know,
$S_n = n/2 [2a + (n-1)d]$

$$S_n = \frac{179}{2}\left[2(105) + (179-1)(5)\right]$$
$$= \frac{179}{2}\left[2(105) + (178)(5)\right]$$
$$= 179\left[105 + (89)5\right]$$
$$= (179)(105 + 445)$$
$$= (179)(550)$$
$$= 98450$$

Therefore, the sum of all natural numbers lying between 100 and 1000, which are multiples of 5, is 98450.

3. In an A.P, the first term is 2 and the sum of the first five terms is one-fourth of the next five terms. Show that 20^{th} term is –112.

Solution:

Given,
The first term (a) of an A.P = 2
Let's assume *d* be the common difference of the A.P.
So, the A.P. will be 2, 2 + *d*, 2 + 2*d*, 2 + 3*d*, …
Then,
Sum of first five terms = 10 + 10*d*
Sum of next five terms = 10 + 35*d*
From the question, we have
$10 + 10d = ¼ (10 + 35d)$
$40 + 40d = 10 + 35d$
$30 = -5d$
$d = -6$
$a_{20} = a + (20 - 1)d = 2 + (19)(-6) = 2 - 114 = -112$
Therefore, the 20^{th} term of the A.P. is –112.

4. How many terms of the A.P. -6, -11/2, -5, …. are needed to give the sum –25?

Solution:

Let's consider the sum of *n* terms of the given A.P. as –25.
We known that,
$S_n = n/2 [2a + (n-1)d]$
where *n* = number of terms, *a* = first term, and *d* = common difference
So here, *a* = –6
$d = -11/2 + 6 = (-11 + 12)/ 2 = 1/2$
Thus, we have

$$-25=\frac{n}{2}\left[2\times(-6)+(n-1)\left(\frac{1}{2}\right)\right]$$
$$-50=n\left[-12+\frac{n}{2}-\frac{1}{2}\right]$$
$$-50=n\left[-\frac{25}{2}+\frac{n}{2}\right]$$
$$-100=n(-25+n)$$
$$n^2-25n+100=0$$
$$n^2-5n-20n+100=0$$
$$n(n-5)-20(n-5)=0$$
$$n=20 \text{ or } 5$$

5. In an A.P., if *p*th term is 1/q and *q*th term is 1/p, prove that the sum of first *pq* terms is ½ (pq + 1) where p ≠ q.

Solution:

We know that the general term of an A.P is given by: $a_n = a + (n - 1)d$

From the question, we have

$$p^{\text{th}} \text{ term} = a_p = a+(p-1)d=\frac{1}{q} \qquad ...(1)$$

$$q^{\text{th}} \text{ term} = a_q = a+(q-1)d=\frac{1}{p} \qquad ...(2)$$

Subtracting (2) from (1), we have

$$(p-1)d-(q-1)d=\frac{1}{q}-\frac{1}{p}$$
$$(p-1-q+1)d=\frac{p-q}{pq}$$
$$(p-q)d=\frac{p-q}{pq}$$
$$d=\frac{1}{pq}$$

Using the value of d in (1), we get

$$a+(p-1)\frac{1}{pq}=\frac{1}{q}$$
$$\Rightarrow a=\frac{1}{q}-\frac{1}{q}+\frac{1}{pq}=\frac{1}{pq}$$
$$S_{pq}=\frac{pq}{2}[2a+(pq-1)d]$$
$$=\frac{pq}{2}\left[\frac{2}{pq}+(pq-1)\frac{1}{pq}\right]$$
$$=1+\frac{1}{2}(pq-1)$$
$$=\frac{1}{2}pq+1-\frac{1}{2}=\frac{1}{2}pq+\frac{1}{2}$$
$$=\frac{1}{2}(pq+1)$$

