Model Reduction of Continuous Time Interval Systems - A Computer Aided Approach

D.Bala Bhaskar,

Asst Professor, Dept of EEE,GVP College of Engg (A) , Visakhapatanam

ABSTRACT: A mixed algorithm is discussed for reducing the complexity of large scale dynamical systems with parameter uncertainty. The reduced model denominator is computed via Mihailov criterion and numerator via application of SRAM. The numerator coefficients are obtained using a simple formula by matching 't' initial time moments and 'm' markov parameters which avoids formation of routh tables. The reduced model obtained by this method preserves and approximates the characteristic properties of original system such as stability, time domain and frequency domain performance indices at most. The computational simplicity and accurate approximation of original system by proposed method is discussed by considering one example from literature, and the results are compared with the other existing techniques in literature.

KEYWORDS: *Interval Systems, Kharitonov's Theorem, Mihailov Criterion, SRAM, Order Reduction, Stability, Dominant Characteristics.*

I. INTRODUCTION

The Differential equations governing most of the practical systems are of complex and very high order, the analysis and controller design for such systems is very tedious. Computing reduced model is essential which leads to an active research area order reduction. The reduced model obtained will retain some dominant characteristics of the original system by reducing computational efforts, memory requirements, and design process. Methods for model reduction of continuous time and discrete time systems available in literature are [1-8], based on aggregation[4], balanced truncation, moment matching, least squares[6], Pade approximation[2], continued fraction expansion[1], Routh approximation[5], stability equation and many others.

In general, majority of industrial control processes, flight vehicle systems, robots, flexible manipulator systems, electric motors, cold rolling mills etc are modeled in continuous time and/or discrete time with uncertain parameters[9], The uncertainties in these subsystems arise from un modeled dynamics, parameters variation, sensor noises, actuator constraints, etc. Generally , these uncertainty considerations do not follow any known probability distribution patterns and mostly evaluated in terms of amplitude or frequency limits. This leads to another area of research termed as continuous-time parametric interval system models [10], with constant coefficients, but unknown within a limited range.

Existing methods in literature for model reduction of such systems - Routh-Pade approximation (Bandyopadhyay et al,1994), which is extension of (Shamash, 1975) has the limitation of not attaining stable reduced model and is highlighted in the work of Hwang and Yang (1999). $\gamma-\delta$ approximation (Bandyopadhyay, et al, 1997), generates stable reduced model but increases computational efforts. Routh approximants (Sastry et al, 2000) proposed an algorithm for deriving the denominator and numerator coefficients by computing only γ parameters. Dolgin and Zeheb(2003) found that the technique proposed by Bandyopadhyay et. al., (1994,1997) may produce unstable interval model even though the high-order interval system is stable. Yang (2005), further, showed that the method proposed by Dolgin and Zeheb (2003) may also result unstable interval model of stable high-order interval system.

In this paper, an algorithm is proposed for order reduction of continuous interval system which is inspired by (BAI-WU WAN,1981),(Sastry et al, 2000), and extension of (Sastry et al, 2010). The reduced model numerator computed by matching initial 't' time moments and 'm' markov parameters of original system(avoids formation of routh table) and Denominator computed via Mihailov criterion. The proposed technique ensures the stability of reduced order models and provides better approximation of higher order discrete-time systems. The paper is arranged as follows: Section 2 covers the problem description and basic interval arithmetic whereas section 3 proposed algorithm discussed. Numerical example, results are discussed in section 4 and conclusions are included in section 5.

2. PROBLEM STATEMENT - INTERVAL SYSTEMS

Let us consider a general high order single input single output interval system of n^{th} order is defined as

$$G_n(s) = \frac{N(s)}{D(s)} = \frac{[c_{20}^-, c_{20}^+] + [c_{21}^-, c_{21}^+]s + ... + [c_{2n-1}^-, c_{2n-1}^+]s^{n-1}}{[c_{10}^-, c_{10}^+] + [c_{11}^-, c_{11}^+]s + ... + [c_{1n}^-, c_{1n}^+]s^n} ....(1)$$

Where $[c_{2i}^-, c_{2i}^+]$; $0 \le$ i $\le$ n-1 and $[c_{1j}^-, c_{1j}^+]$; $0 \le$ j $\le$ n are scalar constants known as interval parameters.

