Reverse Derivations On Prime Rings

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Abstract- In this paper some results concerning to reverse derivations on prime rings are presented. If R be a prime ring with a non-zero reverse derivation d and S be the ideal of R then R is commutative.

Keywords- *Center, Prime ring, Derivation, Reverse derivation.*

I. INTRODUCTION

Bresar and Vukman [1] have introduced the notion of a reverse derivation. The reverse derivations on semi prime rings have been studied by Samman and Alyamani [2].

II. PRELIMINARIES

Throughout, R will represent an associative ring with center $Z(R)$ defined as $Z=\{z\in R/[z,R]=0\}$. We write $[x,y]$ for $xy-yx$. Recall that a ring R is called prime if $aRb=0$ implies $a=0$ or $b=0$. An additive mapping d from R into itself is called a derivation if $d(xy)=d(x)y+x\,d(y)$ for all $x,y\in R$ and is called a reverse derivation if $d(xy)=d(y)x+yd(x)$ for all $x,y\in R$.

III. MAIN RESULT

THEOREM 1: Let R be a prime ring with a nonzero reverse derivation d and let S be an ideal of R Suppose that $d(xr)\in Z,\forall x\in S,r\in R$ where Z denotes the center of R. Then R is commutative.

PROOF: Given $[d(xr),y]=0$, $\forall x\in S\ and\ y,r\in R$

i.e., $[d(r)x+rd(x),y]=0$

$\Rightarrow[d(r)x+rd(x)]y-y[d(r)x+rd(x)]=0$

$\Rightarrow d(r)xy+rd(x)y-yd(r)x-yrd(x)=0$

Adding and subtracting $ryd(x),d(r)yx$,

then we get,

$\Rightarrow d(r)xy+rd(x)y-ryd(x)+ryd(x)-yd(r)x+$
$d(r)yx-d(r)yx-yrd(x)=0$

$\Rightarrow d(r)xy+r[d(x)y-yd(x)]+[ry-yr]d(x)+$
$[d(r)y-yd(r)]x-d(r)yx=0$

$\Rightarrow[d(r)xy-d(r)yx]+r[d(x)y-yd(x)]+$
$[ry-yr]d(x)+[d(r)y-yd(r)]x=0$

$\Rightarrow d(r)[xy-yx]+r[d(x),y]+[r,y]d(x)+[d(r),y]x=0$

$\Rightarrow d(r)[x,y]+r[d(x),y]+[r,y]d(x)+[d(r),y]x=0$

$\Rightarrow d(r)[x,y]+r[d(x),y]+[r,y]d(x)=0$

Put y = x in the above equation, then we get,

$\Rightarrow d(r)[x,x]+r[d(x),x]+[r,x]d(x)=0$

$\Rightarrow r[d(x),x]+[r,x]d(x)=0$

By expanding this equation, we conclude that,

$\Rightarrow r(d(x)x-xd(x))+(rx-xr)d(x)=0$

$\Rightarrow rd(x)x-rxd(x)+rxd(x)-xrd(x)=0$

$\Rightarrow rd(x)x-xrd(x)=0$

$\Rightarrow rd(x)x=xrd(x)$ …………………… (1)

We write zr instead of r in (1) and using this equality, we get,

$\Rightarrow zrd(x)x=xzrd(x)$

$\Rightarrow zxrd(x)=xzrd(x)$

$\Rightarrow zxrd(x)-xzrd(x)=0$

$\Rightarrow[zx-xz]rd(x)=0$

$\Rightarrow[z,x]rd(x)=0,\forall x\in S\ and\ z,r\in R$ ………. (2)

Since R is prime, we have, $[z,x]=0$ or $d(x)=0.$

Since d is a non zero reverse derivation,

we have, $d(x)\neq 0.$

$\Rightarrow[z,x]=0,\ \forall z\in R,x\in S$.

$\Rightarrow[R,S]=0$.

Hence R is commutative. This completes the proof of the theorem.

IV. REFERENCES

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