PhysicsNinja

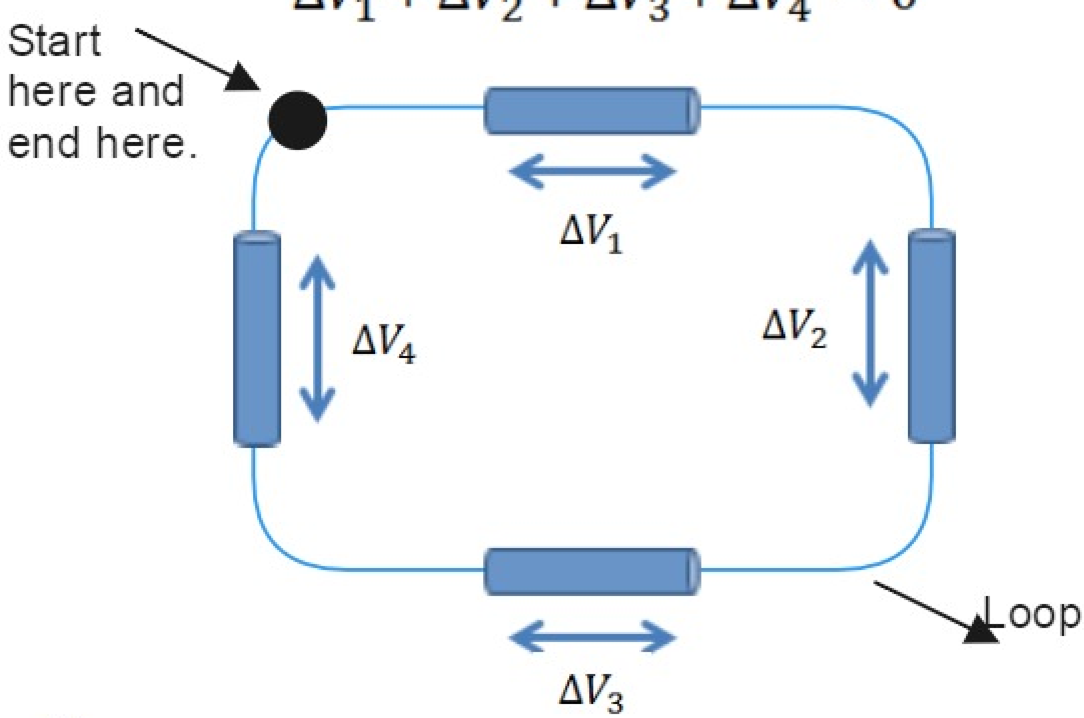
Circuits

Strategy

- Draw a circuit diagram. Label all known and unknown quantities.
- Write one loop equation for each independent loop.
- Write one junction equation for each junction in the circuit
- Solve the equations for the currents and potential differences.

Kirchhoff's loop law

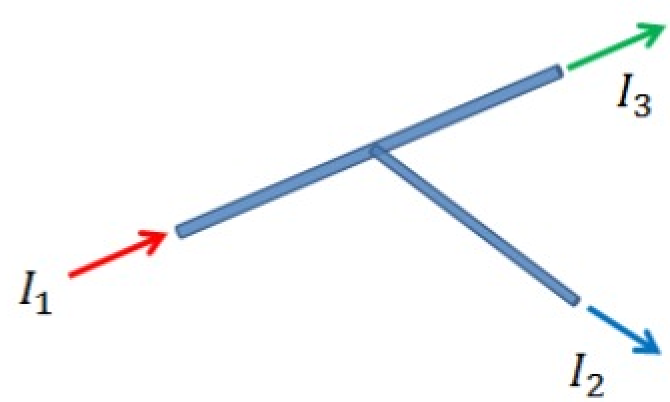
$$\Delta V_1 + \Delta V_2 + \Delta V_3 + \Delta V_4 = 0$$