Therefore, the sum of first pq terms of the A.P is $\frac{1}{2}(pq+1)$

6. Find the sum to *n* terms of the A.P., whose *k*th term is 5*k* + 1.

Solution:
Given, the k^{th} term of the A.P. is $5k + 1$.
k^{th} term $= a_k = a + (k – 1)d$
And,
$a + (k – 1)d = 5k + 1$
$a + kd – d = 5k + 1$
On comparing the coefficient of k, we get $d = 5$
$a – d = 1$
$a – 5 = 1$
$\Rightarrow a = 6$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$= \frac{n}{2}\left[2(6) + (n-1)(5)\right]$$
$$= \frac{n}{2}\left[12 + 5n - 5\right]$$
$$= \frac{n}{2}(5n + 7)$$

7. The ratio of the sums of *m* and *n* terms of an A.P. is m^2: n^2. Show that the ratio of m^{th} and n^{th} term is (2*m* – 1): (2*n* – 1).
Solution:
Let's consider that a and b to be the first term and the common difference of the A.P. respectively.
Then from the question, we have

$$\frac{\text{Sum of m terms}}{\text{Sum of n terms}} = \frac{m^2}{n^2}$$
$$\frac{\frac{m}{2}\left[2a + (m-1)d\right]}{\frac{n}{2}\left[2a + (n-1)d\right]} = \frac{m^2}{n^2}$$
$$\frac{2a + (m-1)d}{2a + (n-1)d} = \frac{m}{n} \quad \text{....... (1)}$$

Putting $m = 2m – 1$ and $n = 2n – 1$ in (1), we get

$$\frac{2a + (2m-2)d}{2a + (2n-2)d} = \frac{2m-1}{2n-1}$$
$$\Rightarrow \frac{a + (m-1)d}{a + (n-1)d} = \frac{2m-1}{2n-1} \quad \text{...... (2)}$$

Now,

$$\frac{m^{th}\text{ term of A.P.}}{n^{th}\text{ term of A.P.}} = \frac{a + (m-1)d}{a + (n-1)d} \quad \text{...... (3)}$$

From (2) and (3), we have

$$\frac{m^{th}\text{ term of A.P}}{n^{th}\text{term of A.P}} = \frac{2m-1}{2n-1}$$

Hence, the given result is proved.
8. If the sum of *n* terms of an A.P. is $3n^2 + 5n$ and its m^{th} term is 164, find the value of *m*.
Solution:
Let's consider a and b to be the first term and the common difference of the A.P. respectively.
$a_m = a + (m – 1)d = 164$ … (1)
We the sum of the terms is given by,

$S_n = n/2 [2a + (n-1)d]$

$$\frac{n}{2} [2a + nd - d] = 3n^2 + 5n$$

$$na + \frac{d}{2}n^2 - \frac{d}{2}n = 3n^2 + 5n$$

$$\frac{d}{2}n^2 + \left(a - \frac{d}{2}\right)n = 3n^2 + 5n$$

On comparing the coefficient of n^2 on both sides, we get

$$\frac{d}{2} = 3$$

$$\Rightarrow d = 6$$

On comparing the coefficient of n on both sides, we get

$$a - \frac{d}{2} = 5$$

$$a - 3 = 5$$

$$a = 8$$

Hence, from (1), we get

$8 + (m - 1) 6 = 164$

$(m - 1) 6 = 164 - 8 = 156$

$m - 1 = 26$

$m = 27$

Therefore, the value of m is 27.

9. Insert five numbers between 8 and 26 such that the resulting sequence is an A.P.

Solution:

Let's assume A_1, A_2, A_3, A_4, and A_5 to be five numbers between 8 and 26 such that 8, A_1, A_2, A_3, A_4, A_5, 26 are in an A.P.

Here we have,

$a = 8$, $b = 26$, $n = 7$

So,

$26 = 8 + (7 - 1) d$

$6d = 26 - 8 = 18$

$d = 3$

Now,

$A_1 = a + d = 8 + 3 = 11$

$A_2 = a + 2d = 8 + 2 \times 3 = 8 + 6 = 14$

$A_3 = a + 3d = 8 + 3 \times 3 = 8 + 9 = 17$

$A_4 = a + 4d = 8 + 4 \times 3 = 8 + 12 = 20$

$A_5 = a + 5d = 8 + 5 \times 3 = 8 + 15 = 23$

Therefore, the required five numbers between 8 and 26 are 11, 14, 17, 20, and 23.

