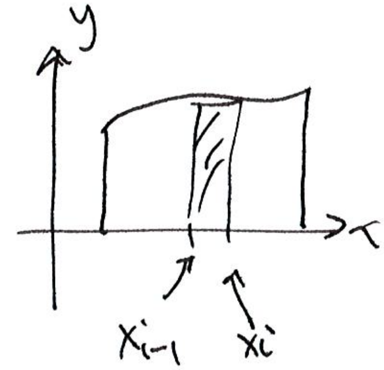
Math 1496 - Calc I

11/9/22 1-4

Last class we define the Riemann Sum

$$\int_a^b f(x) = \lim_{\substack{n\to\infty \\ \Delta x_i \to 0}} \sum_{i=1}^{n} f(x_i^*)\Delta x_i$$



$\Delta x_i = x_i - x_{i-1}$

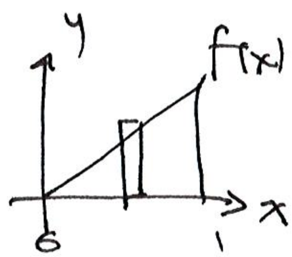
x_i^* in $[x_{i-1}, x_i]$

Fundamental Th^m^ of Calc - If f is cont^s^ [a,b]

$$\int_a^b f(x)dx = F(b) - F(a)$$

F(x) is the anti derivative of f(x) ie - $F'(x) = f(x)$

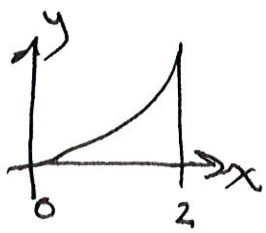
ex1



$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)\cdot\frac{1}{n} = \int_0^1 x\,dx$$

$$= \frac{x^2}{2}\Big|_0^1 = \frac{1}{2} - 0 = \frac{1}{2}$$

ex2



$$\lim_{n\to\infty} \sum_{i=1}^{\infty} \left(\frac{2i}{n}\right)^2\cdot\frac{2}{n} = \int_0^2 x^2 dx = \frac{x^3}{3}\Big|_0^2 = 8/3$$

Properties - Definite Integral

(1) $\int_0^b f(x)+g(x)\,dx = \int_0^b f(x)dx + \int_a^b g(x)dx$

(2) $\int_0^b c\,f(x)dx = c\int_d^b f(x)dx$

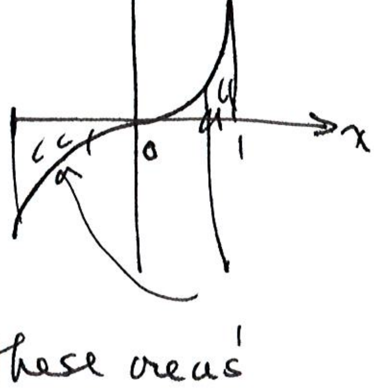
Ex 3 $\int_0^2 (2x+4)\,dx = 2\int_0^2 x\,dx + 4\int_0^2 1\,dx$

$= 2\cdot \frac{x^2}{2}\Big|_0^2 + 4\cdot x\Big|_0^2$

$= 2\left(\frac{4}{2}-0\right) + 4\cdot[(2)-0] = 4+8 = 12$

Ex 4 $\int_{-1}^{1} x^3 dx = \frac{x^4}{4}\Big|_{-1}^{1} = \frac{1^4}{4} - \frac{(-1)^4}{4} = \frac{1}{4}-\frac{1}{4} = 0$

Area Problem



these areas cancel

Note: $\int_{-1}^{6} x^3 dx = \frac{x^4}{4}\Big|_{-1}^{6}$

$= 0 - \frac{(-1)^4}{4} = -\frac{1}{4}$

Finding Area's

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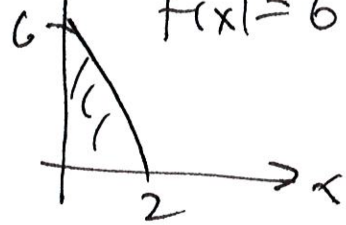
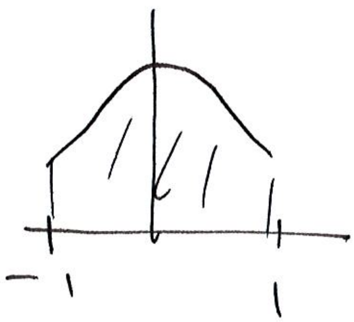