Conservation of Energy for Each Charge

Junction law

$$I_1 = I_2 + I_3$$



Conservation of charge

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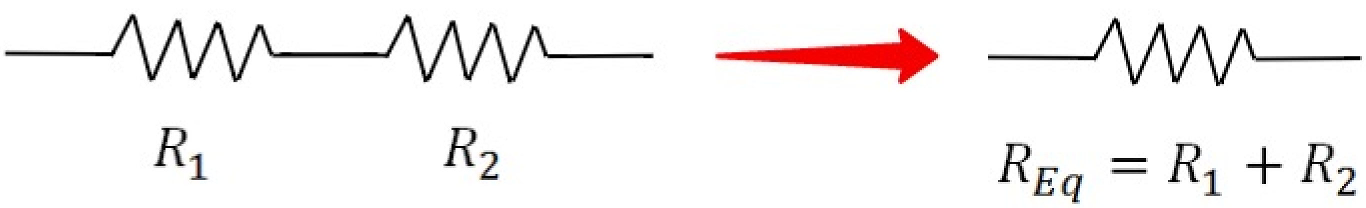
Circuits

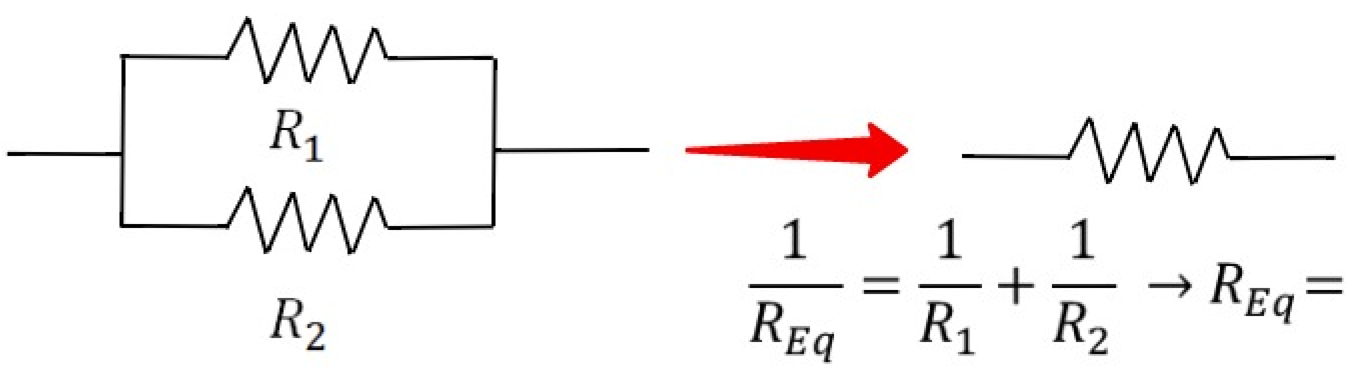
We analyze a circuit by transforming it into a simpler one, containing a single equivalent resistor.
We need to turn combinations of resistors (series and parallel) into one resistor that provides the same overall resistance as the combination.

resistors in series | **single equivalent resistor**



$$R_{Eq} = R_1 + R_2$$

resistors in parallel



$$\frac{1}{R_{Eq}} = \frac{1}{R_1} + \frac{1}{R_2} \rightarrow R_{Eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$R_{Eq} = \frac{R_1 R_2}{R_1 + R_2}$$

For 2 resistors in parallel simply remember
"the product divided by the sum"

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Circuits

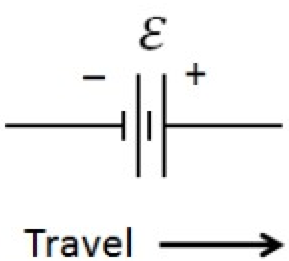
Concepts

Ohm's law

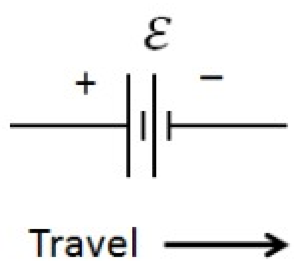
A potential difference between the ends of a conductor with resistance *R* creates a current