The genera k[th] order reduced model is defined as

$$R_k(s) = \frac{[d_{20}^-, d_{20}^+] + [d_{21}^-, d_{21}^+]s + ... + [d_{2,k-1}^-, d_{2,k-1}^+]s^{k-1}}{[d_{10}^-, d_{10}^+] + [d_{11}^-, d_{11}^+]s + ... + [d_{1k}^-, d_{1k}^+]s^k} ...(2)$$

Where $[d_{2i}^-, d_{2i}^+]$; $0 \le$ i $\le$ k-1 and $[d_{1j}^-, d_{1j}^+]$; $0 \le$ j $\le$ k are scalar constants known as interval parameters.

A. INTERVAL ARITHMETIC E. Hansen [10] and E. Hansen et al [11]

Let [c , d] and [e , f] be two intervals then

Addition: [c, d] + [e, f] = [c + e, d + f] ..(3)

Subtraction: [c, d] - [e, f] = [c - f, d - e] .. (4)

Multiplication: [c , d] [e , f] = [Min(ce ,cf, de, df) , Max (ce , cf ,de , df)] ……..(5)

Division: $\frac{[c,d]}{[e,f]} = [c,d][\frac{1}{f}, \frac{1}{e}]$ ………. (6)

B. **KHARITONOV'S THEOREM:** Kharitonov [12] and R. Barmish [13]

It is used to determine the stability of a dynamic system when the physical parameters of the system are not known precisely i.e coefficients of the system are only known to be within specified ranges. Let a family of real interval polynomials

$y(s) = [x_0^-, x_0^+] + [x_1^-, x_1^+]s + .. + [x_n^-, x_n^+]s^n .....(7)$ Where x_j^- represents lower bound and x_j^+ represents upper bound of the coefficients system parameters respectively.

An interval system is stable (i.e if all members of the family are stable) if the following four Kharitonov's polynomials

$$y_1(s) = x_0^- + x_1^- s + x_2^+ s^2 + x_3^+ s^3 + x_4^- s^4 + ...(8)$$

$$y_2(s) = x_0^- + x_1^+ s + x_2^+ s^2 + x_3^- s^3 + x_4^- s^4 + ...(9)$$

$$y_3(s) = x_0^+ + x_1^- s + x_2^- s^2 + x_3^+ s^3 + x_4^+ s^4 + ...(10)$$

$$y_4(s) = x_0^+ + x_1^+ s + x_2^- s^2 + x_3^- s^3 + x_4^+ s^4 + ...(11)$$

Are stable

3. PROPOSED METHOD

Determination of denominator polynomial of k[th] order reduced model

Substituting $s = j\omega$ in $D(s)$ and separating it into real and imaginary parts

$$D(j\omega) = [c_{11}^-, c_{11}^+] + [c_{12}^-, c_{12}^+](j\omega) + ... + [c_{1,n+1}^-, c_{1,n+1}^+](j\omega)^n$$

$$= ([c_{11}^-, c_{11}^+] - [c_{13}^-, c_{13}^+]\omega^2 + ...) + j\omega([c_{12}^-, c_{12}^+]\omega - [c_{14}^-, c_{14}^+]\omega^2 + ...)$$

$$= \xi(\omega) + j\omega\eta(\omega) \quad ...... \quad (12)$$

Where ω is angular frequency, in rad/sec and

$\xi(\omega) = ([c_{11}^-, c_{11}^+] - [c_{13}^-, c_{13}^+]\omega^2 + ...)$ and $\eta(\omega) = ([c_{12}^-, c_{12}^+]\omega - [c_{14}^-, c_{14}^+]\omega^2 + ...)$

