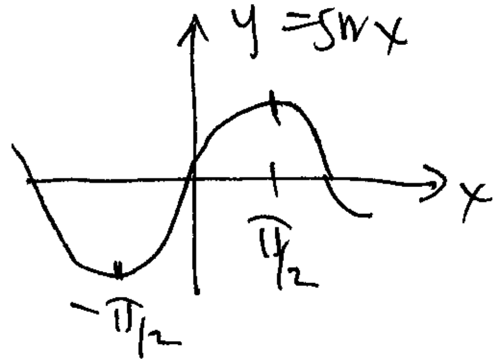
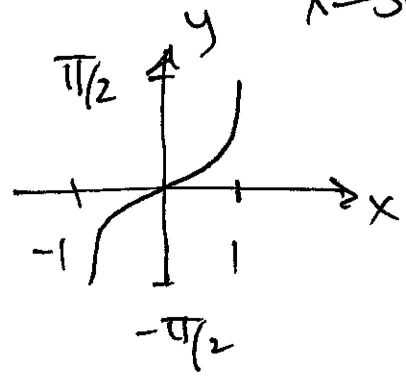
Math 1496 - Calc I

Derivative of inverse Trig functions

$y = \sin^{-1} x$

$x = \sin y \Leftrightarrow y = \sin^{-1} x$





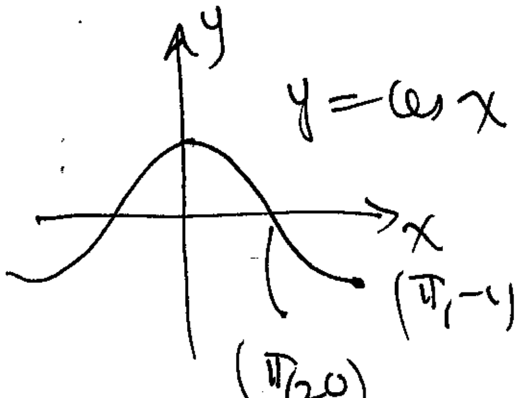
$x = \sin y \qquad 1 = \cos y \, y' \Rightarrow y' = \dfrac{1}{\cos y} = \dfrac{1}{\pm\sqrt{1-\sin^2 y}}$

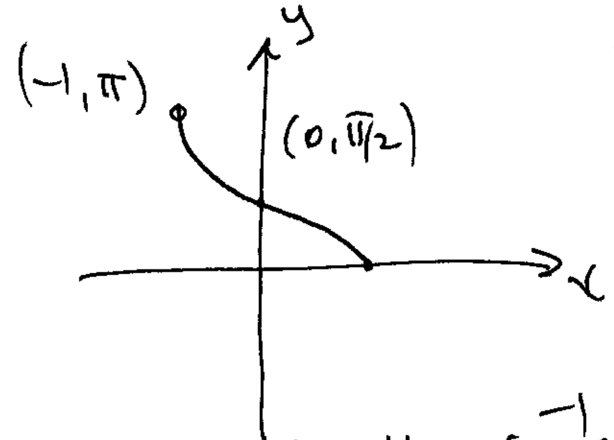
$$y' = \frac{\pm 1}{\sqrt{1-x^2}}$$

which case - since the slopes of tangents are > 0
choose +ve case

$$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$$

$y = \cos^{-1} x$





$$x = \cos y \iff y = \cos^{-1} x$$

$$\frac{d}{dx} x = \frac{d}{dx} \cos y \Rightarrow \quad 1 = -\sin y \, y' \Rightarrow y' = \frac{-1}{\sin y}$$

$$\sin^2 y + \cos^2 y = 1 \Rightarrow \quad \frac{dy}{dx} = \frac{-1}{\pm\sqrt{1-\cos^2 y}} = \frac{\mp 1}{\sqrt{1-x^2}}$$

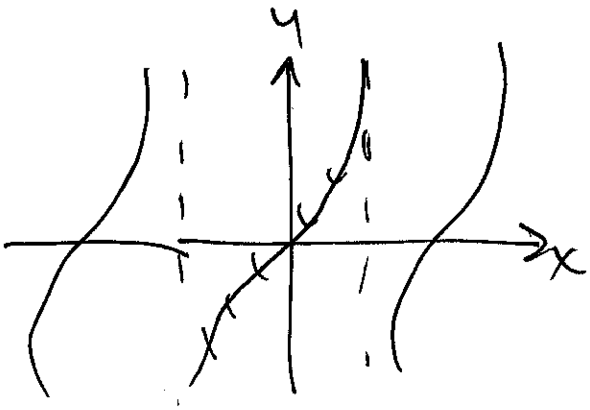
since the slopes (of tangents are −ve) choose −ve