$$I = \frac{\Delta V}{R}$$

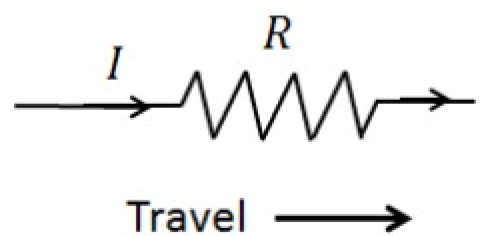
Signs of ΔV for Kirchhoff's loop law



$$\Delta V_{bat} = +\,\varepsilon$$



$$\Delta V_{bat} = -\,\varepsilon$$



$$\Delta V_{Res} = -IR$$

Power in a circuit

The battery *supplies* energy at the rate

$$P_{bat} = I\,\Delta V_{bat}$$

The resistors *dissipate* energy at the rate

$$P_R = I\,\Delta V_R = I^2 R = \frac{(\Delta V_{Res})^2}{R}$$

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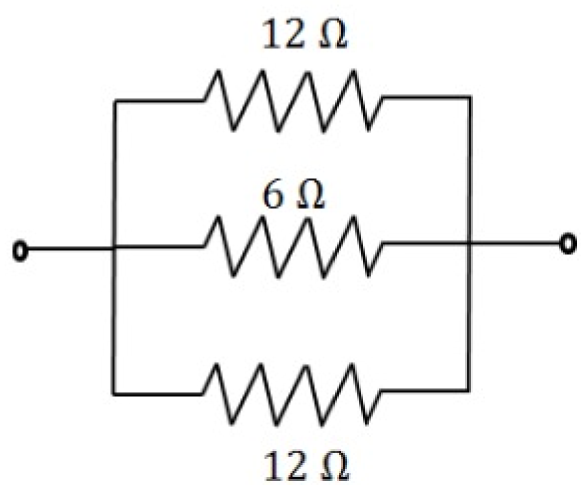
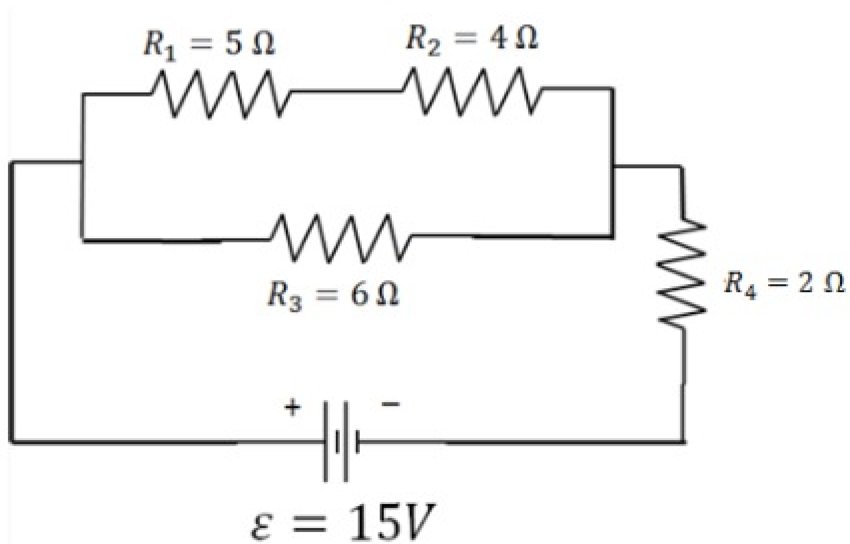
Circuits

Problems:

1) What is the equivalent resistance of the following network of resistors.



2) Find the current through each resistor, the potential difference across each resistor, the power being delivered by the battery and the power dissipated by each resistor.