For $\xi(\omega) = 0$ and $\eta(\omega) = 0$ the frequencies which are intersecting $\omega_0 = 0, \pm[\omega_1^-, \omega_1^+], ... \pm[\omega_{n-1}^-, \omega_{n-1}^+]$ are obtained, where $\left|[\omega_1^-, \omega_1^+]\right| \prec \left|[\omega_2^-, \omega_2^+]\right| \prec ... \prec \left|[\omega_{n-1}^-, \omega_{n-1}^+]\right|$

Similarly substitute $s = j\omega$ in $D_k(s)$ and separating it into real and imaginary parts

$$D_k(j\omega) = ([d_{11}^-, d_{11}^+] - [d_{13}^-, d_{13}^+]\omega^2 + ...) + j\omega([d_{12}^-, d_{12}^+]\omega - [d_{14}^-, d_{14}^+]\omega^2 + ...)$$

$$D_k(j\omega) = \phi(\omega) + j\omega\varphi(\omega) \quad .... (13) \quad \text{where}$$

$\phi(\omega) = [d_{11}^-, d_{11}^+] - [d_{13}^-, d_{13}^+]\omega^2 + ...$ and $\varphi(\omega) = ([d_{12}^-, d_{12}^+]\omega - [d_{14}^-, d_{14}^+]\omega^2 + ...)$

For $\phi(\omega) = 0$ and $\varphi(\omega) = 0$ we get k number of roots must be real and positive and alternatively distributed along the ω axis. Then the first k number of frequencies are $\omega_0 = 0, [\omega_1^-, \omega_1^+], ...[\omega_{k-1}^-, \omega_{k-1}^+]$ are obtained which are kept unchanged, and the roots of $\phi(\omega) = 0$ and $\varphi(\omega) = 0$

$$\phi(\omega) = [\lambda_1^-, \lambda_1^+](\omega^2 - [\omega_1^2, \omega_1^2])(\omega^2 - [\omega_3^2, \omega_3^2])(\omega^2 - [\omega_5^2, \omega_5^2]) ....(14)$$

$$\psi(\omega) = [\lambda_2^-, \lambda_2^+](\omega^2 - [\omega_2^2, \omega_2^2])(\omega^2 - [\omega_4^2, \omega_4^2])(\omega^2 - [\omega_6^2, \omega_6^2]) ....(15)$$

The coefficient values of $[\lambda_1^-, \lambda_1^+]$ and $[\lambda_2^-, \lambda_2^+]$ are calculated from $\xi(0) = \phi(0)$ and $\eta([\omega_1^-, \omega_1^+]) = \psi([\omega_1^-, \omega_1^+])$ substituting these values in equations (14) and (15)

$D_k(j\omega)$ is obtained as

$D_k(j\omega) = \phi(\omega) + j\omega\varphi(\omega)$ now replace jω by 's' then the k[th] order reduced model is

$$D_k(s) = [d_{10}^-, d_{10}^+] + [d_{11}^-, d_{11}^+](s) + ... + [d_{1,k}^-, d_{1,k}^+](s^k) \quad ... (16)$$

NUMERATOR COEFFICIENTS USING SRAM

After obtaining the reduced denominator $D_k(s)$, the numerator of the biased model, which will retain the first't' time moments and 'm' markov parameters is found as follows:

$$N_k(s) = N_{kt}(s) + N_{km}(s) \quad ..... (17) \text{ With } k=t+m.$$

$$= [T_1^-, T_1^+] + [T_2^-, T_2^+]s + ... + [T_t^-, T_t^+]s^{k-m+1} + [M_m^-, M_m^+]s^{k-m} + .. + [M_2^-, M_2^+]s^{k-2} + [M_1^-, M_1^+]s^{k-1} \quad ....18)$$

in general $$[T_t^-, T_t^+] = \frac{[d_{10}^-, d_{10}^+]}{[c_{10}^-, c_{10}^+]} * [c_{2,t-1}^-, c_{2,t-1}^+] \quad ..19)$$