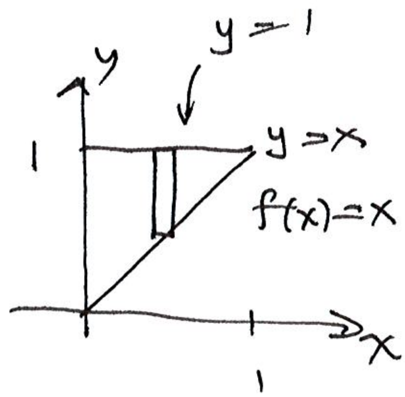
$f(x) = 6 - 3x$

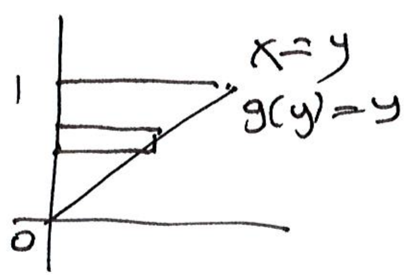
$$\int_0^2 (6-3x)\,dx$$

#20 $f(x) = \dfrac{4}{x^2+2}$



$$\int_{-1}^{1} \frac{4}{x^2+2}\,dx$$





need to use

$y = 1$ & $y = x$

2 curves - later

instead

$$\int_c^d g(y)\,dy = \int_0^1 y\,dy$$

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#14 $\displaystyle\int_{-2}^{-1}\left(u - \frac{1}{u^2}\right)du$ Note: variable is "u"

$$= \int_{-2}^{-1}(u - u^{-2})\,du = \frac{u^2}{2} - \frac{u^{-1}}{-1}\Big|_{-2}^{-1} = \frac{u^2}{2} + \frac{1}{u}\Big|_{-2}^{-1}$$

$$= \left[\frac{(-1)^2}{2} + \frac{1}{-1}\right] - \left[\frac{(-2)^2}{2} + \frac{1}{-2}\right].$$

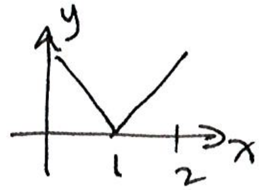
$$= \left(\frac{1}{2} - 1\right) - \left(2 - \frac{1}{2}\right) = -\frac{1}{2} - \frac{3}{2} = -2$$

#18 $\displaystyle\int_{1}^{8}\sqrt{\frac{2}{t}}\,dt$ New variable is "t"

$$\sqrt{2}\int_{1}^{8} t^{-1/2}\,dt = \sqrt{2}\left(\frac{t^{1/2}}{\frac{1}{2}}\Big|_{1}^{8}\right) = 2\sqrt{2}\left(8^{1/2} - 1^{1/2}\right)$$

$$= 2\sqrt{2}\left(2\sqrt{2} - 1\right) = 8 - 2\sqrt{2}$$

#21 $\displaystyle\int_{0}^{2}|x-1|\,dx$



if $x < 1$ $y = 1 - x$

$x > 1$ $y = x - 1$

$$\int_{0}^{1}(1-x)\,dx + \int_{1}^{2}(x-1)\,dx$$

$$= x - \frac{x^2}{2}\Big|_{0}^{1} + \frac{x^2}{2} - x\Big|_{1}^{2} = \left[\left(1 - \frac{1}{2}\right) - (0 - 0)\right] + \left[\left(\cancel{\frac{2^2}{2} - 2}\right)^{0} - \left(\frac{1}{2} - 1\right)\right]$$

$$= \frac{1}{2} + \frac{1}{2} = 1$$