10. If $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is the A.M. between *a* and *b*, then find the value of *n*.

Solution:

The A.M between a and b is given by, (a + b)/2

Then according to the question,

$$\frac{a+b}{2}=\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$$
$$(a+b)(a^{n-1}+b^{n-1})=2(a^n+b^n)$$
$$a^n+ab^{n-1}+ba^{n-1}+b^n=2a^n+2b^n$$
$$ab^{n-1}+a^{n-1}b=a^n+b^n$$
$$ab^{n-1}-b^n=a^n-a^{n-1}b$$
$$b^{n-1}(a-b)=a^{n-1}(a-b)$$
$$b^{n-1}=a^{n-1}$$
$$\left(\frac{a}{b}\right)^{n-1}=1=\left(\frac{a}{b}\right)^0$$
$$n-1=0$$
$$n=1$$
Thus, the value of n is 1.

E.

1. Find the 20th and *n*th terms of the G.P. 5/2, 5/4, 5/8, ………

Solution:

Given G.P. is 5/2, 5/4, 5/8, ………

Here, *a* = First term = 5/2

r = Common ratio = (5/4)/(5/2) = ½

Thus, the 20th term and nth term

$$a_{20}=ar^{20-1}=\frac{5}{2}\left(\frac{1}{2}\right)^{19}=\frac{5}{(2)(2)^{19}}=\frac{5}{(2)^{20}}$$
$$a_n=ar^{n-1}=\frac{5}{2}\left(\frac{1}{2}\right)^{n-1}=\frac{5}{(2)(2)^{n-1}}=\frac{5}{(2)^n}$$

2. Find the 12th term of a G.P. whose 8th term is 192 and the common ratio is 2.

Solution:

Given,

The common ratio of the G.P., $r = 2$

And, let *a* be the first term of the G.P.

Now,

$a_8 = ar^{8-1} = ar^7$

$ar^7 = 192$

$a(2)^7 = 192$

$a(2)^7 = (2)^6 (3)$

So,

$$a=\frac{(2)^6\times 3}{(2)^7}=\frac{3}{2}$$

Hence,

$$a_{12}=ar^{12-1}=\left(\frac{3}{2}\right)(2)^{11}=(3)(2)^{10}=3072$$

3. The 5th, 8th and 11th terms of a G.P. are *p*, *q* and *s*, respectively. Show that $q^2 = ps$.

Solution:

Let's take *a* to be the first term and *r* to be the common ratio of the G.P.

Then according to the question, we have

$a_5 = a r^{5-1} = a r^4 = p$ … (i)

$a_8 = a r^{8-1} = a r^7 = q$ … (ii)

$a_{11} = a r^{11-1} = a r^{10} = s$ … (iii)

Dividing equation (ii) by (i), we get

$$\frac{ar^7}{ar^4} = \frac{q}{p}$$

$$r^3 = \frac{q}{p} \quad ..... \text{(iv)}$$

On dividing equation (iii) by (ii), we get

$$\frac{ar^{10}}{ar^7} = \frac{s}{q}$$

$$r^3 = \frac{s}{q} \quad ..... \text{(v)}$$

Equating the values of r^3 obtained in (iv) and (v), we get

$$\frac{q}{p} = \frac{s}{q}$$

$$q^2 = ps$$

Hence proved.

4. The 4th term of a G.P. is square of its second term, and the first term is –3. Determine its 7th term.

Solution:

Let's consider *a* to be the first term and *r* to be the common ratio of the G.P.

Given, $a = -3$

And we know that,

$a_n = ar^{n-1}$

So, $a_4 = ar^3 = (-3)\, r^3$

$a_2 = a\, r^1 = (-3)\, r$

Then from the question, we have

$(-3)\, r^3 = [(-3)\, r]^2$

$\Rightarrow -3r^3 = 9\, r^2$

$\Rightarrow r = -3$

$a_7 = a\, r^{7-1} = a\, r^6 = (-3)\,(-3)^6 = -\,(3)^7 = -2187$

Therefore, the seventh term of the G.P. is –2187.