$$[M_m^-, M_m^+] = \frac{1}{[c_{1,n}^-, c_{1,n}^+]} \left\{ \begin{array}{l} \sum_{i=1}^{m} ([c_{2,n-i}^-, c_{2,n-i}^+][d_{1,k-m-i}^-, d_{1,k-m-i}^+]) - \\ \sum_{j=0}^{m-1} ([M_j^-, M_j^+][c_{1,n-m+j}^-, c_{1,n-m+j}^+]) \end{array} \right\} \quad \ldots (20)$$

with $[M_0^-, M_0^+] = [0,0]$

4. NUMERUCAL EXAMPLE: Consider the 3rd order system transfer function given by B. Bandyopadhyay [1997] , Dolgin et al [2003], G.V.K. Sastry et al [2000], V. Krishnamurthy et al [1978], Prasad et al [2003]

$$G(s) = \frac{[2,3]s^2 + [17.5,18.5]s + [15,16]}{[2,3]s^3 + [17,18]s^2 + [35,36]s + [20.5,21.5]}$$

Substitute $s = j\omega$ in the denominator

$$D(j\omega) = ([20.5,21.5] - [17,18]\omega^2) + \ldots + j\omega([35,36] - [2,3]\omega^2)$$

The intersecting frequencies are $[\omega_i^-, \omega_i^+] = 0, [1.0929, 1.9081], [3.4641, 4.1833]$

Following procedure $[\lambda_1^-, \lambda_1^+]$ =-[17.0001,18.0007] and $[\lambda_2^-, \lambda_2^+]$ =[31.3826,33.6111]

substituting these values and replacing $j\omega = s$ the reduced denominator is obtained as

$$D_2(s) = [17.0001, 18.0007]s^2 + [31.3826, 33.6111]s + [20.3061, 21.7052]$$

The second order reduced model numerator using SRAM which retaining 't' time moments and

$$[T_1^-, T_1^+] = \frac{[20.3061, 21.7052]}{[20.5, 21.5]} * [15,16] = [14.16705, 16.9406]$$

$$[T_2^-, T_2^+] = \frac{[20.3061, 21.7052]}{[20.5, 21.5]} * [17.5, 18.5] = [16.5282, 19.5876]$$

Numerator by matching two time moments is

$$N_2(s) = [16.5282, 19.5876]s + [14.16705, 16.9406]$$

The proposed reduced model is

$$R_2(s) = \frac{[16.5282, 19.5876]s + [14.16705, 16.9406]}{[17.0001, 18.0007]s^2 + [31.3826, 33.6111]s + [20.3061, 21.7052]}$$

Reduced order transfer function by Sastry et al[2000] is

$$R_2(s) = \frac{[0.94, 1.35]s + [0.8409, 1.168]}{s^2 + [2.0181, 2.4430]s + [1.492, 1.5007]}$$

Reduced order transfer function by D Kranthi et al[2013]

$$R_2(s) = \frac{[0.4838, 1.826]s + [0.8241, 1.1307]}{[1,1]s^2 + [2.0758, 2.3751]s + [1.1896, 1.459]}$$

Reduced order transfer function by A. Jaiswal et al[2014]

$$R_2(s) = \frac{[-0.2438, 4.755]s + [2.345, 3.352]}{[1,1]s^2 + [3.777, 5.99]s + [3.373, 4.295]}$$

Reduced order transfer function by D Kranthi et al[2011]

$$R_2(s) = \frac{[35.6065, 49.4454]s + [14.0331, 17.1024]}{[17.0001, 18.0007]s^2 + [31.3826, 33.6111]s + [20.3061, 21.7052]}$$

