Math 1496 - Calc I

Limits

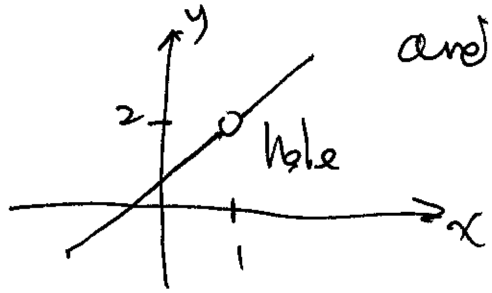
consider the function

$$f(x) = \frac{x^2-1}{x-1},$$

If we sub in $x=1$ we have a problem.
we have "$\frac{0}{0}$" which means nothing so
we wish to explore this function some more
we will look at it

(1) graphically
(2) numerically
(3) analytically

If we graph the function we get
and getting close to $x=1$



$$\lim_{x\to 1} \frac{x^2-1}{x-1} = 2$$

Numerically - then we create a table

x	.5	.9	.99	.999	1	1.001	1.01	1.1
$f(x)$	1.5	1.9	1.99	1.999	?	2.001	2.01	2.1

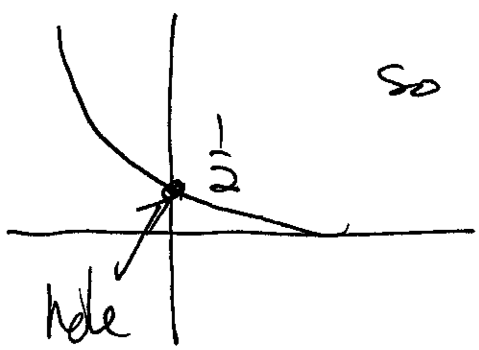
and from the table we see

$$\lim_{x \to 1} \frac{x^2-1}{x-1} = 2 \quad \text{the same}$$

<u>Ex 2</u> $\displaystyle\lim_{x \to 0} \frac{\sqrt{x+1}-1}{x} = \text{"}\frac{0}{0}\text{"}$ "direct substitution"

so we look at $f(x) = \dfrac{\sqrt{x+1}-1}{x}$

graphically & numerical



so the graph says

$$\lim_{x \to 0} \frac{\sqrt{x+1}-1}{x} = \frac{1}{2}$$

Next, we create a table so

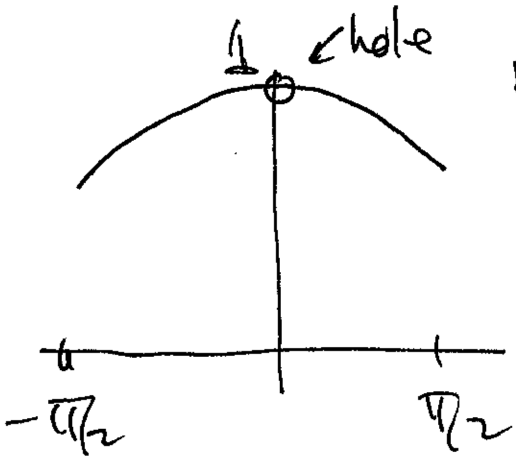
x	$-.1$	$-.01$	$-.001$	0
$f(x)$	$.5132$	$.5013$	$.5001$	?

x	0	$.001$	$.01$	$.1$
$f(x)$	?	$.4999$	$.4988$	$.4881$

and the table says

$$\lim_{x \to 0} \frac{\sqrt{x+1} - 1}{x} = \frac{1}{2}$$

Ex $\lim_{x \to 0} \frac{\sin x}{x} = "\frac{0}{0}"$



From the graph we see $\lim_{x \to 0} \frac{\sin x}{x} = 1$

Table

x	$-.1$	$-.01$	$-.001$	0	$.001$	$.01$	$.1$
$f(x)$	$.9983$	$.9999$	1	$?$	1	$.9999$	$.9983$

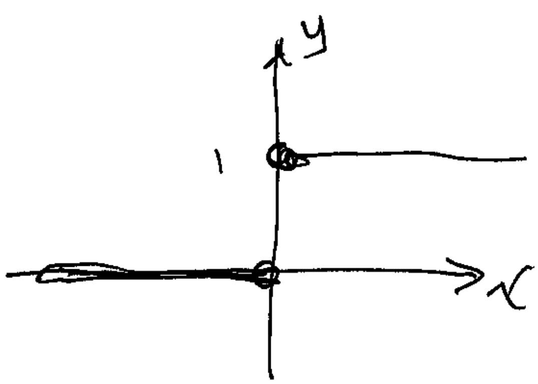
and the table says

$$\lim_{x \to 0} \frac{\sin x}{x}$$

It's import to realize that limits don't always exist. Consider for example

$$f(x) = \begin{cases} 0 & x < 0 \\ 1 & x \geq 0 \end{cases}$$

Here's the graph



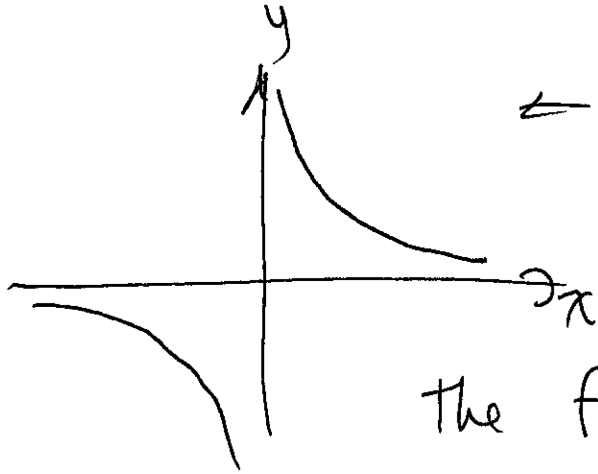
$$\lim_{x \to 0^-} f(x) = 0$$

$$\lim_{x \to 0^+} f(x) = 1$$

Not the same

These are what we called "one" sided limits

Consider $f(x) = \dfrac{1}{x}$



← here is the graph

we see as $x \to 0^+, 0^-$

the function becomes unbounded

so $\displaystyle\lim_{x \to 0^-} \frac{1}{x} = -\infty$ DNE – (does not exist)

$\displaystyle\lim_{x \to 0^+} \frac{1}{x} = +\infty$