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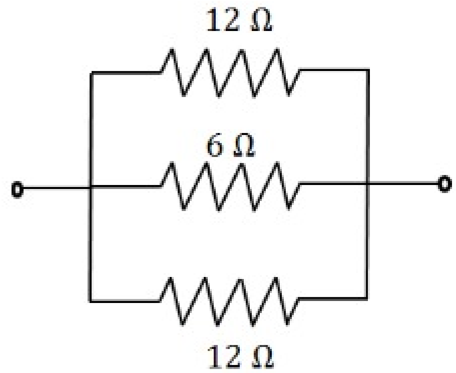
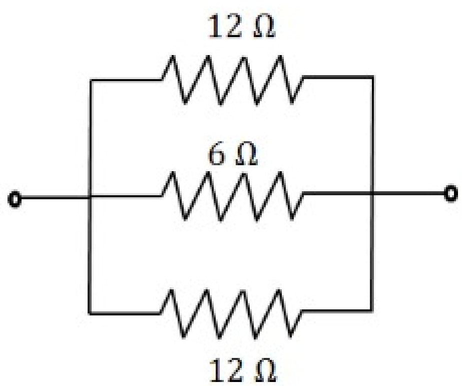
Circuits

Solutions

1) What is the equivalent resistance of the following network of resistors.

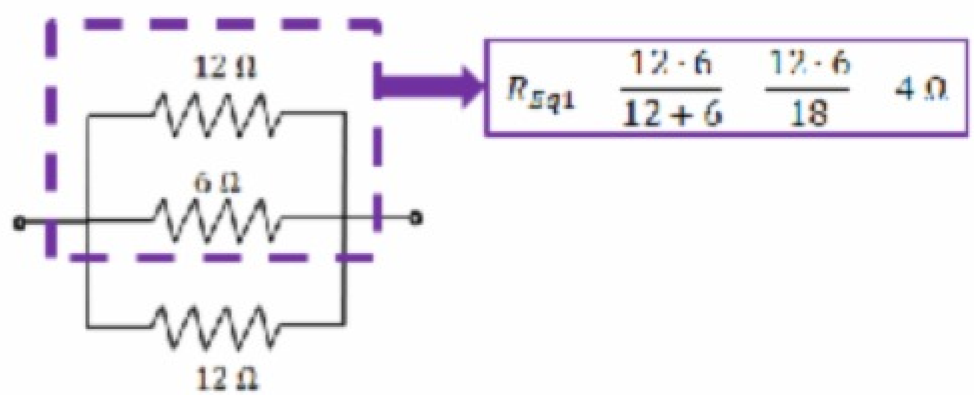


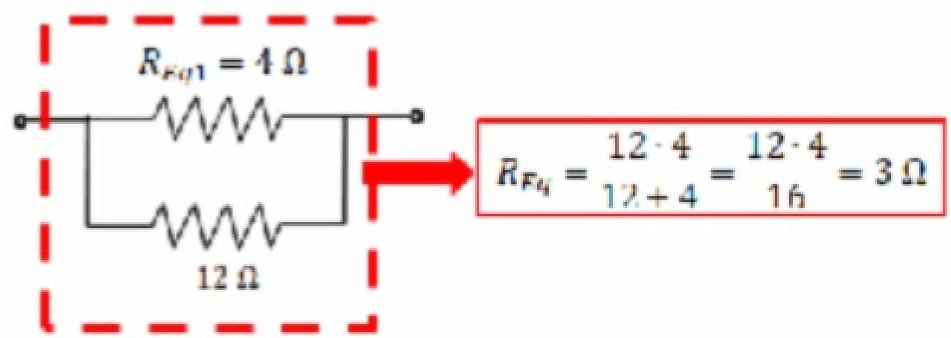


$$\frac{1}{R_{Eq}} = \frac{1}{12} + \frac{1}{6} + \frac{1}{12} = \frac{4}{12} = \frac{1}{3}$$