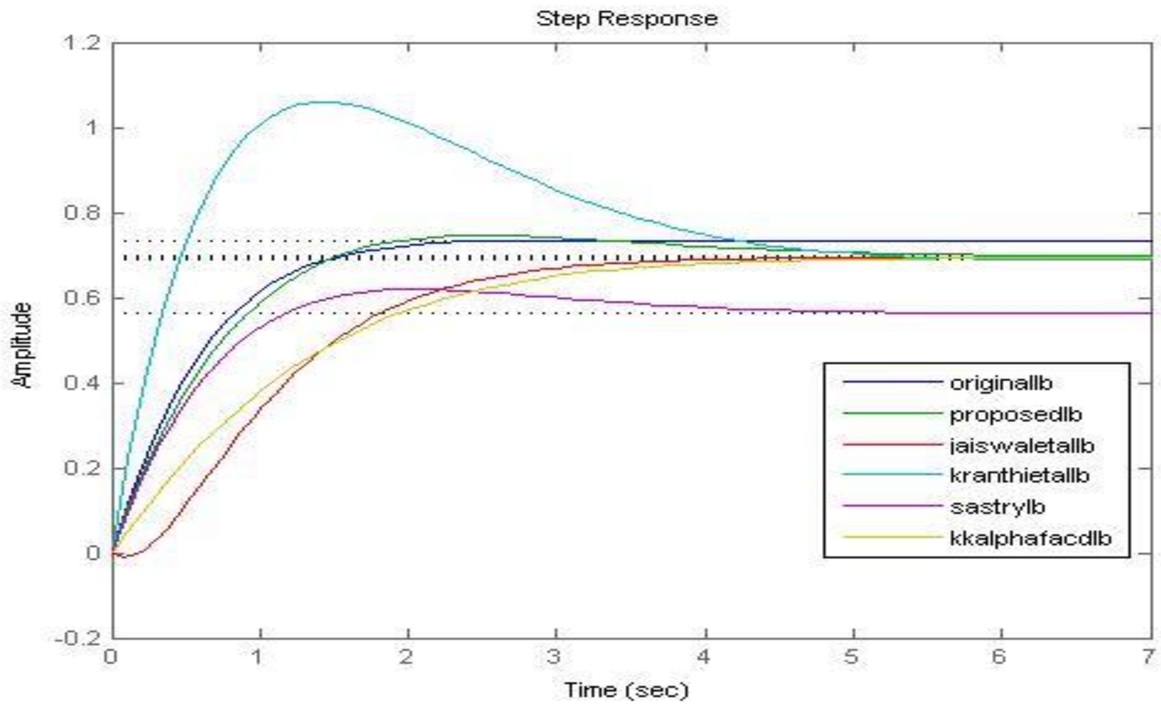

Figure 1 Comparison of Step responses lower Bound

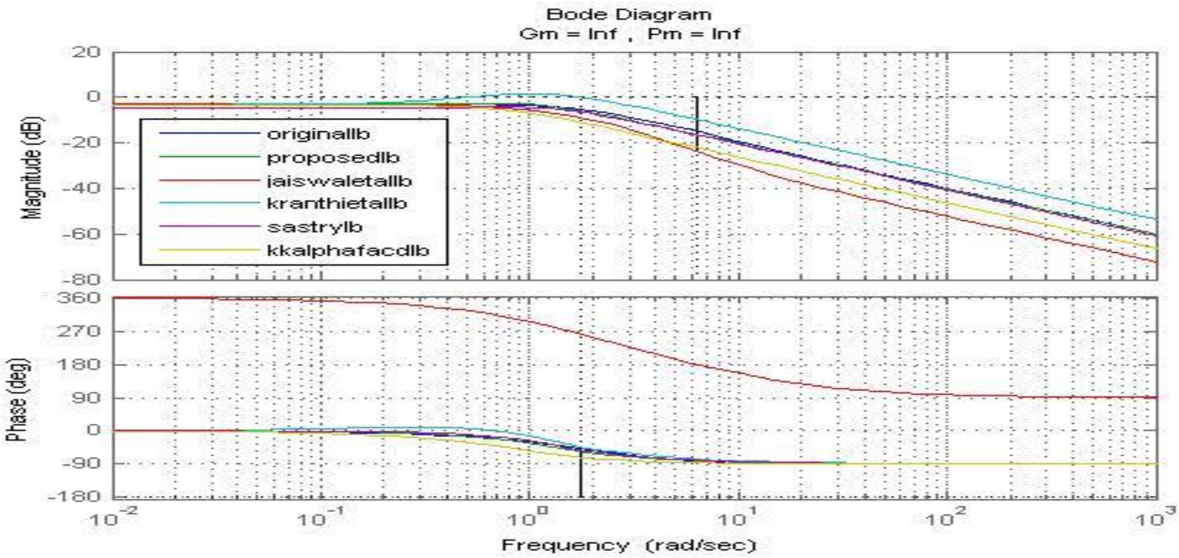

Fig. 2 Comparison of frequency responses of lower Bound

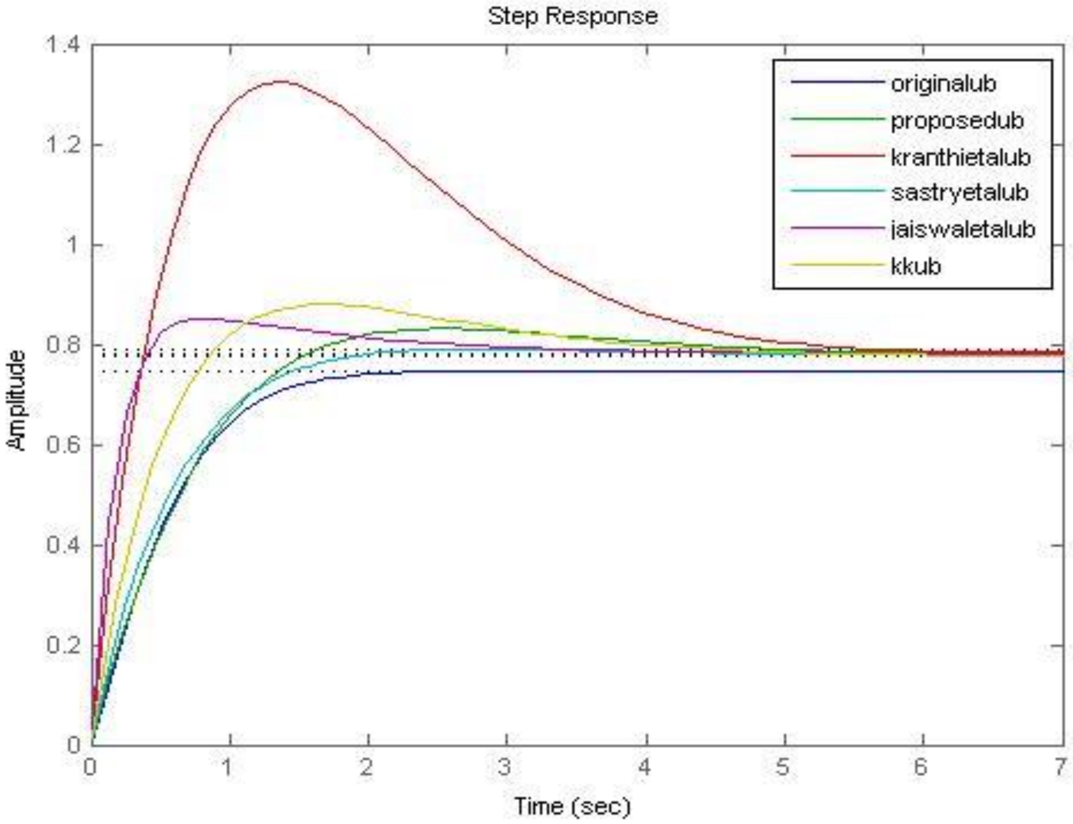

Figure 3 Comparison of Step responses of upper bound

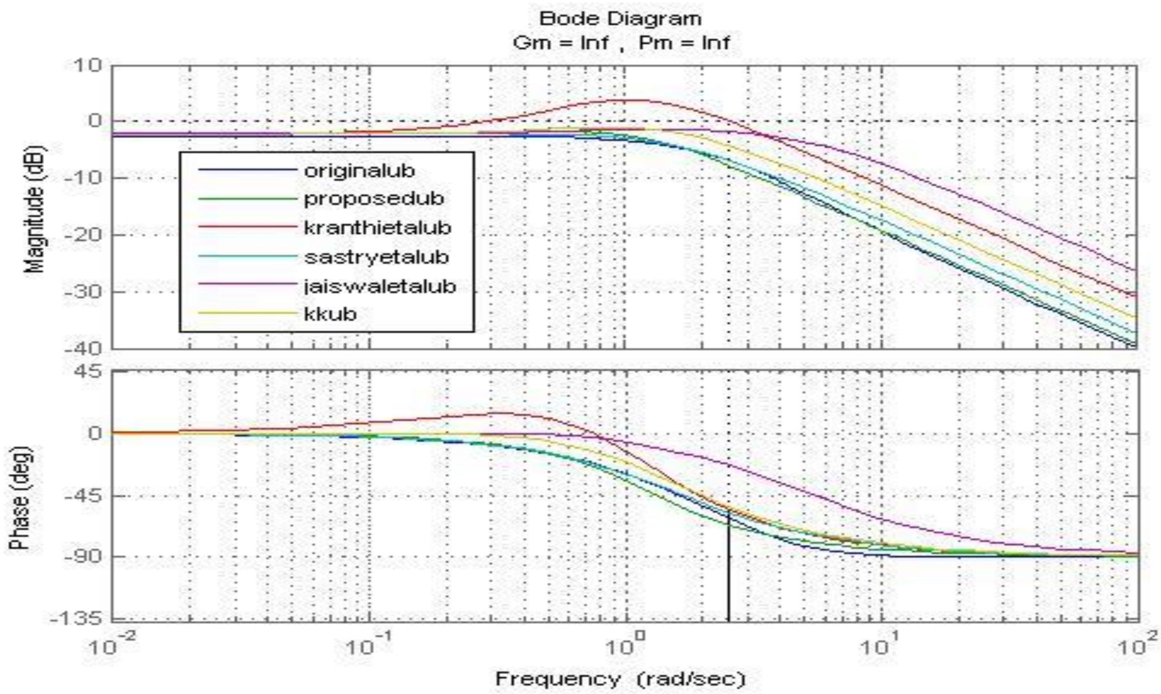

Fig. 4 Comparison of frequency responses of Upper Bound

INTEGRAL SQUARE ERROR

The integral square error is determined between transient part of step responses of original and reduced systems. $ISE = \int_0^\infty [y(t) - y_r(t)]^2 dt$ where y(t) is the step response of original system, $y_r(t)$ step response of reduced system. It can be observed that ISE with proposed technique is small compared to other methods

5. CONCLUSIONS:

