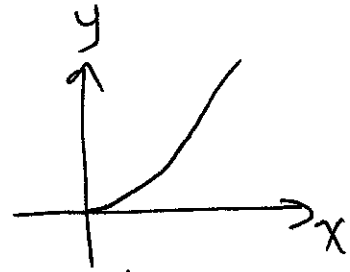
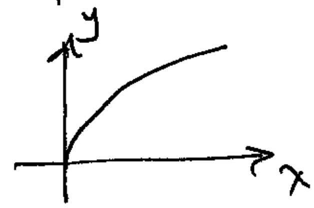
Math 1496 – Calc I

4.4 Concavity

Consider $f(x) = x^2$

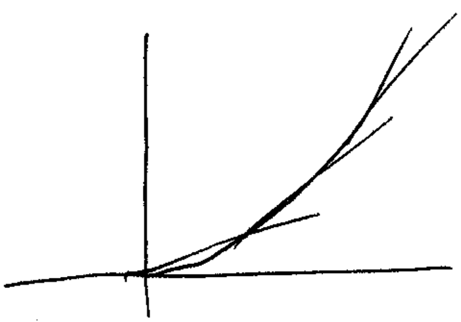


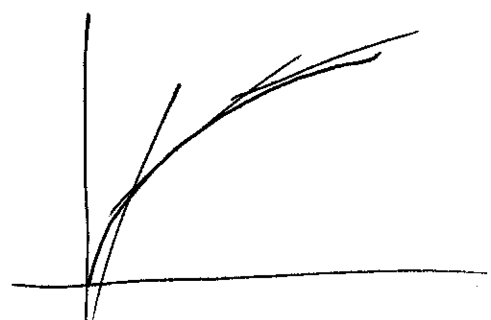
$f(x) = \sqrt{x}$



Now $f'(x) = 2x$

$f'(x) = \dfrac{1}{2\sqrt{x}}$) for $x > 0$ both $f' > 0$





Notice the slopes of the tangents are increasing

and here the slopes of the tangents are decreasing

if f' slope of tangent

f'' rate of change of slopes

Concavity

Let $f(x)$ be a function that has 2nd order derivatives (they exist) on some interval

if $f''(x) > 0$ on I, f is concave upward

$f''(x) < 0$ on I, f is concave downward

ex 1 Determine when f is increasing/decreasing & concave ↑/↓ for the following

$$f(x) = x^3 - 6x^2 + 9x$$

$$f'(x) = 3x^2 - 12x + 9$$

$$= 3(x-1)(x-3)$$

$$f''(x) = 6x - 12$$

$$= 6(x-2)$$

concave ↓

$(-\infty, 2)$

concave ↑

$(2, \infty)$

x		1		2		3	
$x-1$	−	0	+	+	+	+	+
$x-3$	−	−	−	−	−	0	+
$(x-1)(x-3)$	+	0	−	−	−	0	+
slope	/	—	\	\	\	—	/
$x-2$	−	−	−	0	+	+	+
fn							

← these should agree

←

<u>Def^n</u> Point of inflection (PI)

when a graph changes its concavity
(concave ↑ to ↓ or ↓ to ↑)
we say that pt is a pt of inflection

ex $f(x) = x^3$ $f'(x) = 3x^2$ $f''(x) = 6x$

$x < 0$ $f'' < 0$ $x > 0$ $f'' > 0$ so $x = 0$ is a PI

ex $y = 3x^5 - 15x^4$

$y' = 15x^4 - 60x^3$

$= 15x^3(x-4)$ $y' = 0$ $x = 0, 4$ crit #

$y'' = 60x^3 - 180x^2$

$= 60x^2(x-3)$ $y'' = 0$ when $x = 0, 3$
possible PI

⇒ go to sign chart

x		0		3		4	
x^3	$-$	0	$+$	$+$	$+$	$+$	$+$
$x-4$	$-$	$-$	$-$	$-$	$-$	0	$+$
$x^3(x-4)$	$+$	0	$-$	$-$	$-$	0	$+$
slope	/	—	\	\	\	_	/
x^2	$+$	0	$+$	$+$	$+$	$+$	$+$
$x-3$	$-$	$-$	$-$	0	$+$	$+$	$+$
$x^2(x-3)$	$-$	0	$-$	0	$+$	$+$	$+$
$\cup/\cap$	$\cap$	X	$\cap$	Pi	$\cup$	$\cup$	

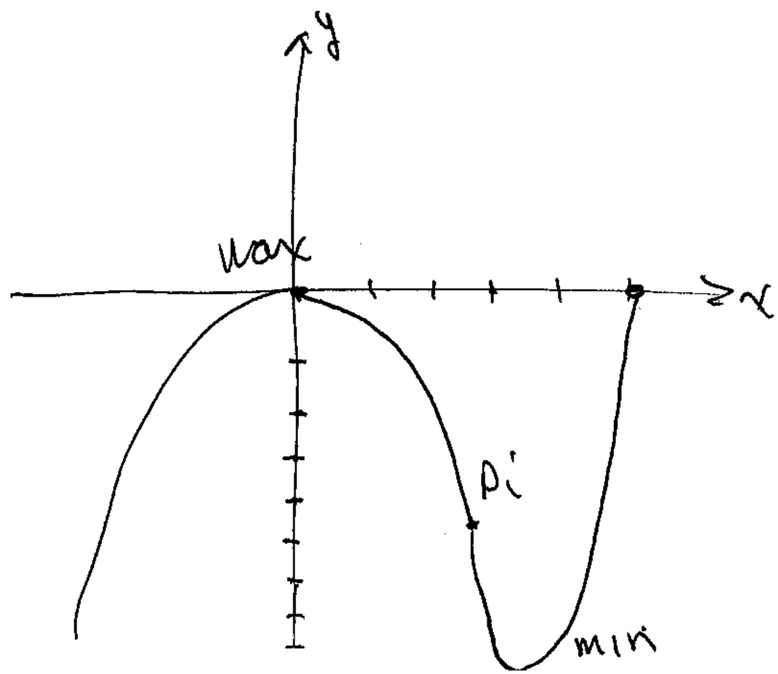
increasing $(-\infty, 0)$ $(4, \infty)$ max $(0,0)$

decreasin $(0,4)$ min $(4,-768)$

concave ↓ $(-\infty,0)$ $(0,3)$

concave ↑ $(3,\infty)$

Pi $(3,-486)$



2nd Derivative Test

Let $f(x)$ be such that $f'(c) = 0$ & f'' exists

If $f''(c) > 0$ then f has a relative min at $x = c$

If $f''(c) > 0$ then f has a relative max at $x = c$

If $f''(c) = 0$ test fails

Ex $f(x) = 2x^3 + 3x^2$

$f'(x) = 6x^2 + 6x = 6x(x+1)$ $\quad f' = 0 \quad x = 0, -1$ (crit #'s)

$f''(x) = 12x + 6$

$= 6(2x+1)$

$f''(0) = 6 > 0$ so min

$f''(-1) = -6 < 0$ so max

Ex $f(x) = x^4$ $\quad f' = 4x^3$ $\quad f' = 0$ when $x = 0$

$f'' = 12x^2$ $\quad f''(0) = 0$ so test fails