Fuzzy Set Theory Approach for Reliability Evaluation of Fault-Tolerant Cube Based Computer Interconnection Systems

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Abstract- This paper presents a new and simple method for evaluating the fuzzy reliability of fault-tolerant cube based computer interconnection systems. Through the proposed method, first all the maximal incomplete sub cubes present in a faulty cube are identified by taking maximum fault tolerance level equal to the system dimension. Then a new and simple algorithm has been proposed to evaluate the fuzzy reliability of the cube based topology. Here the system fuzzy reliability is expressed in terms of fuzzy probability of the disjoint terms of all path sets. The proposed method only uses two operations i.e. multiplication and complementation in evaluating system reliability. The proposed method is well illustrated through an example of a 3-dimensinal hypercube. Then Fuzzy reliability of some important fault-tolerant cube based computer interconnection networks have been evaluated.

Keywords- Fuzzy set, Hypercube, Reliability, Interconnection network

Notations

IC_n	*n-cube Interconnection n/w*
HC_n	*Hypercube Interconnection n/w*
s	*Source node*
t	*Destination node*
$\otimes$	*Discarding operation*
n	*System dimension*
u,v,w	*Adjacent nodes of source node*
$\bar{v}$, $\bar{w}$	*Antipodal nodes of v, w*
X	*a set containing a space of points in the probability domain*
x	*an element of X*
p_i	*fuzzy probability of an event i*
$\bar{p}_i$	*complement of fuzzy probability of an event i*
$\mu_{p_i}(p)$	*membership function of fuzzy probability p_i*
N	*number of nodes of the cube*
R	*fuzzy reliability of cube*
G	*reliability logic graph*
V	*vertex set*
E	*edge set*
S	*system success containing all paths between the source node(s) to destination node (t)*
P_i	*path at the i^{th} step*
$W, \overline{W}$	*indicator variables*

Assumptions:-

1. Node failures are statistically independent of each other.
2. Repair facility is not available.

I. INTRODUCTION

Uncertainty affects decision-making and appears in a number of different forms. The concept of information is fully connected with the concept of uncertainty. The most fundamental aspect of this connection is that the uncertainty involved in any problem solving situation is a result of some information deficiency, which may be incomplete, imprecise, fragmentary, not fully reliable, vague, contradictory, or deficient in some other way. So the effects of uncertainty in a system can be handled in a better way by using fuzzy Set Theory [10]. A major part of a parallel computer network is it's interconnection system, which is used to interconnect a large number of standalone processors. Therefore a wide variety of interconnection systems have been proposed, one of the widely used topology is the n-cubes [1]. Due to attractive properties like regularity, symmetry, small diameter, strong connectivity, recursive construction and partition ability the n-cube topology has enjoyed the largest popularity [2].

Whenever a fault arises, an n-cube may operate in a gracefully degradable manner due to the execution of parallel algorithms in smaller fault free sub cubes[3], which are comprises of healthy nodes. In order to maintain cube topology in the presence of faults, researchers have proposed addition of spare nodes thereby replacing the failed components with spares. This results in a much larger system than what is attained by any conventional reconfiguration scheme which identifies only complete sub cube [5]. Also fault tolerance can be achieved by reconfiguring the larger system to smaller sized system after the occurrence of fault [4]. Unlike a complete one, an incomplete cube can be of any arbitrary size, i.e. can be used to interconnect systems with any numbers of processors, making it possible to finish a given batch of jobs faster than it's complete counterpart alone by supporting simultaneous execution of multiple jobs of different sizes by assigning more nodes to execute the job cooperatively. Similar research work can be found in literature [6] and [7].Thus reconfiguring a faulty n-cube in to a maximal incomplete cube tends to lower potential performance degradation .