An algorithm was discussed for modeling of linear dynamic systems with parameter uncertainties. The micro models obtained by proposed method will guarantee the stability and approximate performance characteristics of original systems at most. The denominator is obtained by using Mihailov criterion and numerator is obtained using SRAM by matching first't' time moments. The proposed method is simple mathematically; computer oriented and requires less number of computations. A numerical example is discussed, and the results are compared with other existing methods in literature proposed recently. The comparison of step and frequency shows that, the reduced models obtained by proposed algorithm will better approximate and retain the important characteristic feature of original system compared to other existing methods.

Reduction method	Lower Bound (lb)	Upper Bound (ub)	Rise Time (sec)		Peak time (sec)		Settling Time(sec)		Peak over Shoot(%)	
			lb	ub	lb	ub	lb	ub	lb	ub
Original	--	---	1.15	1.05	3.23	2.9	1.86	1.68	0.413	0.146
Proposed	0.0085	0.0098	1.07	1.08	2.53	2.5	4.49	4.45	6.88	6.45
Sastry et al[2000]	0.2256	0.0095	0.847	1.09	2	2.78	4.1	1.64	9.91	1.49
Kranthi Kumar et al[2013]	0.0125	0.0082	2.41	0.618	---	1.7	3.96	4.42	----	13.4
Kranthi Kumar et al[2011]	0.0124	0.0169	0.364	0.303	1.38	1.36	4.93	4.99	53.5	67.9
A Jaiswal et al [2014]	0.0095	0.0107	1.9	0.28	--	0.763	3.44	2.86	--	9.08

Table .1 COMPARISION OF ISE and TIME DOMAIN SPECIFICATIONS

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