$$R_{Eq} = 3\,\Omega$$

or





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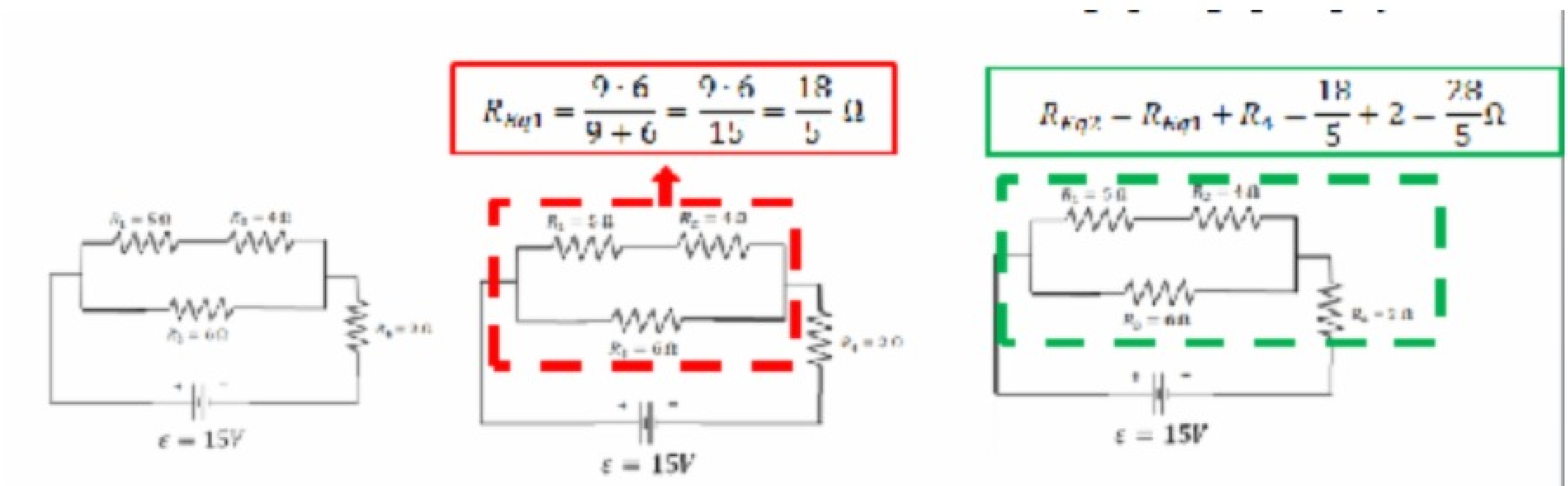
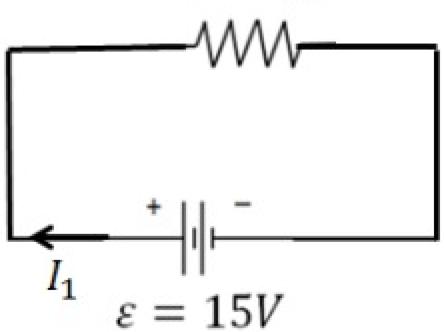
2) Find the current through each resistor, the potential difference across each resistor, the power being delivered by the battery and the power dissipated by each resistor.

For many circuits with a single battery, it's often easiest to find the current through the battery first (and in this case the current through resistance R_4) by simplifying the circuit. Using equivalent resistance we find;



$$R_{Eq2} = \frac{28}{5}\Omega$$



$$+\varepsilon - I_1 R_{Eq2} = 0$$

$$I_1 = \frac{\varepsilon}{R_{Eq2}} = \frac{15}{28/5} = \frac{75}{28}\ \Omega$$

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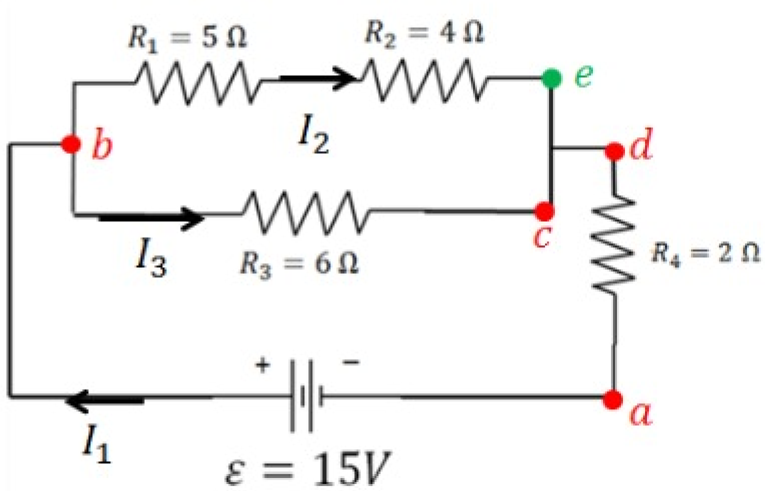
Circuits