With the increase in size, the complexity of the interconnection network increases there by corresponding increase in computational power to maintain acceptable performance under reliable conditions [8] and [9]. For this the reliability prediction of a cube network is quite essential. However, there lies a large degree of uncertainty in system failure and therefore, the conventional methods [11] and [12] of reliability evaluation for large computer systems may not be appropriate to get a realistic value. Under such condition, one of the tools to cope with imprecision of available information in reliability analysis is *fuzzy set theory* [10]

Tanaka et al [13] and Misra and Weber [14] showed how fuzzification can be carried out for the quantitative analysis of fault tree. Chowdhury and Mishra [15] evaluated the reliability of a non-series parallel network. Patra et al [16] presents a method for evaluating fuzzy reliability of a communication network with fuzzy element capacities and probabilities. Tripathy et al [17] have proposed a method to evaluate fuzzy reliability of MINs. But none of methods discussed above considers the cube based interconnection system and suggests a general method of evaluating fuzzy reliability of it. So, there is always a need to search for a general and efficient method to evaluate the fuzzy reliability of such systems.

In this paper, a general and efficient method has been proposed to find an expression of fuzzy system reliability taking in to consideration of the special requirements of fuzzy sets. This method is supported by an efficient algorithm and well illustrated through a 3-dimensinal hypercube.

II. METHODOLOGY FOR FINDING SUB CUBES

Definition 1:- A discarded region in an interconnection network is the smallest sub cube comprises of a faulty node and the antipodal nodes of the (n-1) fault free adjacent nodes.

For a faulty n-cube IC_n and a given source node s. It is possible to identify systematically every fault free sub cube which involves the source node s. This is expressed by set P = {Pi/Pi is a fault free sub cube in IC_n and Pi involve node s}.This can be done by determining the region which never contribute to any fault free sub cube containing the node s. Each fault results in one such regions known as discarded region which is the smallest sub cube involving both the faulty and the antipodal nodes of adjacent (n-1) nodes. A discarded region is addressed by performing ⊗ operation on the labels of the faulty node and the antipodal node where ⊗ is the bit operation defined as: it yields 0 (or 1) if the two corresponding bits are "0" (or "1"),and it is ∗ if the two corresponding bits differ.

III. PROPOSED METHOD FOR FUZZY RELIABILITY EVALUATION

Proposed Approach

The current section proposed a method to evaluate the fuzzy reliability of cube based computer interconnection systems.

Fuzzy Probability

Fuzzy probability represents a fuzzy number between zero and one, assigned to the probability of an event. One can chose different types of membership functions for fuzzy probability. For instance, a fuzzy probability may have a trapezoidal membership function. The fuzzy probabilities of an event i can then be denoted by a four parameter function i.e.

$$p_i = (\alpha_{i1}, \alpha_{i2}, \beta_{i2}, \beta_{i1}) \quad (1)$$

The membership function is given by

$$\mu_{p_i}(p) = \begin{cases} 0, & 0 \le p \le \alpha_{i1} \\ 1 - \dfrac{\alpha_{i2} - p}{\alpha_{i2} - \alpha_{i1}} & \alpha_{i1} \le p \le \alpha_{i2} \\ 1 & \alpha_{i2} \le p \le \beta_{i2} \\ 1 - \dfrac{p - \beta_{i2}}{\beta_{i1} - \beta_{i2}} & \beta_{i2} \le p \le \beta_{i1} \\ 0 & \beta_{i1} \le p \le 1 \end{cases} \quad (2)$$

Operation used in computing fuzzy reliability

Let p_i and p_j be two fuzzy sets that have membership functions given by $\mu(p_i)$ & $\mu(p_j)$, respectively. The operations used in fuzzy reliability evaluation, i.e. multiplication and complementation can be defined as follows:

1. *Multiplication-*

$p_i.p_j$ = *product of* p_i *and* p_j

$$= \mu_{p_i p_j}(p) = \mu_{p_i}(p).\mu_{p_j}(p) \quad (3)$$

How ever, Tanka et al [13] provided an approximation of the multiplication procedure by defining

$$p_{ij} = p_i.p_j = (\alpha_{i1}.\alpha_{j2}, \alpha_{i2}.\alpha_{j2}, \beta_{i2}.\beta_{j2}, \beta_{i1}\beta_{j1}) \quad (4)$$