5. Which term of the following sequences:

(a) 2, 2√2, 4,… is 128 ? (b) √3, 3, 3√3,… is 729 ?

(c) 1/3, 1/9, 1/27, … is 1/19683 ?

Solution:

(a) The given sequence, 2, 2√2, 4,…

We have,

a = 2 and r = 2√2/2 = √2

Taking the nth term of this sequence as 128, we have

$$a_n = ar^{n-1}$$

$$(2)\left(\sqrt{2}\right)^{n-1} = 128$$

$$(2)(2)^{\frac{n-1}{2}} = (2)^7$$

$$(2)^{\frac{n-1}{2}+1} = (2)^7$$

$$\frac{n-1}{2} + 1 = 7$$

$$\frac{n-1}{2} = 6$$

$$n - 1 = 12$$

$$n = 13$$

Therefore, the 13th term of the given sequence is 128.

(ii) Given sequence, √3, 3, 3√3,…
We have,
a = √3 and r = 3/√3 = √3
Taking the n^{th} term of this sequence to be 729, we have

$$a_n = ar^{n-1}$$
$$\therefore ar^{n-1} = 729$$
$$(\sqrt{3})(\sqrt{3})^{n-1} = 729$$
$$(3)^{\frac{1}{2}}(3)^{\frac{n-1}{2}} = (3)^6$$
$$(3)^{\frac{1}{2}+\frac{n-1}{2}} = (3)^6$$

Equating the exponents, we have

$$\frac{1}{2}+\frac{n-1}{2} = 6$$
$$\frac{1+n-1}{2} = 6$$
$$\therefore n = 12$$

Therefore, the 12^{th} term of the given sequence is 729.
(iii) Given sequence, 1/3, 1/9, 1/27, …
a = 1/3 and r = (1/9)/(1/3) = 1/3
Taking the n^{th} term of this sequence to be 1/19683, we have

$$a_n = ar^{n-1}$$
$$\therefore ar^{n-1} = \frac{1}{19683}$$
$$\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)^{n-1} = \frac{1}{19683}$$
$$\left(\frac{1}{3}\right)^{n} = \left(\frac{1}{3}\right)^{9}$$
$$n = 9$$

Therefore, the 9^{th} term of the given sequence is 1/19683.
6. For what values of *x*, the numbers -2/7, x, -7/2 are in G.P?
Solution:
The given numbers are -2/7, x, -7/2.
Common ratio = x/(-2/7) = -7x/2
Also, common ratio = (-7/2)/x = -7/2x

$$\therefore \frac{-7x}{2} = \frac{-7}{2x}$$
$$x^2 = \frac{-2\times 7}{-2\times 7} = 1$$
$$x = \sqrt{1}$$
$$x = \pm 1$$

Therefore, for *x* = ± 1, the given numbers will be in G.P.
7. Find the sum to 20 terms in the geometric progression 0.15, 0.015, 0.0015 …
Solution:
Given G.P., 0.15, 0.015, 0.00015, …
Here, *a* = 0.15 and r = 0.015/0.15 = 0.1

We know that, $S_n = \dfrac{a(1-r^n)}{1-r}$

$$\therefore S_{20} = \frac{0.15\left[1-(0.1)^{20}\right]}{1-0.1}$$
$$= \frac{0.15}{0.9}\left[1-(0.1)^{20}\right]$$
$$= \frac{15}{90}\left[1-(0.1)^{20}\right]$$
$$= \frac{1}{6}\left[1-(0.1)^{20}\right]$$

8. Find the sum to *n* terms in the geometric progression √7, √21, 3√7, ….

Solution:

The given G.P is √7, √21, 3√7, ….