2) Find the current through each resistor, the potential difference across each resistor, the power being delivered by the battery and the power dissipated by each resistor.



$$I_1 = \frac{75}{28}\ \Omega$$

junction rule

$$I_1 = I_2 + I_3$$

Loop rule

$$abcda \rightarrow +15 - I_3R_3 - I_1R_4 = 0.$$

$$abeda \rightarrow +15 - I_2R_1 - I_2R_2 - I_1R_4 = 0.$$

The only unknows in the loops rules are the currents. Solving them gives,

$$I_3 = \frac{15 - I_1R_4}{R_3} = \frac{15 - \frac{75}{28}\cdot 2}{6} = \frac{45}{28}\ \text{Amps}$$

$$I_2 = \frac{15 - I_1R_4}{R_1 + R_2} = \frac{15 - \frac{75}{28}\cdot 2}{9} = \frac{45}{42} = \frac{15}{14}\ \text{Amps}$$

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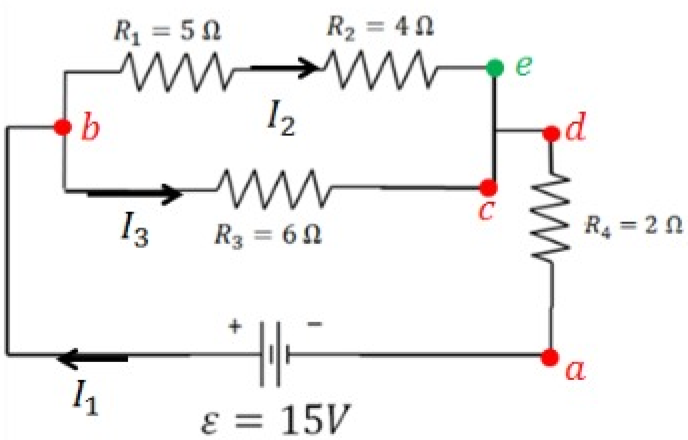
Circuits

2) Find the current through each resistor, the potential difference across each resistor, the power being delivered by the battery and the power dissipated by each resistor.



Voltage across each resistor

$$\Delta V_1 = R_1 I_2 = 5\cdot\frac{15}{14} = \frac{75}{14}\,V$$

$$\Delta V_2 = R_2 I_2 = 4\cdot\frac{15}{14} = \frac{60}{14}\,V$$

$$\Delta V_3 = R_3 I_3 = 6\cdot\frac{45}{28} = \frac{135}{14}\,V$$

$$\Delta V_4 = R_4 I_1 = 2\cdot\frac{75}{28} = \frac{75}{14}\,V$$

Power dissipated by each resistor

$$P_{R_1} = R_1 I_2^2 = 5\cdot\left(\frac{15}{14}\right)^2 = \frac{1125}{196}\,W$$

$$P_{R_2} = R_2 I_2^2 = 4\cdot\left(\frac{15}{14}\right)^2 = \frac{900}{196}\,W$$

$$P_{R_3} = R_3 I_3^2 = 6\cdot\left(\frac{45}{28}\right)^2 = \frac{12150}{784}\,W$$

$$P_{R_4} = R_4 I_1^2 = 2\cdot\left(\frac{75}{28}\right)^2 = \frac{11250}{784}\,W$$

$$P_{Dissipated} = P_{R_1} + P_{R_2} + P_{R_3} + P_{R_4} = \frac{1125}{28}\,W$$

Power supplied by the battery

$$P_{Battery} = \varepsilon I_1 = 15\cdot\frac{75}{28} = \frac{1125}{28}\,W$$

$$P_{Battery} = P_{Dissipated}$$