2. *Complementation*

The complementation of any fuzzy set p_i will be given by

$$\bar{p}_i = 1 - \mu_{p_i} \quad (5)$$

for example, in case of trapezoidal membership function, one could obtain

$$\bar{p}_i = (1 - \alpha_{i1}, 1 - \alpha_{i2}, 1 - \beta_{i2}, 1 - \beta_{i1}) \quad (6)$$

Mathematical Basis

Let S_k denote the kth minimal path set and p_k be the fuzzy probability associated with S_k. Also let R_k be the fuzzy reliability at *kth* step of the sum of fuzzy path probabilities. The reliability expression can be found out by a recursive formula:

$$R_k = R_{k-1} + \Pr\left\{ S_k \cap \overline{\left(\bigcup_i^{k-1} S_i \right)} \right\}$$
$$= R_{k-1} + \Pr\{S_k \cap \bar{S}_1 \cap \bar{S}_2 \cap \cdots \cap \bar{S}_{k-1}\} \quad (7)$$

Proposed Algorithm

1. Convert distributed network in to a probabilistic graph (G(N, E))
2. Find the maximal incomplete cube of G using the proposed method
3. Generate the fuzzy probabilities of links $e \in E'$ using Eq. 2
4. Enumerate all the minimal path sets from the source to destination node.
5. Rearrange the total number of path sets (h) according to increasing order of cardinality.
6. Set k=1 and R_0=0
7. Repeat 6-7 for k=1 to h
8. The reliability expression can be given as
$$R_k = (R_{k-1} + S_k)_{dis}$$
9. $p_k = R_k - R_{k-1}$
10. *Compute the fuzzy reliability of the system as*
$$R = 1 - \prod_{k=1}^{h} \bar{p}_k$$

Efficiency:
Finding the minimal paths using the above proposed algorithm requires $O(N^3)$ operations. So, the running time of proposed algorithm is $O(N^3)$ which is polynomial.

IV. ILLUSTRATION

The proposed method is illustrated through the following example.

Example:
Using the proposed method, the hypercube interconnection system of Fig.1 (a) is viewed as a connected undirected graph *G (N, E)* where *N* is the set of nodes (processing elements) and *E* is the set of edges (links).

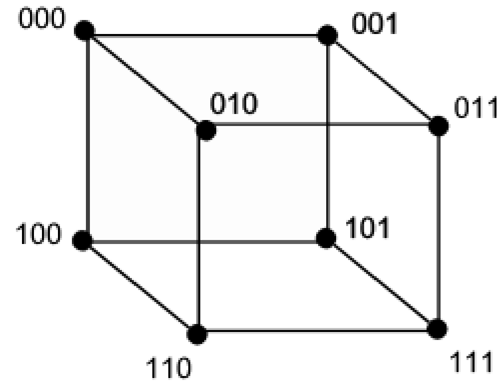

Fig.1(a): Hypercube (n=3).

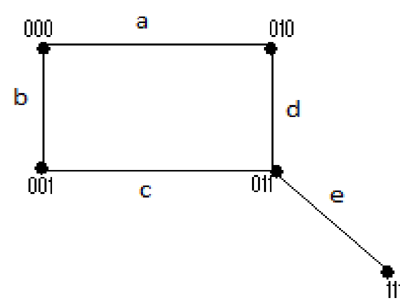

Fig.1(b): Maximal incomplete subcube

Assuming the trapezoidal membership function as given in Equation 2, the fuzzy probabilities of links *a*, *b*, *c*, *d*, *e* are given as follows
p_a=(0.2,0.8, 0.95,0.99), p_b=(0.1, 0.7, 0.85,0.96), p_c=(0.2, 0.37, 0.6,0.79), p_d=(0.1,0.6,0.85,0.96), p_e=(0.17, 0.2, 0.3,0.6),