Here,

a = √7 and

$$r = \frac{\sqrt{21}}{\sqrt{7}} = \sqrt{3}$$
$$S_n = \frac{a(1-r^n)}{1-r}$$
$$\therefore S_n = \frac{\sqrt{7}\left[1-(\sqrt{3})^n\right]}{1-\sqrt{3}}$$
$$= \frac{\sqrt{7}\left[1-(\sqrt{3})^n\right]}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}} \qquad \text{(By rationalizing)}$$
$$= \frac{\sqrt{7}(1+\sqrt{3})\left[1-(\sqrt{3})^n\right]}{1-3}$$
$$= \frac{-\sqrt{7}(1+\sqrt{3})}{2}\left[1-(3)^{\frac{n}{2}}\right]$$
$$= \frac{\sqrt{7}(1+\sqrt{3})}{2}\left[(3)^{\frac{n}{2}}-1\right]$$

9. If the pth, qth and rth terms of a G.P. are *a, b* and *c*, respectively. Prove that $a^{q-r} b^{r-p} c^{p-q} = 1$

Solution:

Let's take *A* to be the first term and *R* to be the common ratio of the G.P.

Then according to the question, we have

$AR^{p-1} = a$

$AR^{q-1} = b$

$AR^{r-1} = c$

Then,

$a^{q-r} b^{r-p} c^{p-q}$

$= A^{q-r} \times R^{(p-1)(q-r)} \times A^{r-p} \times R^{(q-1)(r-p)} \times A^{p-q} \times R^{(r-1)(p-q)}$

$= Aq^{-r+r-p+p-q} \times R^{(pr-pr-q+r)+(rq-r+p-pq)+(pr-p-qr+q)}$

$= A^0 \times R^0$

$= 1$

Hence proved.

10**. If *a, b, c* and *d* are in G.P. show that $(a^2 + b^2 + c^2)(b^2 + c^2 + d^2) = (ab + bc + cd)^2$.**

Solution:

Given, a, b, c, d are in G.P.

So, we have

$bc = ad$ … (1)

$b^2 = ac$ … (2)

$c^2 = bd$ … (3)

Taking the R.H.S. we have

R.H.S.

$= (ab + bc + cd)^2$

$= (ab + ad + cd)^2$ [Using (1)]

$= [ab + d (a + c)]^2$

$= a^2b^2 + 2abd (a + c) + d^2 (a + c)^2$

$= a^2b^2 +2a^2bd + 2acbd + d^2(a^2 + 2ac + c^2)$

$= a^2b^2 + 2a^2c^2 + 2b^2c^2 + d^2a^2 + 2d^2b^2 + d^2c^2$ [Using (1) and (2)]

$= a^2b^2 + a^2c^2 + a^2c^2 + b^2c^2 + b^2c^2 + d^2a^2 + d^2b^2 + d^2b^2 + d^2c^2$

$= a^2b^2 + a^2c^2 + a^2d^2 + b^2 \times b^2 + b^2c^2 + b^2d^2 + c^2b^2 + c^2 \times c^2 + c^2d^2$

[Using (2) and (3) and rearranging terms]

$= a^2(b^2 + c^2 + d^2) + b^2 (b^2 + c^2 + d^2) + c^2 (b^2+ c^2 + d^2)$

$= (a^2 + b^2 + c^2) (b^2 + c^2 + d^2)$

= L.H.S.

Thus, L.H.S. = R.H.S.

Therefore,

$(a^2 + b^2 + c^2)(b^2 + c^2 + d^2) = (ab + bc + cd)^2$

11. Insert two numbers between 3 and 81 so that the resulting sequence is G.P.

Solution:

Let's assume G_1 and G_2 to be two numbers between 3 and 81 such that the series 3, G_1, G_2, 81 forms a G.P.

And let a be the first term and r be the common ratio of the G.P.

Now, we have the 1^{st} term as 3 and the 4^{th} term as 81.

$81 = (3) (r)^3$

$r^3 = 27$

$\therefore r = 3$ (Taking real roots only)

For $r = 3$,

$G_1 = ar = (3) (3) = 9$

$G_2 = ar^2 = (3) (3)^2 = 27$

Therefore, the two numbers which can be inserted between 3 and 81 so that the resulting sequence becomes a G.P are 9 and 27.