A Directed Workshop on *Insight*, Chapter 1: Elements

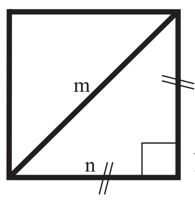
Ninth Session—November 14th, 2018—Lonergan Center, TS Department, Concordia

Examples of Inverse Insights

Although this distinct kind of insight is relatively rare, they are very important--often pivotal--in enhancing our understanding. To highten our awareness of this, we continue exploring Lonergan's examples: surds, uncountability, and invariance. And keep in mind the three features of an inverse insight: there is a positive object, a spontaneous anticipation of intelligibility, and a negation of that intelligibility. The result is a negation of the question anticipating intelligibility. (N.B. I've had to rewrite sections as my initial exploration had flaws. So best to work this stuff out on your own.)

Surds

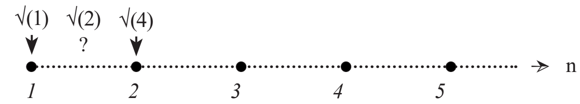
In the world of numbers, one would expect that all numbers are associated with a specific quantity, e.g., "32" represents 32 instances of unity, etc. And then early Greek mathematicians ran into an unexpected consequence of Pythagoras' theorem: (any right angle triangle, the square of the hypotenuse is equal to the sum of the squares of the two sides)—but what happens if the two sides are equal, and equal to one? The hypotenuse must be equal to $\sqrt{(2)}$. But what exactly is the $\sqrt{(2)}$? And so the fun begins.



A right angle triangle with two equal sides.

For this reason, $\sqrt{(2)}$ cannot be a "rational" number reducible to the form m/n. In other words, it can never be given an exact place on the number line. The $\sqrt{(2)}$ is a surd, in the sense that it can never be an actually number number. Yet it is "real" in the sense that it is the result of a logical extension of operations in the same way that -1 or $\sqrt{(-1)}$ are real (though irrational).

➛ *On the number line, $\sqrt{(2)}$ must lie between 1 and 2, since the $\sqrt{(1)} = 1$ and the next highest pure root, $\sqrt{(4)} = 2$.*



➛ *Now all rational numbers, i.e., intelligible numbers, can be reduced to the form m/n, where there exists no common denominator between the two (m is prime when compared to n).*

➛ *Take a right angle triangle whose sides are equal to 1, assign the number n to each side and m to the hypotenuse, and consider the number m/n.*

➛ *By Pythagoras' theorem, $m^2 = n^2 + n^2$.*

$m^2 = 2n^2$

$m^2/n^2 = 2$

If m is prime to n, as a rational number, then m^2 must be prime to n^2.

But m^2/n^2 is equal to 2, which means there must be a common denominator.

Hence the paradox: $m^2/n^2 = 2$ and $m^2/n^2 \neq 2$ are both true.

Non-countable Numbers

In the world of numbers, we would expect it to be possible to at least theoretically count all possible numbers. Are we wrong?

➛ *Let's consider the space between 1 and 2 along the number line.*

➛ *Our first observation is that while there is an infinity of rational numbers of the form m/n between these two numbers the set is in fact countable. A simple algorithm proves the point. Let m & n = 1 (m/n or 1/1 gives us our starting point). Holding n = 1, then increment m by 1. The result is 2/1, which gives us the other end point in our number line. Bring m back to 1, and increment n by 1. This gives us 1/2. Increment n by another 1 gives us 1/3, etc. Although this gives an infinite sequence, we can consider counting another with n set to 2 and m starting at 1 and incrementing by 1. This gives us 1/2 and 2/2 before we exceed our boundaries. Then increment n by another 1, which gives is 1/3, 2/3 and 3/3. Etc. Etc. This means that while can never arrive at the exact number of rational numbers between one and two, they are at least countable.*

➛ *Now let's consider what happens once irrational numbers are similarly placed on the number line. We already know that $\sqrt{(2)}$ must lie on that line, but we will never know its exact position only that it must lie between an ever smaller gap. So to does $\sqrt{(3)}$. Even so, we could probably counter such irrational numbers. But what is a problem numbers with repeating digits.*

→*Repeating digits are numbers whose digits continue on indefinitely, e.g., 0.99999999 ... A example would be 20/6, which gives 3.3333333 ... The problem is that it would be impossible to count the entire set of such numbers even on such a small segment of the number line that lies between 1 & 2. The problem is any kind of repeating digits provides no fixed point on a number line. So what is the next number adjacent to 0.99999 ...? It is impossible to provide an answer to this question, since the next number cannot be determined. Therefore, this set of all numbers with repeated digits cannot be calculated.*

➛*The same applies to any definition of a sequence, e.g., the Fibonacci sequences creates an integer sequence whose elements are the sum of the previous two elements. So:*

1+1=2; 1+2=3; 2+3=5; 3+5=8; 5+8=13; 8+13=21; etc. Now make this into a single number such as 1.1123581321 ...

The problem is that it is possible to created an infinite set of such numbers, but it is a set that cannot possibly be counted since no counting algorithm could ever be created to insure an accurate count.

Invariance

In Einstein's relativity, what is perceived to happen depends on the location of the observer. A simple example of this is the position of the observer with respect to a moving train whose engineer is blowing the whistle. If the observer is standing in front of the train, the pitch of the whistle will be higher than that experienced by the engineer; an observer standing after the trained has passed will observe a drop in pitch. This is the well-known Doppler effect that has a particular application in determining whither or not a star is approaching or receding.

The point is that although all three observations of the whistle's pitch are different they can be shown to be the same whistle. This is invariant over any particular person's observation point.

In Einstein's theory, the speed of light is a constant that remains the same no matter the speed at which the observer is moving. So, if the observer is moving close to the speed of light, he or she still remains in a familiar world even if another observer is not moving.

The problem is that who is moving and who is staying still? Take a person falling in a falling elevator: is he or she falling or floating? When compared to the earth, both elevator and person are falling. But the person, when compared to the elevator, is floating in space. The point is that we can make such transformations quite easily; the event is invariant over changes in the person's observational position.

There's an interesting correspondence with the notion of horizons. First level horizons are simply the results of one person's observations. So a student has one perspective on a university, a professor another, and a member of the populace yet another. Yet, second level horizons pull them all together into a structure that remains invariant across all first level horizons. An example of a second level horizon would be a theoretical model of a university constructed around interlocking and recurring schemes of operation.

Interestingly, this is the core of the argument Lonergan uses in countering Kant's idea that time and space are a priori. We construct time and space out of the process of finding invariant structures from a multitude of personal observations of time and space.