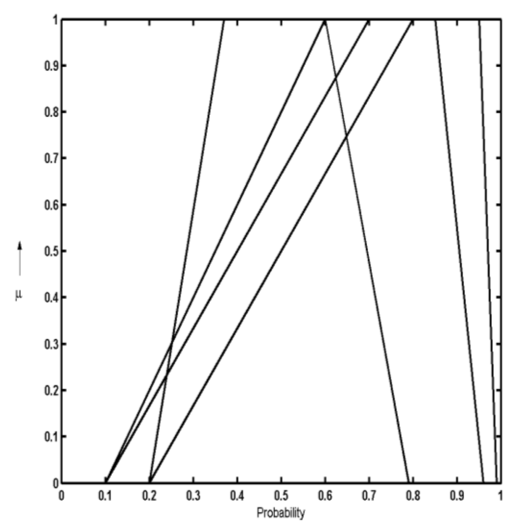

Fig.2(a): Fuzzy probabilities of links *a, b, c, d*

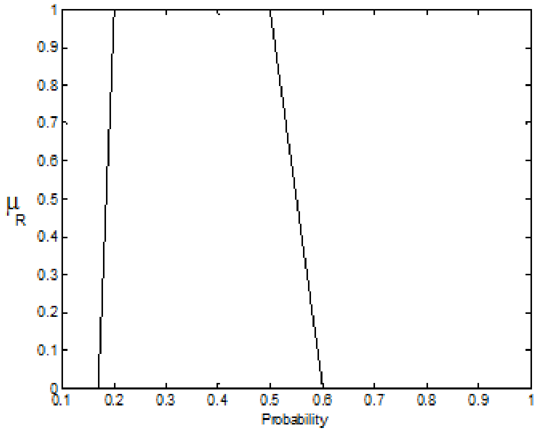

Fig.2(b): Fuzzy probability of link e

The path set is given by

P= {ade, bce}

Using the proposed algorithm, the membership function of the fuzzy reliability of the Hypercube network is given by

$$\mu_R = \{0.4317, 0.5, 0.8, 0.9461\}$$

V. RESULTS AND DISCUSSIONS

The fuzzy reliability of five important cube based computer interconnection systems viz. hypercube (HC), Crossed cube (CQ), Fault-tolerant hypercube (FTH), Varietal hypercube (VH) and the Folded hypercube (FH) have been evaluated by the proposed method. The membership functions of the said parallel computer interconnection networks are plotted against the probability (Figs.). The parameter functions of the said parallel computers are presented in Table 1. The inference that can be drawn from Table 1 is that the fuzzy reliability of hypercube lies between the limits 0.5-0.8 with a 100% possibility. Similarly the fuzzy reliability of FTH, CQ, VH and FH lie between the limits 0.62-0.81, 0.6-0.8, 0.35-0.8, 0.6-0.81 respectively with a 100% possibility. From this, it has been observed that FTH has better Fuzzy reliability in compared to other cube based systems.

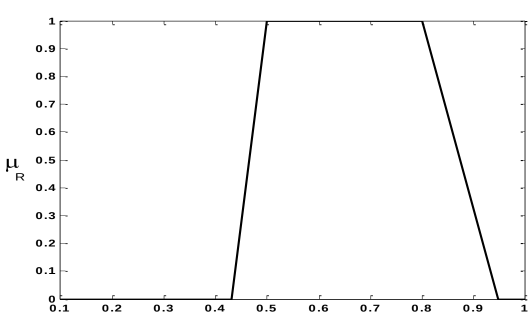

Fig.3: Fuzzy reliability of Hypercube

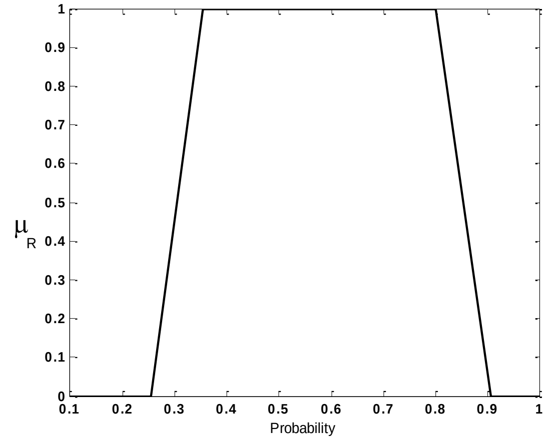

Fig.4(a): Fuzzy reliability of Varietal Hypercube (*n=3*)