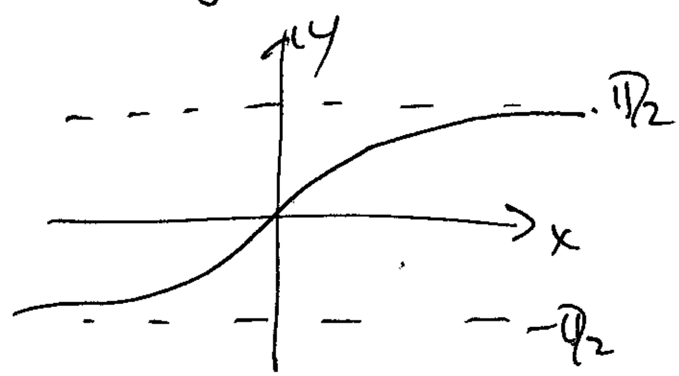
$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}}$$

or $\dfrac{d}{dx} \cos^{-1} x = \dfrac{-1}{\sqrt{1-x^2}}$

$y = \tan^{-1} x$

$x = \tan y$ or $y = \tan^{-1} x$





$x = \tan y$ so $1 = \sec^2 y \, y'$ $\quad y' = \dfrac{1}{\sec^2 y}$

Now $1 + \tan^2 y = \sec^2 y$ so

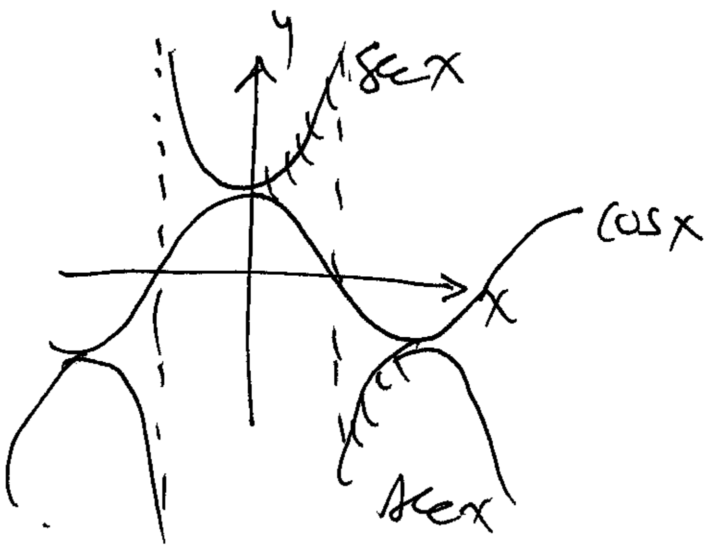
$$y' = \frac{1}{1+\tan^2 y} = \frac{1}{1+x^2}$$

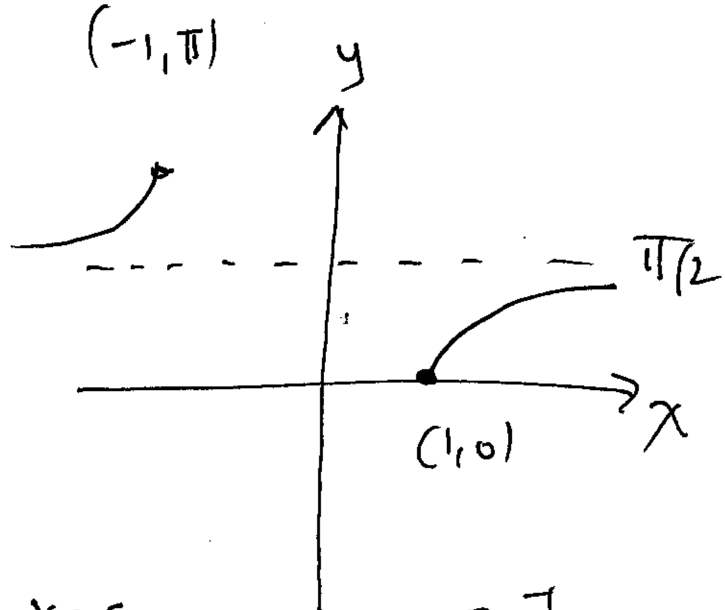
so $\dfrac{d}{dx} \tan^{-1} x = \dfrac{1}{1+x^2}$

Similarly for $y = \cot^{-1} x$

$$\frac{d}{dx} \cot^{-1} x = \frac{-1}{1+x^2}$$

$y = \sec^{-1} x$





$x = \sec y$ or $y = \sec^{-1} x$

pt $(0, 1) \to (1, 0)$

pt $(\pi, -1) \to (-1, \pi)$

$\frac{d}{dx} x = \frac{d}{dx} \sec y \Rightarrow 1 = \sec y \tan y \, y' \Rightarrow y' = \frac{1}{\sec y \tan y}$

$1 + \tan^2 y = \sec^2 y$

$y' = \frac{\pm 1}{x\sqrt{\sec^2 y - 1}} = \frac{\pm 1}{x\sqrt{x^2 - 1}}$ From graph slopes > 0 for all x

$x > 1$ $x < -1$

when $x > 0$ $y' = \frac{1}{x\sqrt{x^2-1}}$

$x < 0$ $y' = \frac{-1}{x\sqrt{x^2-1}}$

$\frac{d}{dx} \sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$

so we have the following

$$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\cos^{-1}x = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2} \qquad \frac{d}{dx}\cot^{-1}x = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}\sec^{-1}x = \frac{1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}\csc^{-1}x = \frac{-1}{|x|\sqrt{x^2-1}}$$