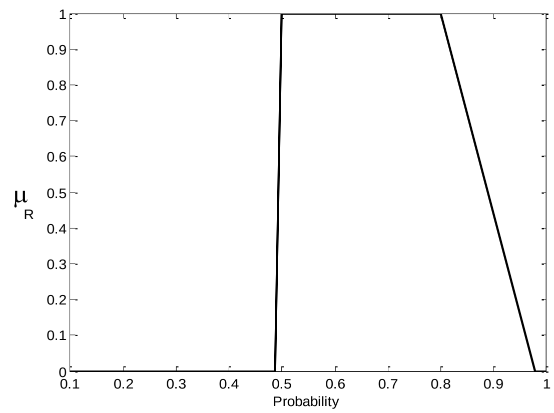

Fig.4(b): Fuzzy reliability of Crossed cube (*n=3*)

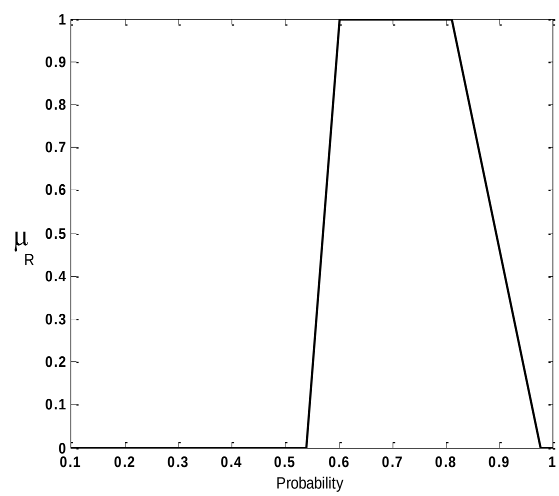

Fig. 4(c): Fuzzy reliability of Folded hypercube (*n=3*)

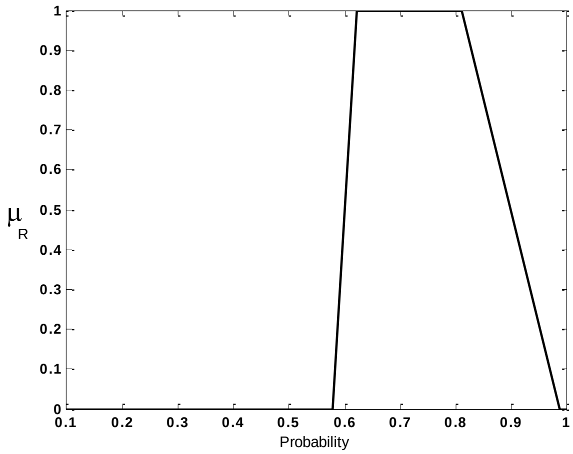

Fig.4(d): Fuzzy reliability of Fault-tolerant Hypercube (*n=3)*

Table 5.1 Parameter Functions of Distributed Interconnection Networks

Networks	α_1	α_2	β_2	β_1
HC	0.4317	0.5000	0.8000	0.9461
VH	0.2500	0.3500	0.8000	0.9018
CQ	0.4873	0.6000	0.8000	0.9761
FH	0.5390	0.6000	0.8106	0.9791
FTH	0.5790	0.6208	0.8106	0.9861

VI. CONCLUSION

This paper introduced fuzzy reliability measure for cube based computer interconnection systems. The importance of fuzzy reliability and its evaluation methods have been presented. Here we proposed two new approaches for fuzzy reliability evaluation. The first one is meant for finding fault-free cube from a faulty one and the second one is for evaluating the fuzzy reliability. Basically, the proposed methods use the path enumeration technique in evaluating fuzzy reliability. The algorithm enumerates all the path sets from the source node to destination node. Then the system fuzzy reliability is expressed in terms of fuzzy probability of the disjoint terms of all path sets. Using the proposed techniques the fuzzy reliability of five number of cube based computer interconnection systems are evaluated and compared. The methods proposed here can also be extended further to evaluate all categories of parallel computer systems.

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